# EC204: Networks & Systems
# Solutions to Problem Set 5

1. (a) See Figure below. $X_1(\omega) = \text{sinc}\left(\frac{\omega}{20000\pi}\right)$.

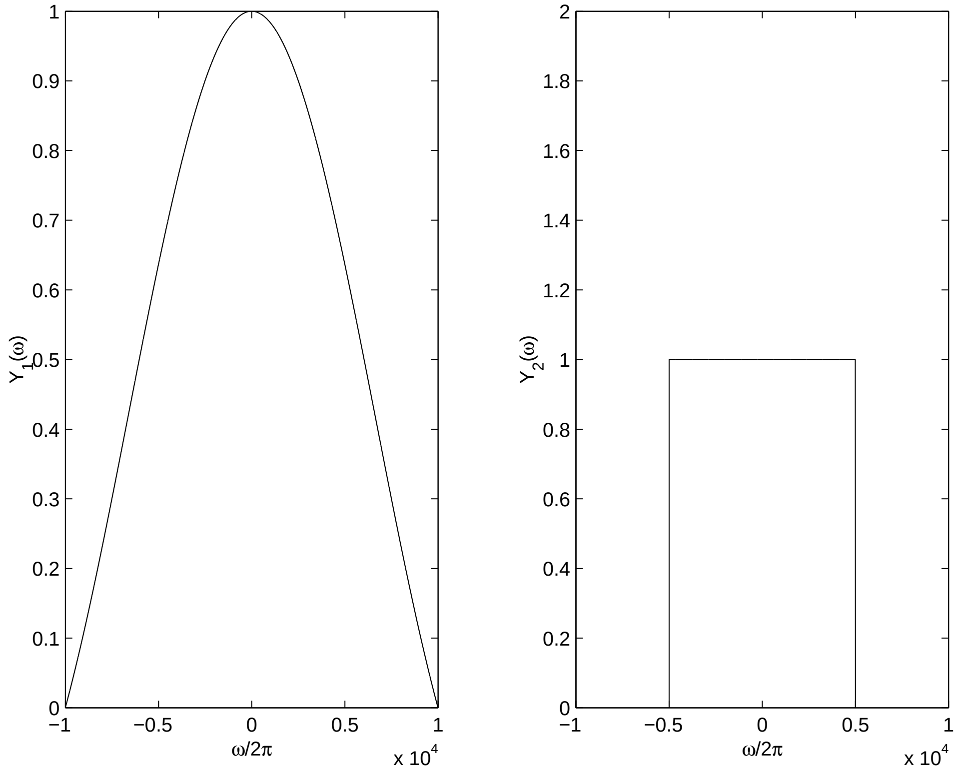


Figure 1: Problem 1

(b) Bandwidth of $y_1(t) = 10000$ Hz, bandwidth of $y_2(t) = 5000$ Hz, bandwidth of $y(t) =$ bandwidth of $y_1(t) +$ bandwidth of $y_2(t) = 15000$ Hz.

2.

$$\frac{V_0(\omega)}{I_s(\omega)} = ?$$

$$I_s(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

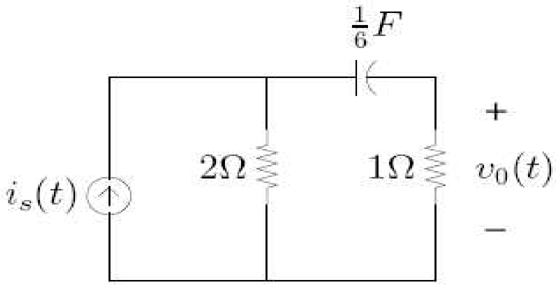


Figure 2: Problem 2

$$\frac{V_0(\omega)}{1} + \frac{\left(V_0(\omega) + \frac{V_0(\omega)}{\jmath\frac{\omega}{6}}\right)}{2} = I_s(\omega)$$

$$\frac{V_0(\omega)}{I_s(\omega)} = \frac{1}{\frac{3}{2} + \frac{1}{\jmath\frac{\omega}{3}}} = \frac{\jmath\omega\frac{2}{3}}{\jmath\omega + 2}$$

$$\Longrightarrow V_0(\omega) = \frac{2/3}{\jmath\omega + 2} + \pi[0] = \frac{2/3}{2 + \jmath\omega}$$

$$\Longrightarrow v_0(t) = \frac{2}{3}e^{-2t}u(t)$$

3. $f(t)$ is a periodic signal with period 4. It can be expanded using the Fourier series. Since $f(t) = -f(t + T_0/2)$ (half-wave symmetry), only odd harmonics are present. Therefore, the frequencies (in Hz) in $f(t)$ are $1/4, 3/4, 5/4, 7/4, \cdots$ (since the fundamental frequency is 1/4 Hz). $y(t)$ is a cosine function of frequency 1/2 Hz.

   Since we want $\cos \pi t$ at the output of the LTI system, the LTI system needs to remove all the components due to $f(t)$ and scale the cosine at 0.5 Hz by 1/2. Therefore, the magnitude response of the proposed LTI system should satisfy:

$$|H(\omega)| = 0 \quad \text{for} \;\; 0.5\pi, 1.5\pi, 2.5\pi, \cdots$$

   and $|H(\omega)| = 0.5$ for $\omega = \pi$. One possible magnitude response for the LTI system is shown in the figure below.

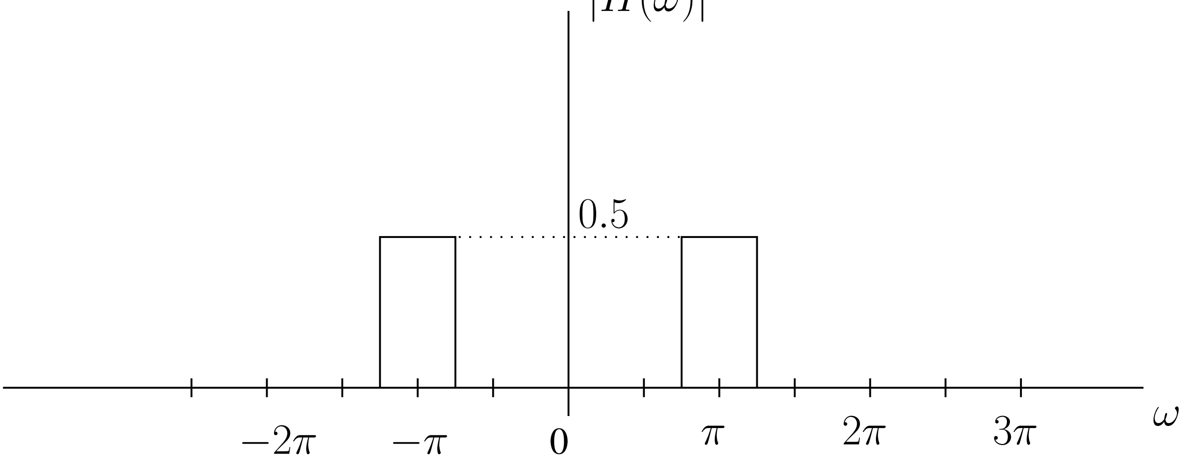


Figure 3: Problem 3

4.

$$z_1(t) = x(t)\cos(\omega_1 t) + y(t)\cos(\omega_2 t)$$

$$\omega_1 = 5W, \omega_2 = 7W$$

$$z_4(t) = x(t)$$

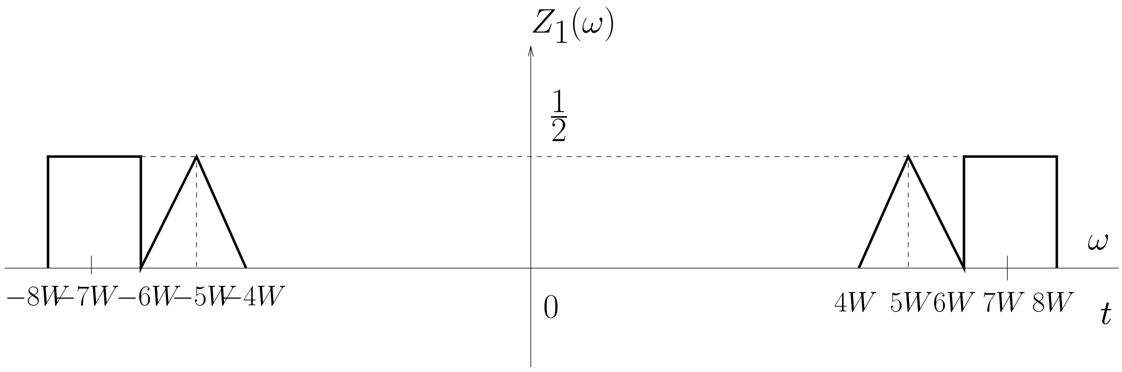


Figure 4: Problem 4

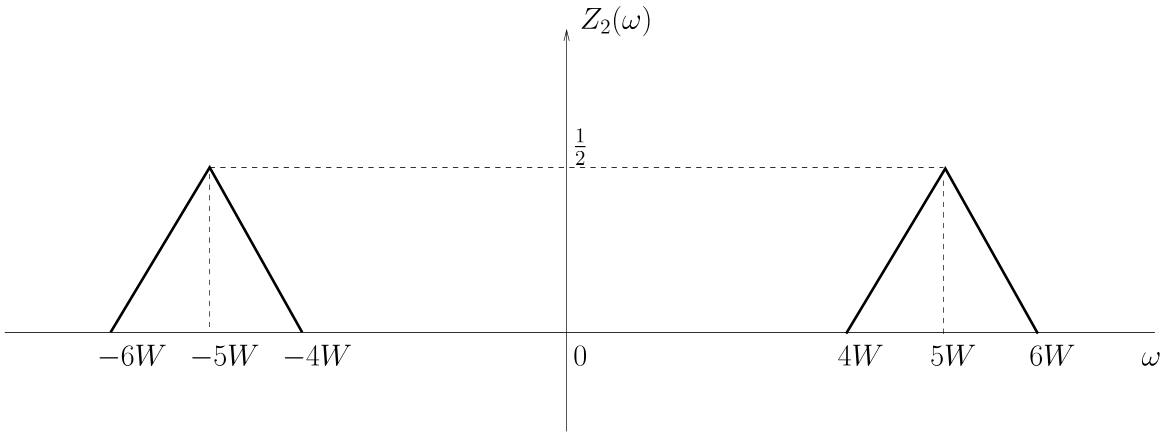


Figure 5: Problem 4

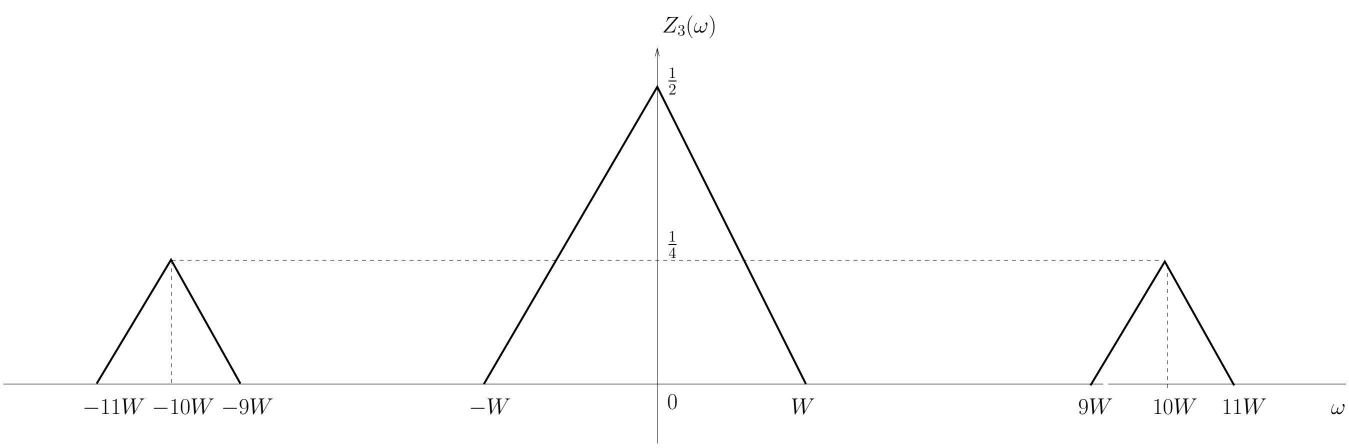


Figure 6: Problem 4

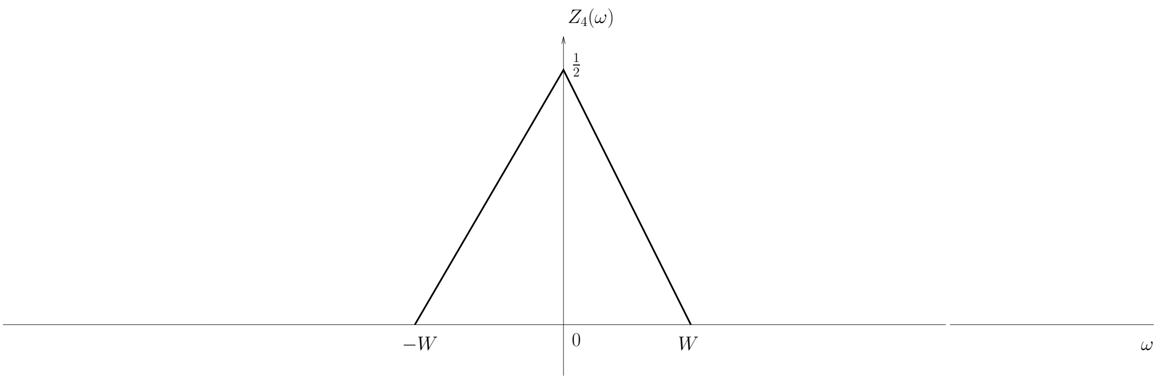


Figure 7: Problem 4