Matched Filter and Timing Offset Estimation

1 Introduction

The study of transmission of digital data over a bandlimited baseband channel is important for a good understanding of data communication. Digital data have broad spectrum with significant low-frequency content. This requires that, for near-distortionless transmission, the low-pass channel used for transmission have a bandwidth large enough to accommodate the essential spectrum of the data. Data transmission over a dispersive channel (low pass channel) results in inter-symbol interference (ISI) and is a major source of bit errors at the receiver. ISI is minimized by optimal signal design. Bit errors are also introduced by receiver and channel noise. Thus the received waveform contains both ISI and noise. The detection of a pulse signal of known shape which is immersed in additive white noise is an important and well-studied problem in communication. The optimum detection of such a pulse is carried out by a *matched filter*, so called because its impulse response is matched to the impulse response of the transmit filter. The matched filter is a linear, time-invariant system.

When the receiver is switched on (and after that, during continuous operation), it needs to know where to sample the output of the matched filter in order to make a decision. Estimation of the sampling *epoch* by the receiver using the received waveform itself – and not using any pilot timing signal – is the *problem of timing recovery.*

In this experiment our objective is to study the working of the matched filter, bit error rate (BER), and the timing offset estimation during transmission. Further, in this experiment we will also get familiar with practical pulse shapes and the effect of pulse shaping in data transmission. The timing recovery part of the experiment provides a basic introduction to the topic of *synchronization.* This part deals with an implementation of a sub-optimal timing recovery algorithm and its performance in the presence of noise.

2 Background Theory

In the presence of noise, the optimal receive filter is one that is matched to the transmit filter. If $p(t)$ and $g(t)$ are the impulse responses of the transmit and receive filters respectively, matched filter theory requires that $g(t) = p(-t)$. A matched filter averages noise (and hence reduces it), correlates the signal with its noisy replica, adds all frequency components in phase and maximizes the SNR.

To ensure zero ISI at the receiver's sampling instant, the combined frequency response of the transmit and receive filters should satisfy the Nyquist condition for zero ISI. One commonly used shape that satisfies this condition is the *Raised Cosine* filter. Let $h(t)$ be the impulse response of the Raised Cosine filter and $H(f)$ its frequency response. Then $H(f)$ and $h(t)$ are given by,

$$H(f) = \begin{cases} T & |f| \leq \frac{1-\alpha}{2T} \\ \frac{T}{2}\left[1+\cos\left\{\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right\}\right] & \frac{1-\alpha}{2T} < |f| \leq \frac{1+\alpha}{2T} \\ 0 & |f| > \frac{1+\alpha}{2T} \end{cases}$$

and

$$h(t) = \text{sinc}\left(\frac{t}{T}\right) \cdot \frac{\cos(\pi\alpha t/T)}{1-(2\alpha t/T)^2}.$$

$h(t)$ retains the zero crossing property of the sinc function. α is the excess bandwidth factor ($0 \leq \alpha \leq 1$). T is the symbol period. The zero crossings of $h(t)$ are the points where the received waveform has zero ISI and are the optimal sampling instants for data detection.

The output of the receive filter is sampled at symbol rate in order to make decisions about transmit symbols. Therefore, the transmit and receive filter combination at this point should have the zero-crossing property. Since $H(f)$ mentioned above has this desirable property, we want the combined frequency response to be $H(f)$. One way to achieve this is to choose $\sqrt{H(f)}$ as the magnitude response for both filters. It can be shown that the corresponding impulse response is given by

$$g(t) = \frac{4\alpha}{\pi\sqrt{T}}\left[\frac{\cos((1+\alpha)\frac{\pi t}{T}) + \frac{T}{4\alpha t}\sin((1-\alpha)\frac{\pi t}{T})}{1-(4\alpha t/T)^2}\right].$$

When a root-raised-cosine filter is used as a matched filter, sampling its output at the correct sampling epoch achieves twin objectives – it maximizes SNR and eliminates ISI. Since raised-cosine is a band-limited pulse, using Nyquist-rate samples of it, it is possible to interpolate and obtain the value of the raised-cosine pulse that is close to the ideal sampling epoch. The accuracy with which this value can be estimated is a function of the upsampling factor and SNR. An algorithm to estimate the sampling time is discussed below in the next section.

3 DSP-based Transmitter and Receiver

Many of the operations of the transmitter and receiver are performed in discrete-time using a Digital Signal Processor (DSP). Digital-to-analog converters (DACs) used at the transmitter make the signal suitable for communication over analog channels and analog-to-digital converters (ADCs) are used at the receiver to get samples of the received waveform for discrete-time processing. Figure 1 shows the discrete-time equivalent system without DACs and ADCs. $p(n)$ and $g(n)$ are the samples of the impulse responses of the transmit and receive filters which in our case are samples of the root raised cosine filter and a filter matched to it. $h(n)$ is the samples of the impulse response of the raised cosine filter. The impulse responses are sampled with a period T_s whose relationship to T varies depending on the location of a filter in the transmitter-receiver chain. The relationship between the two will be made explicit in the figure.

The discrete-time algorithm for timing recovery assumes that the raised-cosine pulse has been sampled with an *unknown timing offset* Δ, i.e., $h(t)$ is sampled with $T_s = T/N + \Delta$ to get N

samples per symbol period, instead sampling at $T_s = T/N$. While the latter samples $h(t)$ at $t = 0$ (the zero ISI point) the former misses it (except for the trivial case when Δ is an integer multiple of T/N).

In order to estimate Δ, the output of the matched filter will be suitably upsampled and interpolated to obtain M samples per symbol period. Maintain M sums such as

$$\text{sum}(r) = \sum_n |q(Mn + r)| \quad \text{for} \;\; r = 0, \ldots, M-1$$

where $q(n)$ is the received, matched-filtered waveform, n is the sumber of symbols used to estimate timing epoch. Perform the max operation over $\text{sum}(r)$.

$$\max_{r \in (0, M-1)} \text{sum}(r)$$

The index of the maximum of $\text{sum}(r)$ over $r \in (0, M-1)$ indicates the location of the timing epoch. This is shown in figure 2.

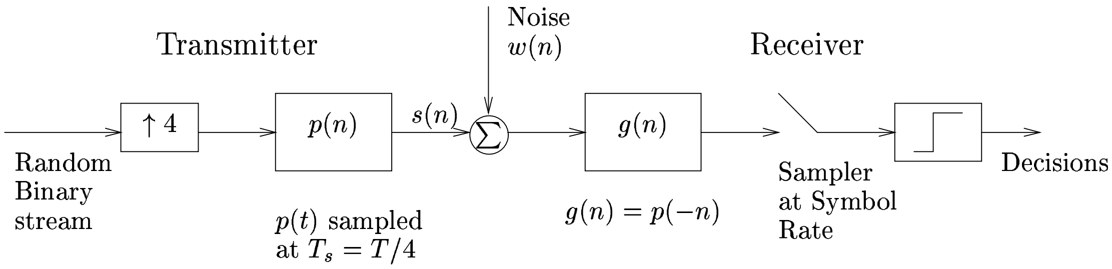

Figure 1: Block diagram of Matched filter receiver.

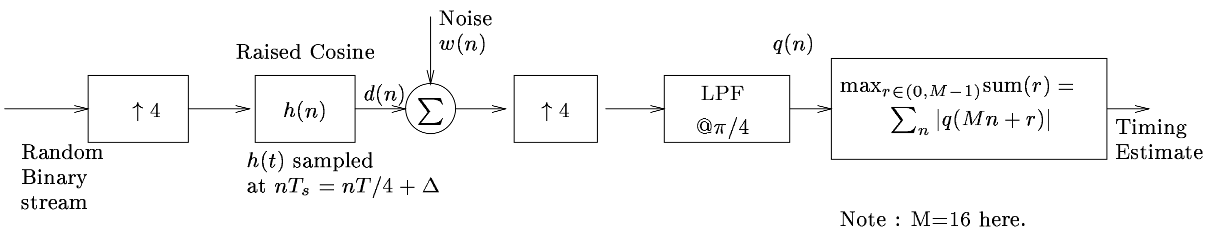

Figure 2: Discrete-time Transmitter and Receiver with timing offset Δ.

4 Tasks to be performed

4.1 Matched Filter

1. Generate three random bit sequences of ± 1 of lengths 100, 1000, and 10000.

2. Generate $p(n)$ (samples of $p(t)$) with $-4T < t < 4T$, choosing T=1, $\alpha = 0.3$ and $t = nT_s$ and $T_s = 1/4$.

   This generates samples of $p(t)$ sampled at four times the symbol rate. Explain how.

3. Plot $p(n)$. Convolve $p(n)$ with itself. Normalise its peak to unity and compare the result of the convolution with $h(n)$. Under what circumstances would convolution of $p(n)$ with itself be equal to (or be a close approximation to) $h(n)$? Note down your observations. Repeat this for different values of α and for different pulse durations. Try durations other than $(-4T, 4T)$.

4. The bit sequence generated above is convolved after upsampling by 4 (why?) with $p(n)$ generating the samples of transmit signal $s(n)$. Note that $\uparrow N$ implies adding $N-1$ zeros between adjacent samples.

5. At the receiver, convolve this waveform with a matched filter and plot the resulting output.

6. Given the output of the receive filter, where would you sample it to decide about the transmitted bit sequence? What is the BER?

7. Now add $w(n)$, random gaussian noise of zero mean and variance (vary from 0.1 to 1.0) to $s(n)$. Estimate BER for each value of noise variance.

4.2 Timing Offset Estimation

1. To introduce timing offset and its subsequent estimation, generate $p(n)$ by sampling $p(t)$ at $t = nT/4 + \Delta$ where $\Delta = T/10$ (see Figure 2).

   Convince yourself that doing this introduces timing offset.

2. Generate a random bit sequence as before of length n (in our experiment $n = 30$).

3. Generate noisy input (with variance = 0.5 or higher).

4. Upsample the output of the receive filter by a factor of 4 ($\uparrow 4$) and interpolate (which is done by using low-pass filter with cutoff at $\pi/4$ whose impulse response is a truncated $\sin(\frac{\pi}{4}n)/(\pi n)$ defined for $-N \le n \le N$. Choose $N = 20$). These operations are shown in the figure by an upsampler and an LPF.

5. There are now M samples for every symbol ($M = 16$ here). Maintain M sums of absolute values of the noisy output (summing averages noise and using magnitude makes it independent of the sign of the input bit). The summing operation is peformed over the total number of input bits n.

6. The index of the maximum of M sums over the total number of symbols transmitted is an estimate of the timing *epoch*.

7. Repeat this for different timing offsets and compare your estimates with the actual offsets.