EC204: Networks & Systems
Solutions to Problem Set 8

1. We have

$$Y(s) = K\frac{(s-a)(s-b)}{(s+1-j1)(s+1+j1)}$$

$$I(s) = V(s)Y(s)$$

$$Y(s)|_{s=0} = 0 = \frac{Kab}{2}$$

$$\implies a = 0 \text{ or } b = 0$$

Without loss of generality, we take $a = 0$.

According to the second condition specified, the output for $\sin t$ V is

$$0.6\sin t + 0.8\cos t$$
$$= \sin(t+\theta),\ \tan\theta = 4/3$$

$$\therefore \qquad \sin t \longrightarrow \sin(t+\theta) \text{ in steady state}$$

$$\therefore \qquad Y(s)|_{s=j1} = e^{j\theta} = \cos\theta + j\sin\theta$$

$$\implies \qquad \frac{Kj(j-b)}{(1+2j)(1)} = 0.6 + j0.8$$

$$-K - jKb = -1 + j2$$

$$\therefore \qquad K = 1, \qquad b = -2$$

At $s = j2$, we will have

$$Y(s)|_{s=j2} = \left.\frac{s(s+2)}{(s+1+j1)(s+1-j1)}\right|_{s=j2}$$

$$= \frac{j2(2+j2)}{(1+j3)(1+j1)} = \frac{j2}{5}(1-j3) = 1.2 + j0.4$$

$$= \frac{2\sqrt{10}}{5}e^{j\phi} = 1.2649e^{j\phi}, \qquad \tan\phi = \frac{1}{3}$$

Hence an input of $\sin 2t$ V will produce a steady state current of $1.2649\sin(2t+\phi)$ A, with $\tan\phi = 1/3$ □

2.

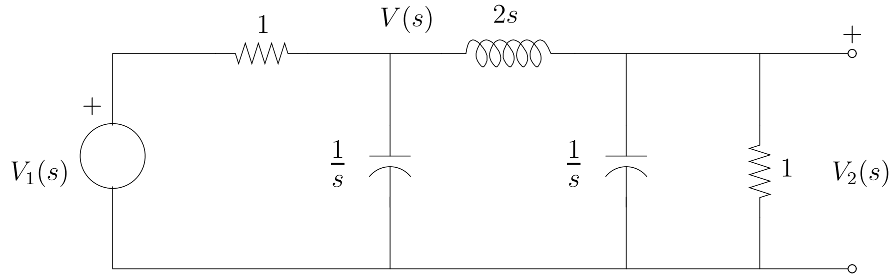

Figure 1: Circuit for Problem 2

Writing out the current equations at the nodes with voltages $V(s)$ and $V_2(s)$, we have

$$V(s) - V_1(s) + sV_2(s) + \frac{V(s) - V_2(s)}{2s} = 0 \tag{1}$$

$$V_2(s) + sV_2(s) + \frac{V_2(s) - V(s)}{2s} = 0 \tag{2}$$

Equation (2) gives

$$2s(1+s)V_2(s) + V_2(s) - V(s) = 0$$

$$\Longrightarrow V(s) = V_2(s)[2s^2 + 2s + 1]$$

Now, using equation (1) and plugging this in, we get

$$2sV(s) - 2sV_1(s) + 2s^2V(s) + V(s) - V_2(s) = 0$$

$$\text{i.e. } V_s[2s^2 + 2s + 1] - 2sV_1(s) - V_2(s) = 0$$

$$\implies V_2(s)[(2s^2 + 2s + 1)^2 - 1] = 2sV_1(s)$$

$$\therefore \qquad \frac{V_2(s)}{V_1(s)} = \frac{2s}{(2s^2 + 2s + 1)^2 - 1} = H(s)$$

$$\implies H(s) = \frac{2s}{4s(s+1)(s^2+s+1)} = \frac{1}{2(s+1)(s^2+s+1)} \quad \square$$

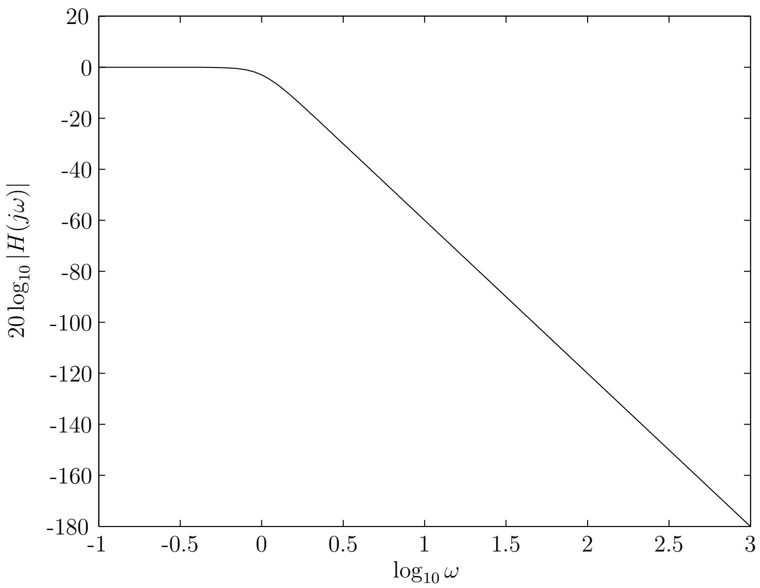

Figure 2: Magnitude plot of $H(j\omega)$ (dB) vs. $\log_{10}(\omega)$

3. By definition, we have

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore \qquad V_i(s) - I_1 R_0 = z_{11} I_1 + z_{12} I_2$$

$$V_0(s) = z_{21} I_1 + z_{22} I_2 = -I_2 R_L \qquad (*)$$

$$\therefore \qquad \frac{V_0(s)}{V_i(s)} = \frac{-I_2 R_L}{V_1 + I_1 R_0} = \frac{-I_2 R_L}{(z_{11} + R_0) I_1 + z_{12} I_2}$$

From (*), we have

$$I_2 = \frac{-z_{21} I_1}{(z_{22} + R_L)}$$

$$\therefore \qquad \frac{V_0(s)}{V_i(s)} = \frac{-R_L}{z_{12} - \dfrac{(z_{11} + R_0)(z_{22} + R_L)}{z_{21}}}$$

$$= \frac{R_L z_{21}}{(z_{11} + R_0)(z_{22} + R_L) - z_{12} z_{21}} \quad \square$$

4.

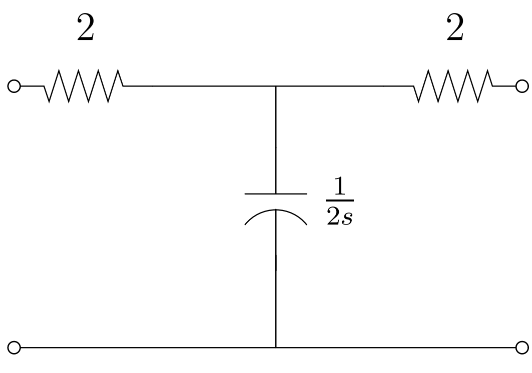

Figure 3: Network 1

From figure (3), we get

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{2 + \dfrac{2.\frac{1}{2s}}{2 + \frac{1}{2s}}} = \frac{4s+1}{8s+4}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{2 + \dfrac{2.\frac{1}{2s}}{2 + \frac{1}{2s}}} . \frac{1/2s}{2 + 1/2s} = \frac{-1}{8s+4}$$

$$y_{12} = y_{21} = \frac{-1}{8s+4}$$

$$y_{22} = y_{11} = \frac{4s+1}{8s+4}$$

$$\therefore \qquad Y_1 = \begin{bmatrix} \dfrac{4s+1}{8s+4} & \dfrac{-1}{8s+4} \\ \dfrac{-1}{8s+4} & \dfrac{4s+1}{8s+4} \end{bmatrix}$$

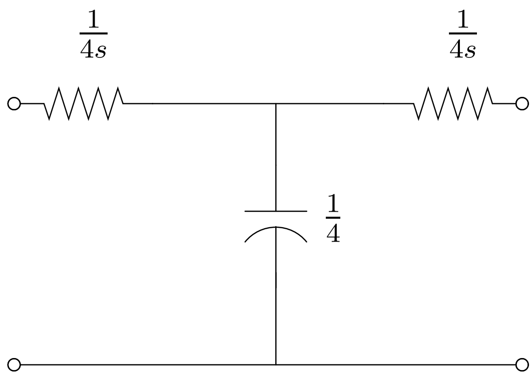

Figure 4: Network 2

From figure (4), we get

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{\dfrac{1}{4s} + \dfrac{1}{4s+4}} = \frac{4s(s+1)}{2s+1}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{4s(s+1)}{2s+1} . \frac{1/4}{1/4 + 1/4s} = \frac{-4s^2}{2s+1}$$

$$y_{12} = y_{21} = \frac{-4s^2}{2s+1}$$

$$y_{22} = y_{11} = \frac{4s(s+1)}{2s+1}$$

$$\therefore \qquad Y_2 = \begin{bmatrix} \dfrac{4s(s+1)}{2s+1} & \dfrac{-4s^2}{2s+1} \\ \\ \dfrac{-4s^2}{2s+1} & \dfrac{4s(s+1)}{2s+1} \end{bmatrix}$$

Hence, the y-parameters of the overall 2-port network are given by

$$Y = Y_1 + Y_2 = \begin{bmatrix} \left(\dfrac{4s+1}{8s+4} + \dfrac{4s(s+1)}{2s+1}\right) & \left(\dfrac{-1}{8s+4} - \dfrac{4s^2}{2s+1}\right) \\ \\ \left(\dfrac{-1}{8s+4} - \dfrac{4s^2}{2s+1}\right) & \left(\dfrac{4s+1}{8s+4} + \dfrac{4s(s+1)}{2s+1}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{16s^2+20s+1}{8s+4} & -\dfrac{16s^2+1}{8s+4} \\ \\ -\dfrac{16s^2+1}{8s+4} & \dfrac{16s^2+20s+1}{8s+4} \end{bmatrix} \quad \square$$

5. It is known that

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{in}(s) = \frac{V_1}{I_1}$$

$$V_1 = -rI_2 = -r.\frac{-V_2}{Z_L(s)} = \frac{rV_2}{Z_L(s)}$$

$$I_1 = \frac{V_2}{r}$$

$$\therefore \qquad Z_{in}(s) = \frac{r^2}{Z_L(s)}$$

If $Z_L(s) = 1/Cs$, we have $Z_{in} = r^2Cs$ which is the Laplace transform of a purely inductive impedance. Hence the network behaves like an inductor across the input terminals.