EE5040: Adaptive Signal Processing
Problem Set 2: Linear least-mean-squares estimation

1. (Sayed Example 4.3) IID symbols $\{S_i\}$ are transmitted over the FIR channel $C(z) = 1 + 0.5z^{-1}$. Each symbol is either $+1$ or -1 with equal probability. The output of the channel is corrupted by zero-mean additive white Gaussian noise V_i of unit variance. The noise and symbols are independent of each other. We want to estimate $\mathbf{X} = [S_0 \;\; S_1]^T$ from the observation vector $\mathbf{Y} = [Y_0 \;\; Y_1]^T$, where

$$Y_0 = S_0 + V_0 \quad \text{and} \quad Y_1 = S_1 + 0.5S_0 + V_1$$

   assuming that transmission starts at time 0 and $S_{-1} = 0$. Find the optimal linear least-mean-squares estimator for $\mathbf{X}$.

2. (Sayed Example 4.4: Linear channel equalization) Consider a setting similar to the previous problem. Assume that symbol transmissions are happening for all $i > -\infty$ (rather than start at time 0). Therefore, we have

$$Y_i = S_i + 0.5S_{i-1} + V_i$$

   for all i. Design a linear equalizer with 3-taps such that the output of the equalizer at time i given by

$$\alpha_0 Y_i + \alpha_1 Y_{i-1} + \alpha_2 Y_{i-2}$$

   is the optimal linear MMSE estimator for $S_{i-\Delta}$. Obtain the result for $\Delta = 0$ and $\Delta = 1$. Formulate the linear model and the equations to obtain the optimal linear MMSE estimate of $S_{i-\Delta}$ from the L observations Y_i, Y_{i-1}, $\cdots$, Y_{i-L+1}.