EC204: Networks & Systems
Solutions to Problem Set 9

1. (a) To solve for $i(t)$, $t \geq 0$ using Thevenin's theorem, we first transform the given network to the Laplace domain, as in figure (1).

   We obtain values for the Thevenin impedance $Z_0(s)$ and voltage source $V_{oc}(s)$ as

$$Z_0(s) = \frac{1}{\frac{1}{s} + \frac{1}{1+1/(1+s)}}$$

$$= \frac{s(s+2)}{s^2+2s+2}$$

$$V_{oc}(s) = \frac{1}{s} \qquad \text{(absence of current)}$$

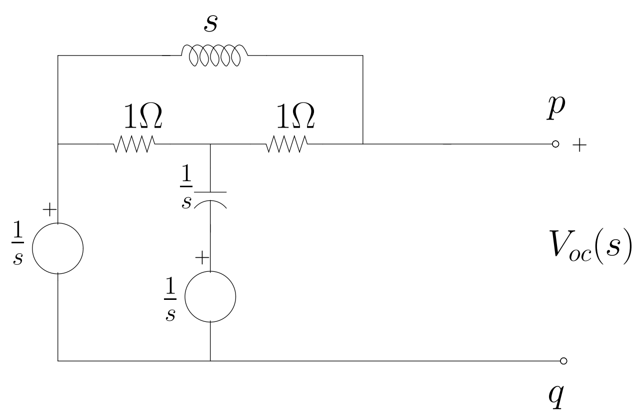

Figure 1: (*Problem 1(a)*) Laplace-transformed network

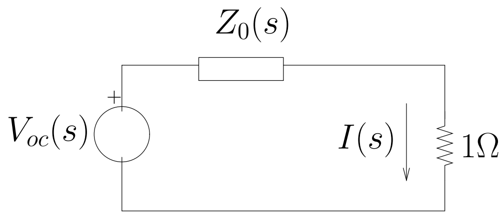

Figure 2: (*Problem 1(a)*) Thevenin equivalent network

From figure (2) we get

$$I(s) = \frac{V_{oc}(s)}{Z_0(s)+1} = \frac{1/s}{\frac{s^2+2s}{s^2+2s+2}+1}$$

$$= \frac{s^2+2s+2}{s(2s^2+4s+2)}$$

$$= \frac{1}{s} - \frac{1}{2}\left[\frac{1}{s+1} + \frac{1}{(s+1)^2}\right]$$

$$\Longrightarrow \qquad i(t) = \{1 - \frac{1}{2}\left[e^{-t} + te^{-t}\right]\}u(t) \quad \square$$

(b) By using the substitution theorem, the Laplace-transformed network can be expressed as shown in figures (3) and (4). Then, using superposition, we can treat the network as the sum of two independent networks as shown in figure (5). As a consequence, we obtain

$$I(s) = \frac{V_{oc}(s)}{Z_0(s)+1} = \frac{s^2+2s+2}{s(2s^2+4s+2)} \quad \square$$

2. The bridge is balanced when $R_x = 600\Omega$, i.e. $I_g = 0$. The incremental networks for $R_x = 630\Omega$ and $R_x = 570\Omega$ are as shown in figures (6) and (7) respectively. These resistive networks can be solved for the galvanometer currents to give

$$I_{g1} = -39.2\mu A, \qquad I_{g2} = 40.8\mu A$$

Therefore, by the linearity of the entire network, we can conclude that the range of I_g is from $-39.2\mu A$ to $40.8\mu A$ $\square$.

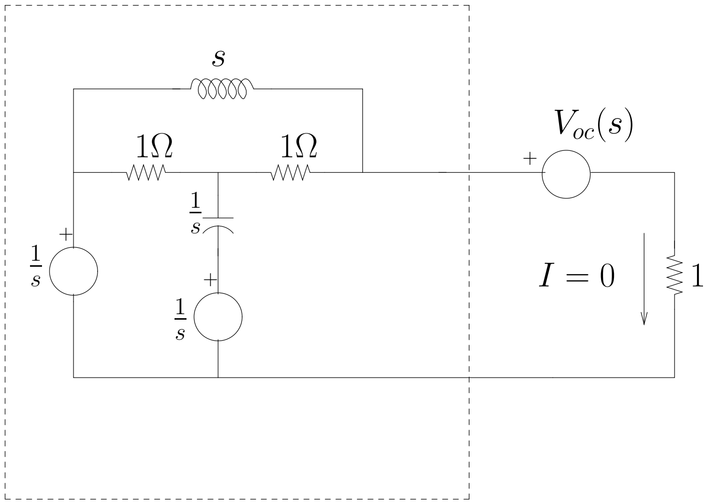

Figure 3: (*Problem 1(b)*) With switch open

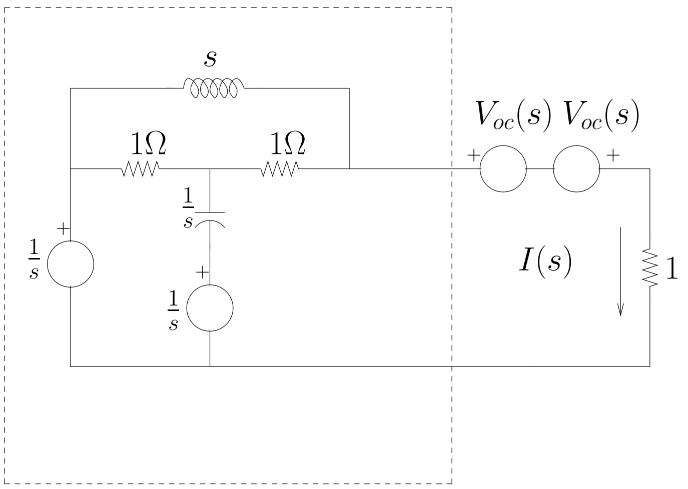

Figure 4: (*Problem 1(b)*) With switch closed

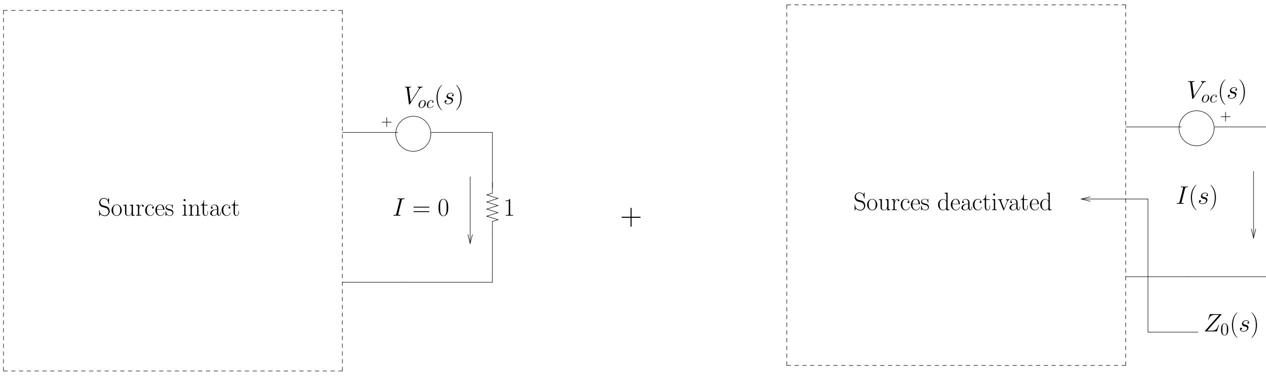

Figure 5: (*Problem 1(b)*) Applying the superposition principle

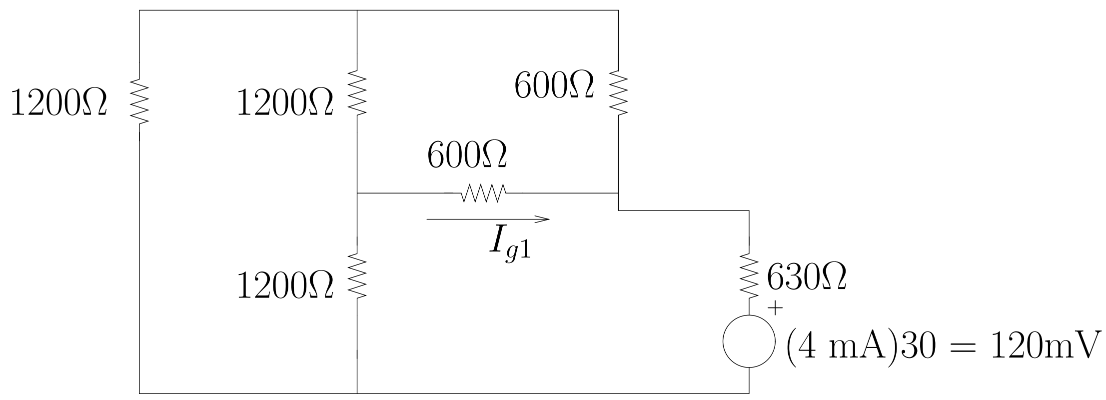

Figure 6: (*Problem 2*) Incremental network for $R_x = 630\Omega$

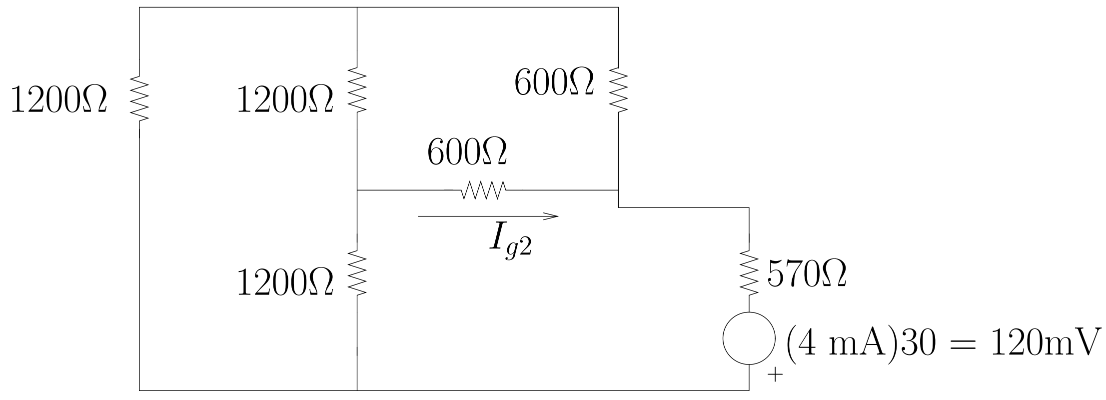

Figure 7: (*Problem 2*) Incremental network for $R_x = 570\Omega$

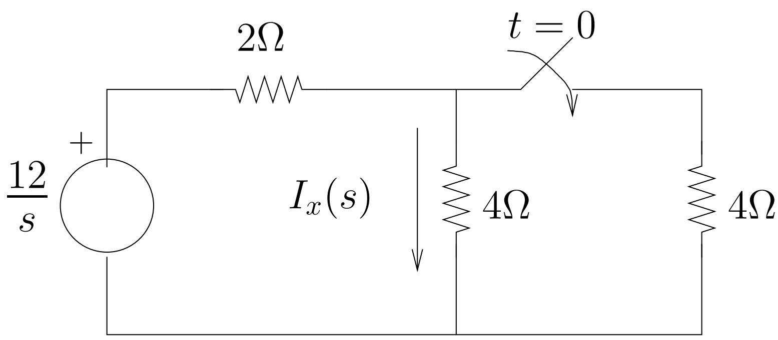

Figure 8: (*Problem 3*) Original network in the Laplace domain

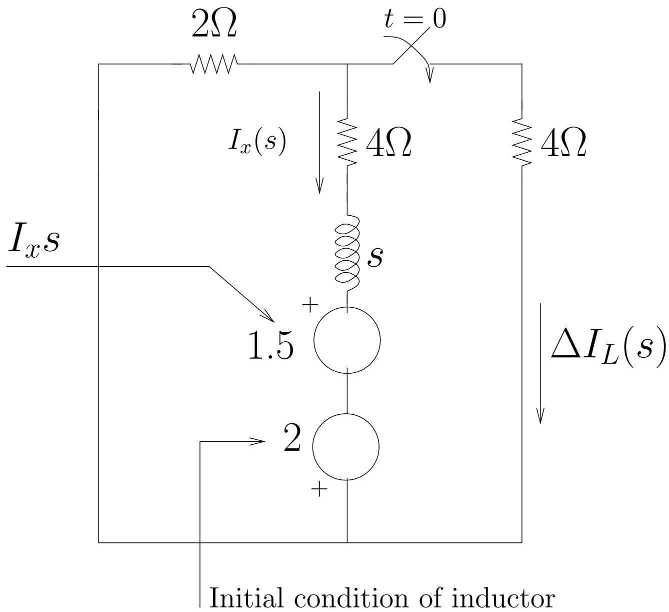

Figure 9: (*Problem 3*) Incremental network in the Laplace domain

3. In the original transformed network (figure (8)), we have

$$I_L(s) = \frac{1.5}{s}, \qquad I_x(s) = \frac{1.5}{s}$$

In the incremental network (figure (9)), we account for the inductor's initial current at $t = 0$ and the compensation voltage $V_x(s)$ given by

$$V_x(s) = sI_x(s) = 1.5$$

We have,

$$\Delta I_L(s) = \frac{-0.5}{s+4+\frac{4}{3}} \times \frac{2}{2+4}$$

$$= \frac{-0.5}{3s+16}$$

$$\Longrightarrow \qquad \Delta i_L(t) = \frac{-0.5}{3} e^{-16t/3} u(t)$$

$$\therefore \qquad i_L(t) = \mathcal{L}^{-1}(\frac{1.5}{s}) + \Delta i_L(t)$$

$$= [1.5 - \frac{0.5}{3} e^{-16t/3}] u(t) \quad \square$$

4. (a) By substitution theorem, the network in case (ii) is equivalent to network (a) in Figure 10. Now, using superposition theorem, the solution for network (a) is the same as the superposition of the solutions for networks (b) and (c).

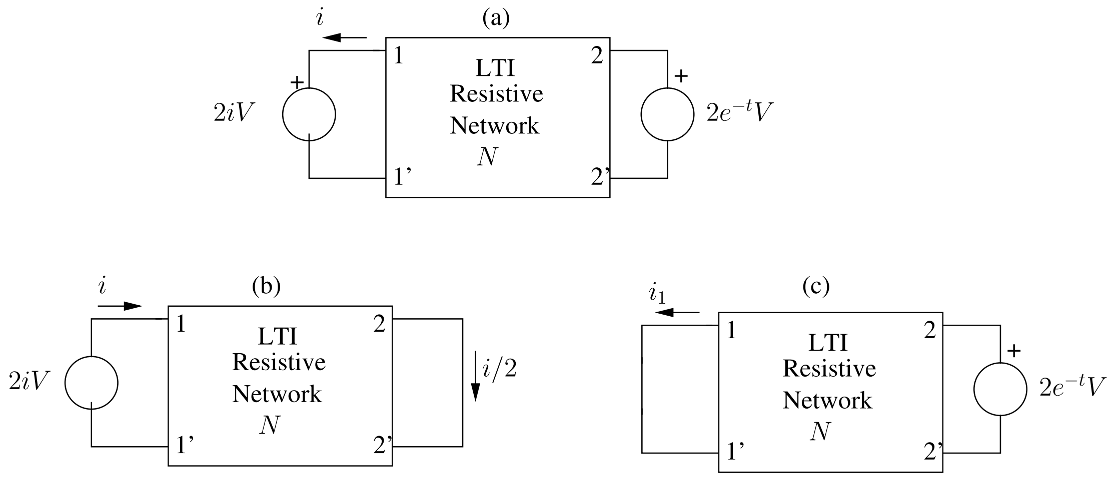

Figure 10: *Problem 4*

By linearity (using case (i)), the current entering terminal 1 in network (b) is i. Similarly, using reciprocity theorem, i_1 in network (c) is equal to $e^{-t}/2$ A. Using superposition, we have $i = i_1 - i$. Therefore, we have

$$i = i_1/2 = e^{-t}/4 \ A.$$

(b) Since $z_{11} = z_{22}$, $y_{11} = y_{22}$. Also, for this network $y_{12} = y_{21}$. Therefore, we can easily write down the y-parameters (from cases (i) and (ii)) as

$$Y = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.5 \end{bmatrix}.$$

Finally, we have

$$Z = Y^{-1} = \left[\begin{array}{cc} 8/3 & 4/3 \\ 4/3 & 8/3 \end{array} \right].$$