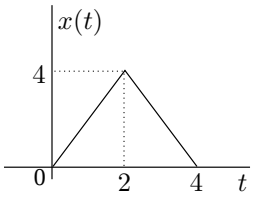
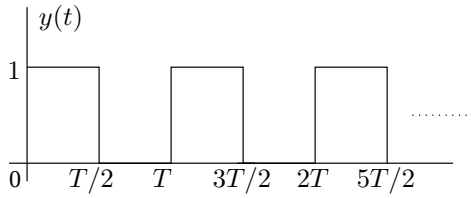
EC204: Networks & Systems
Problem Set 6

1. Find the Laplace transform of $x(t)$ and $y(t)$ shown below.





2. Consider the partial fraction expansions shown below.

$$\frac{1}{(s+a)(s-p_1)(s-p_1^*)} = \frac{K_1}{s+a} + \frac{K_2}{s-p_1} + \frac{K_3}{s-p_1^*}$$

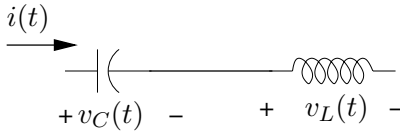
$$\frac{s+b}{(s+a)(s-p_1)(s-p_1^*)(s-p_2)(s-p_2^*)} = \frac{K_1}{s+a} + \frac{K_2}{s-p_1} + \frac{K_3}{s-p_1^*} + \frac{K_4}{s-p_2} + \frac{K_5}{s-p_2^*}$$

Show that $K_3 = K_2^*$ and $K_5 = K_4^*$. Assume that a and b are real.

3. Obtain the inverse L-transforms of (a) $\dfrac{s+2}{s^2+8s+15}$, (b) $\dfrac{s+1}{(s+2)^2(s+3)}$, (c) $\dfrac{2s^2+s+1}{s(s+2)}$, (d) $\dfrac{2s+1}{(s+2)(s^2+1)^2}$, and (e) $\dfrac{1}{10^4 s^2 + 10^2 s + 1}$.

4. Determine $f(t) \star g(t)$ using L-transforms given that $g(t) = e^{-2t}u(t)$ and $f(t) = 1$ for $0 \le t \le 2$ and 0 otherwise.

5. A capacitor and inductor are connected in series as shown below. If the Laplace transform



of the voltage across a capacitor $V_C(s) = \mathcal{L}\left[v_C(t)\right]$ is given by $V_C(s) = \dfrac{as+b}{s^2+cs+d}$, find (a) $v_C(0^+)$, (b) $i(0^+)$, and (c) $v_L(0^+)$.

6. The error $e(t)$ of a controlled quantity in a control system has the L-transform

$$E(s) = \frac{s+1}{(s+1)^2+4}.$$

Without explicitly finding $e(t)$, evaluate (a) $e(\infty)$, (b) $\displaystyle\int_0^\infty e(t)dt$, and (c) $\displaystyle\int_0^\infty te(t)dt$.