EC305 Problem Set 6

1. See Examples 3.1 to 3.10

2. [Problem 3.6] A person enters a bank and finds all the four clerks busy serving the customers. There are no other customers in the bank, so the person will start service as soon as one of the customers in service leaves. Customers have independent, identical, exponential distribution of service time.

   (a) What is the probability that the person will be the last to leave the bank assuming that no other customers arrive?

   (b) If the average service time is 1 minute, what is the average time the person will spend in the bank?

   (c) Will the answer in part(a) change if there are some additional customers waiting in a common queue and customers begin service in the order of their arrival.?

3. [Problem 3.7] A communication line is divided into two identical channels each of which will serve a packet traffic stream where all the packets have equal transmission time T and equal interarrival rate $R > T$. Consider, alternatively, statistical multiplexing of the two traffic streams by combining the two channels into a single channel with transmission time $T/2$ for each packet. Show that the average system time of a packet will be decreased from T to something between $T/2$ and $3T/4$, while the variance of waiting time in queue will be decreased from T to something between $T/2$ and $3T/4$, while the variance of waiting time in queue will be increased from 0 to as much as $T^2/16$

4. [Problem 3.9] A communication line capable of transmitting at a rate of 50 Kbits/sec will be used to accommodate 10 sessions each generating Poisson traffic at a rate 150 packets/min. Packet lengths are exponentially distributed with mean 1000 bits.

   (a) For each session, find the average number of packets in queue, the average number in the system, and the average delay per packet when the line is allocated to the sessions by using:

      i. 10 equal-capacity time-division multiplexed channels. (Answer: $N_Q = 5$, N = 10, $T = 0.4$ sec.)

      ii. Statistical multiplexing. (Answer: $N_Q = 0.5$, N = 1, $T = 0.04$ sec.)

   (b) Repeat part (a) for the case where five of the sessions transmit at a rate of 250 packets/min while the other five transmit at a rate of 50 packets/min. (Answer N_Q = 21, N = 26, $T = 1.038$ sec.)

5. [Problem 3.19] A telephone company establishes a direct connection between two cities expecting Poisson traffic with rate 30 calls/min. The durations of calls are independent and exponentially distributed with mean 3 min. The interarrival times are independent of call durations. How many circuits should the company provide to ensure that an attempted call is blocked (because all circuits are busy) with probability less than 0.01? It is assumed that blocked calls are lost (*i.e.*, blocked calls are not attempted again).

6. [Problem 3.21] Consider the $M/M/1/m$ system which is the same as $M/M/1$ except that there can be no more than m customers in the system and customers arriving when the

system is full are lost. Show that the steady-state occupancy probabilities are given by

$$p_n = \frac{\rho^n(1-\rho)}{1-\rho^{m+1}}, \qquad 0 \le n \le m$$