EC305 Problem Set 4

1. White noise of power spectral density $N_0/2$ is filtered using an ideal low pass filter of bandwidth B. What is the variance of the output noise process?

2. A stationary Gaussian process X_t with zero-mean and power spectral density $S_X(f)$ is applied to a linear filter with impulse response as shown in Figure 1. A sample Y is taken of the random process at the filter output at time T. a) Determine the mean and variance of Y. b) What is the probability density function of Y?. c) If $S_X(f) = N_0/2$, what is the joint PDF of samples of the output random process taken at $t = T$ and $t = 3T/2$?

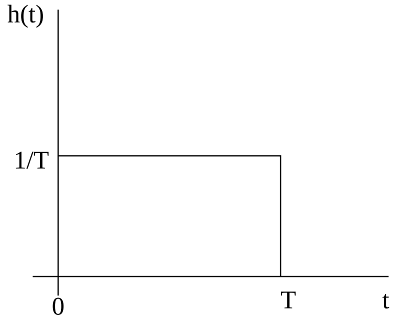

Figure 1:

3. The *effective aperture* A of a dish antenna is given by $(\lambda^2/4\pi)G$, where G is the antenna gain at wavelength λ. The *aperture efficiency* is the ratio of the effective aperture A to its physical aperture (the physical perture is equal to the area of the circle defined by the antenna radius).

   A radio link uses a pair of 2 m dish antennas with an efficiency of 60% each, as transmitting and receiving antennas. Other specifications of the link are: Transmitted power = $1dBW$, Carrier frequency = $4GHz$, Distance of the receiver from the transmitter = $150m$. Calculate the free-space loss, the power gain of each antenna, and the received power in dBW.

   Repeat the calculation for a carrier frequency of 12 GHz.

4. Consider the receiver shown in Figure 2, which consists of a low-noise RF amplifier, frequency down-coverter, and IF amplifier. The figure includes the noise figures and power gains of these components. The antenna temperature is 298K. Calculate the equivalent noise temperature of each component, assuming a room temperature of 298K. Calculate the effective noise temperature of the whole receiver (all three components together).

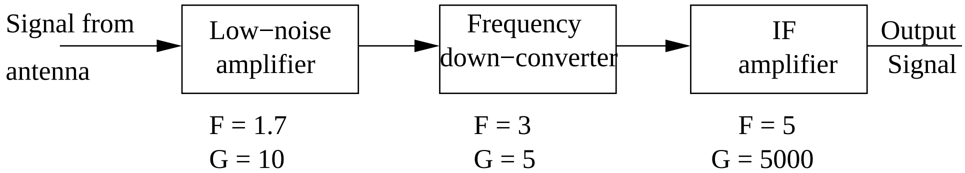

Figure 2:

5. A cellular base-station transmits 8W of power through an antenna with 6 dBi gain at a carrier frequency of 900 MHz. The signal bandwidth is 200 kHz, and is narrow enough that the channel can be approximated by a scaling constant. Assume that the path loss is equal to Kd^{-4}, where d is the distance between the base-station and the receive antenna on the cellphone, and $K = 50$. Assume that the cellphone antenna gain is 0 dBi and the noise figure is 6 dB. If the cellphone antenna is at room temperature, and the minimum SNR required for reliable reception is 20 dB, what is the maximum distance from the base-station at which the cellphone will work reliably?

6. Consider a cellular system with ideal hexagonal cells. Let R be the length of one side of the hexagon.

   (a) Specify a frequency assignment pattern for a frequency reuse factor of 1/7 (i.e., each frequency is used exactly once in a cluster of 7 cells) such that neighbouring cells do not use the same frequency.

   (b) Find the distance between the centers of 2 cells that are assigned the sam e frequency. For a given cell, how many cells that use the same frequency are at this distance?

   (c) Repeat part (a) and (b) for a frequency reuse factor of 1/4.