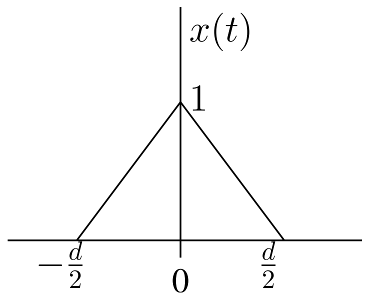
EC204: Networks & Systems
Problem Set 4

1. A signal $x(t)$ can be expressed as the sum of even and odd components as $x(t) = x_e(t) + x_o(t)$. (a) If $x(t) \iff X(\omega)$, show that for real $x(t)$, $x_e(t) \iff Re[X(\omega)]$ and $x_o(t) \iff j\,Im[X(\omega)]$. (b) Verify these results for $x(t) = e^{-at}u(t)$.

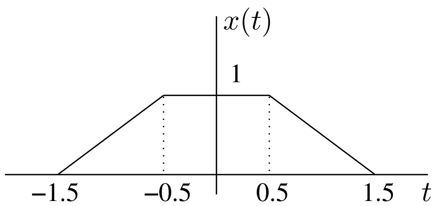
2. Sketch the following functions: (a) $rect(t/2)$, (b) $rect((t-10)/8)$, and (c) $\text{sinc}(t/5)rect(t/10)$.

3. If $x(t) \iff X(\omega)$, then show that $X(0) = \int_{-\infty}^{\infty} x(t)dt$ and $x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)d\omega$. Also show that $\int_{-\infty}^{\infty} \text{sinc}(x)dx = \int_{-\infty}^{\infty} \text{sinc}^2(x)dx = 1$.

4. Find the Fourier transform of $x(t)$ shown below in three different ways: (i) directly through integration, (ii) using the time-differentiation property of the Fourier transform, and (iii) using the Fourier transform of the $rect(\cdot)$ function and the convolution property of the Fourier transform. Sketch the Fourier transform $X(\omega)$.



5. Find the Fourier transform $X(\omega)$ of the signal $x(t)$ shown below.



6. Find the energy of the signal $x(t) = e^{-at}u(t)$. Determine the frequency W (in rad/s) so that the energy contributed by the spectral components of all the frequencies below W is 95% of the signal energy E_x.

7. Use Parseval's theorem to show that the energy of the signal $x(t) = 2a/(t^2 + a^2)$ is $2\pi/a$.

8. If $f_1(t) = 2\,rect\,(t/4)$ and $f_2(t) = rect\,(t/2)$, then find $g(t) = f_1(t) \star f_2(t)$ (where $\star$ represents convolution) and $G(\omega)$. Sketch the maginitude and phase of $G(\omega)$.

9. A signal $x(t)$ is bandlimited to B Hz. Show that $x^n(t)$ is bandlimited to nB Hz.

10. Prove the frequency-differentiation property (dual of the time-differentiation property): $-jtx(t) \Longleftrightarrow \dfrac{dX(\omega)}{d\omega}$. Using this property determine the Fourier transform of $te^{-at}u(t)$.

11. The Fourier tranform of the triangular pulse $x(t)$ shown below is $X(\omega) = \dfrac{1}{\omega^2}(e^{-j\omega} + j\omega e^{-j\omega} - 1)$. Using this, find the Fourier transforms of $x_1(t)$ and $x_2(t)$ shown below.