Downlink Pareto Optimal Beamforming with Limited Cooperation

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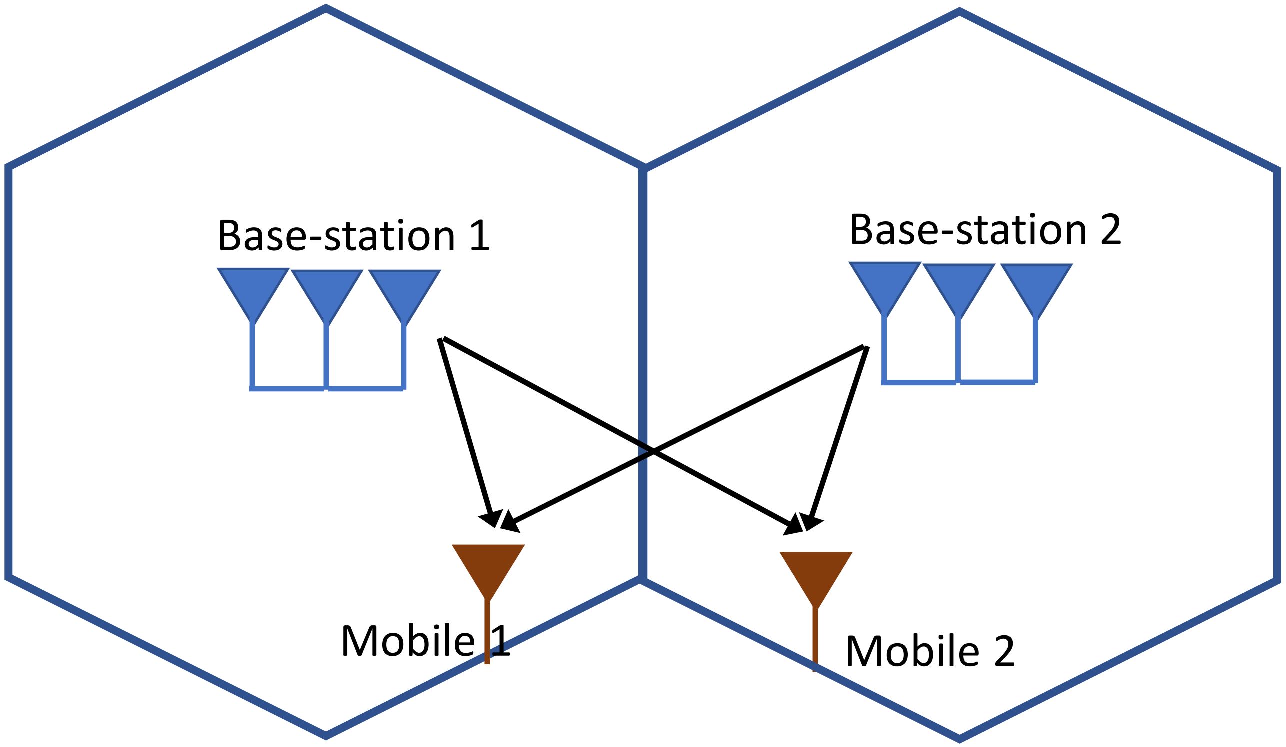
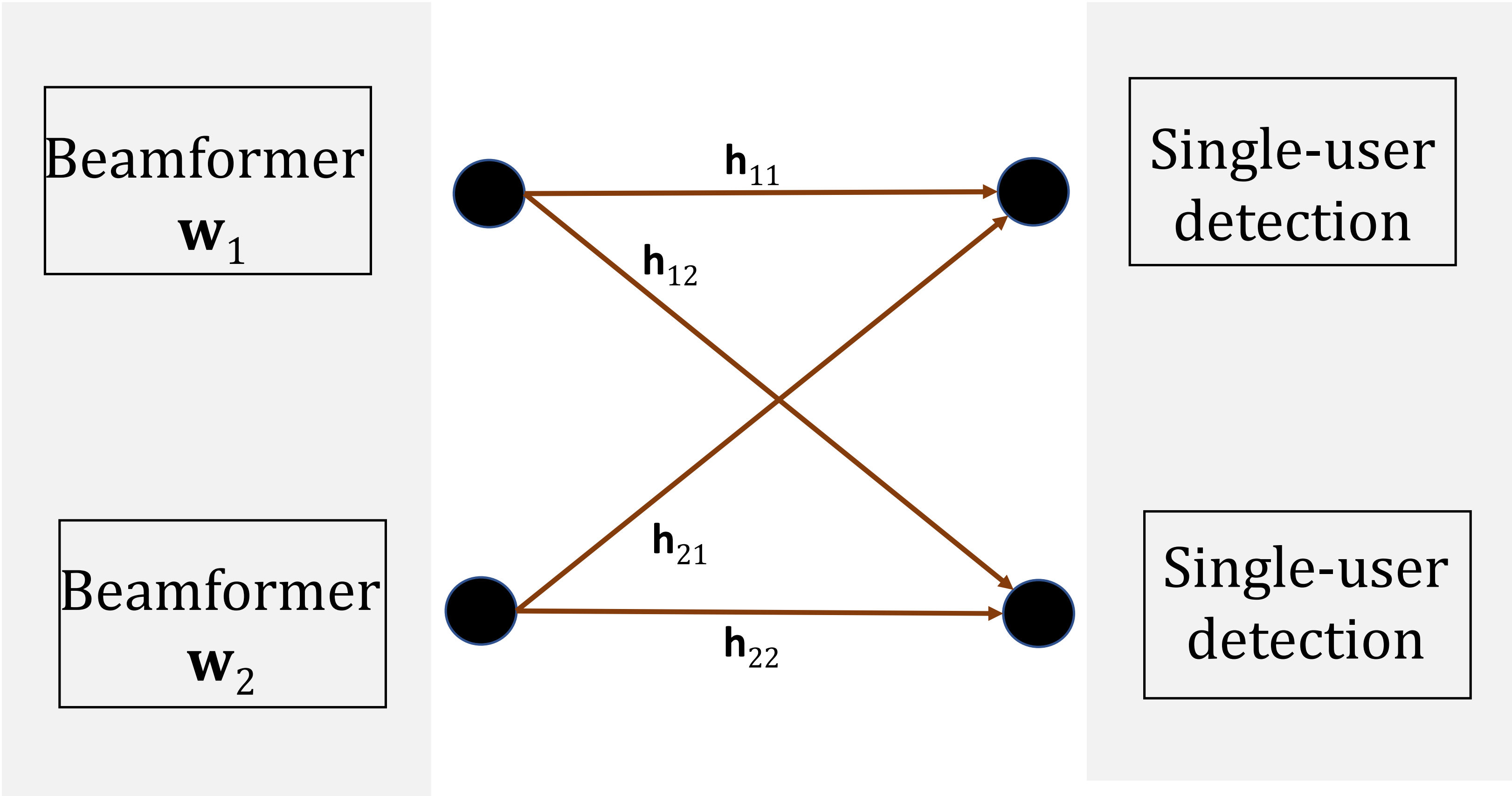
Multicell Downlink Beamforming



- Problem: Design beamforming vectors at each BS
- Distributed solution with limited exchange of information

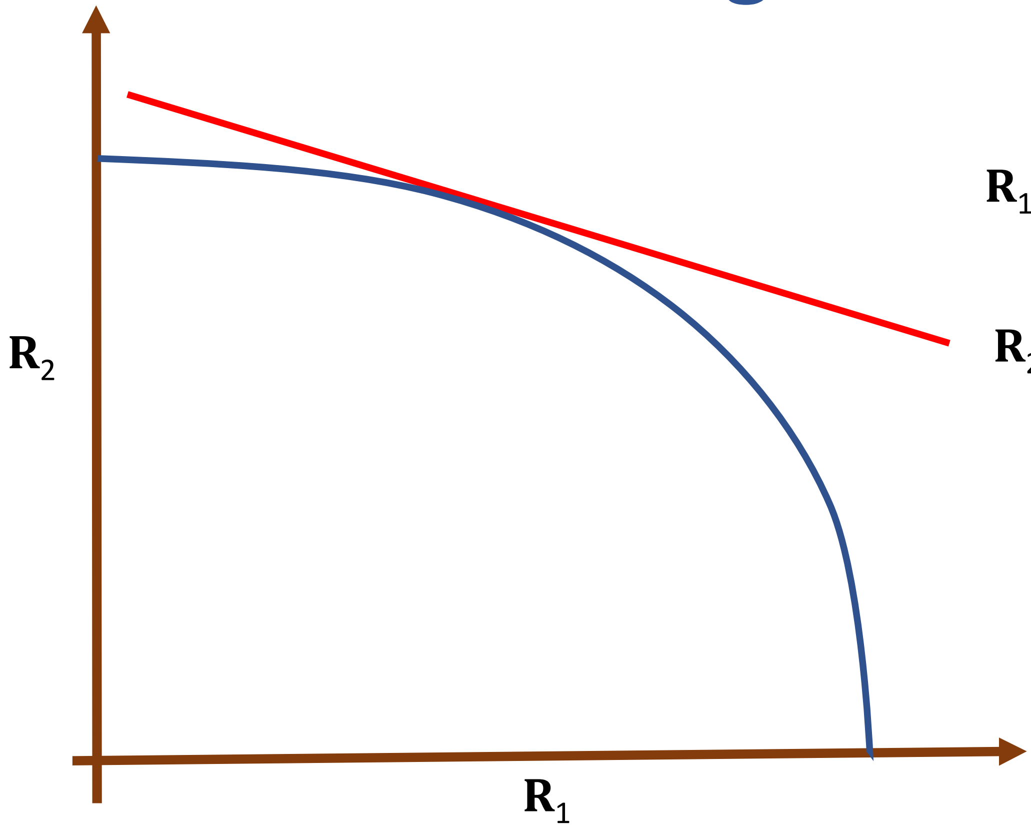
MISO Interference Channel Model



Beamforming optimal under Gaussian codebooks + single-user detection

Zhang & Cui 2010, Shang, Chen & Poor 2009

Achievable Rate Region



$$\mathbf{R}_1 = \log\left(1 + \frac{\left|\mathbf{h}_{11}^H \mathbf{w}_1\right|^2}{\left|\mathbf{h}_{21}^H \mathbf{w}_2\right|^2 + \sigma_1^2}\right)$$

$$\mathbf{R}_2 = \log\left(1 + \frac{\left|\mathbf{h}_{22}^H \mathbf{w}_2\right|^2}{\left|\mathbf{h}_{12}^H \mathbf{w}_1\right|^2 + \sigma_2^2}\right)$$

- Can be non-convex
- Boundary points to be determined
  - Pareto optimal rate vector: Not possible to improve any component without decreasing at least one other component

Finding the beamforming vectors

- Weighted sum rate maximization

$$\beta_1 \log\left(1 + \frac{\left|\mathbf{h}_{11}^H \mathbf{w}_1\right|^2}{\left|\mathbf{h}_{21}^H \mathbf{w}_2\right|^2 + \sigma_1^2}\right) + \beta_2 \log\left(1 + \frac{\left|\mathbf{h}_{22}^H \mathbf{w}_2\right|^2}{\left|\mathbf{h}_{12}^H \mathbf{w}_1\right|^2 + \sigma_2^2}\right)$$

- Power constraints

$$\|\mathbf{w}_1\|^2 \leq P_1$$
$$\|\mathbf{w}_2\|^2 \leq P_2$$

- Centralized solution

Distributed solution with limited coordination

BS 1

$$\log\left(1+\frac{\left|\mathbf{h}_{11}^H\mathbf{w}_1\right|^2}{\Gamma_{21}+\sigma_1^2}\right)$$

$$\left|\mathbf{h}_{12}^H\mathbf{w}_1\right|^2\le\Gamma_{12}$$

$$\|\mathbf{w}_1\|^2\le P_1$$

Γ_{21} →

← Γ_{12}

BS 2

$$\log\left(1+\frac{\left|\mathbf{h}_{22}^H\mathbf{w}_2\right|^2}{\Gamma_{12}+\sigma_2^2}\right)$$

$$\left|\mathbf{h}_{21}^H\mathbf{w}_2\right|^2\le\Gamma_{21}$$

$$\|\mathbf{w}_2\|^2\le P_2$$

- There exist interference thresholds corresponding to each boundary point
- Local channel information

Zhang & Cui 2010

Solution for given interference thresholds

$$\max_{\gamma_1, \delta_1, \theta_1, \phi_1} \gamma_1$$

$$\gamma_1^2 + \delta_1^2 \leq P_1$$

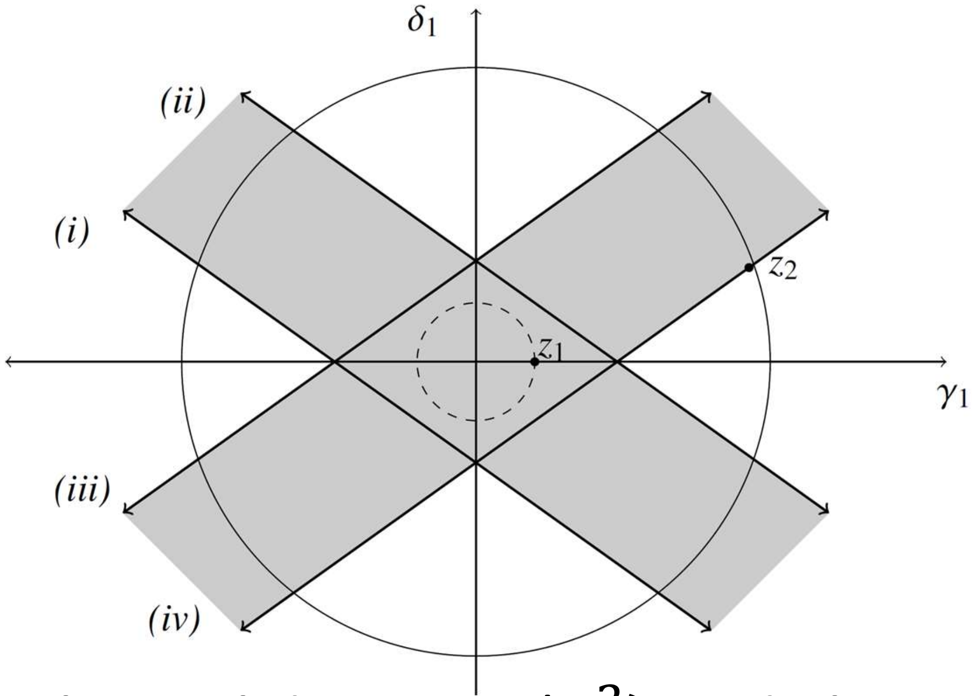
$$a\,\gamma_1^2 + b\delta_1^2 + 2ab\gamma_1\delta_1 \cos(\theta_1 - \phi_1) \leq \Gamma_{12}$$

- Power along channel direction (γ_1^2) and along orthogonal direction (δ_1^2)

M. Vishnu Narayanan, S. Bhashyam, *Pareto Optimal Distributed Beamforming for the Multi-band Multi-cell Downlink,* Proceedings of IEEE Global Communications Conference (GLOBECOM 2017), Singapore, Dec. 2017.

Solution for given interference thresholds

- Closed form solution



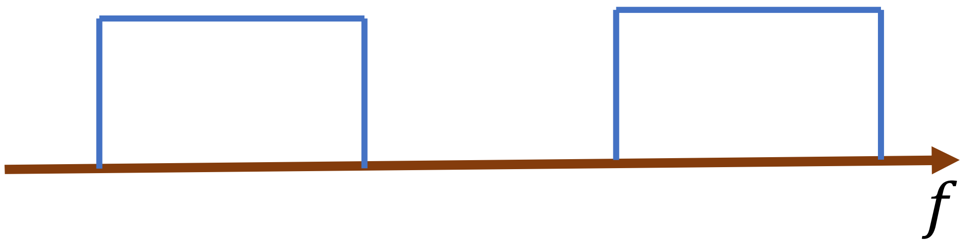
- Power along channel direction (γ_1^2) and along orthogonal direction (δ_1^2)

Weighted sum rate maximization

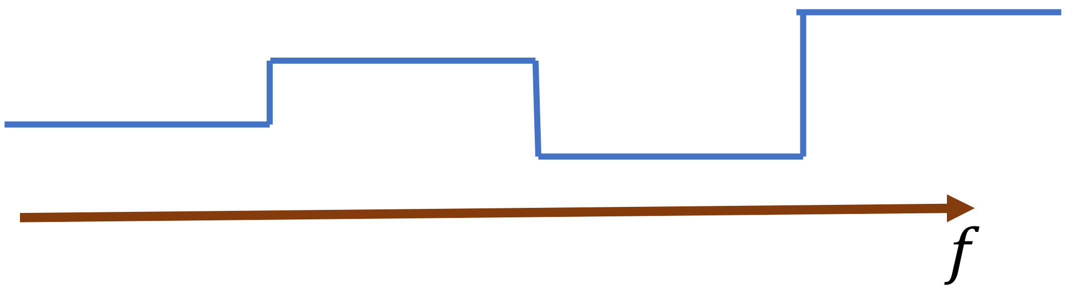
- Update interference thresholds using gradient ascent
- Use closed form solution for given thresholds

Multiple band case

Flat fading model so far



Multiple bands or resource blocks



Frequency selective channel

Power allocation + Beamforming

$$\max_{\{\mathbf{w}_{ik}\}} \sum_i \beta_i \sum_k \log\left(1 + \frac{\left|\mathbf{h}_{iik}^H \mathbf{w}_{ik}\right|^2}{\left|\mathbf{h}_{jik}^H \mathbf{w}_{jk}\right|^2 + \sigma_{ik}^2}\right)$$

$$\sum_k \|\mathbf{w}_{ik}\|^2 \leq P_i \text{ for all } i$$

- Sum power constraint over all bands
- Beamforming vector for each band

Pareto boundary: *k*-band & 1-band

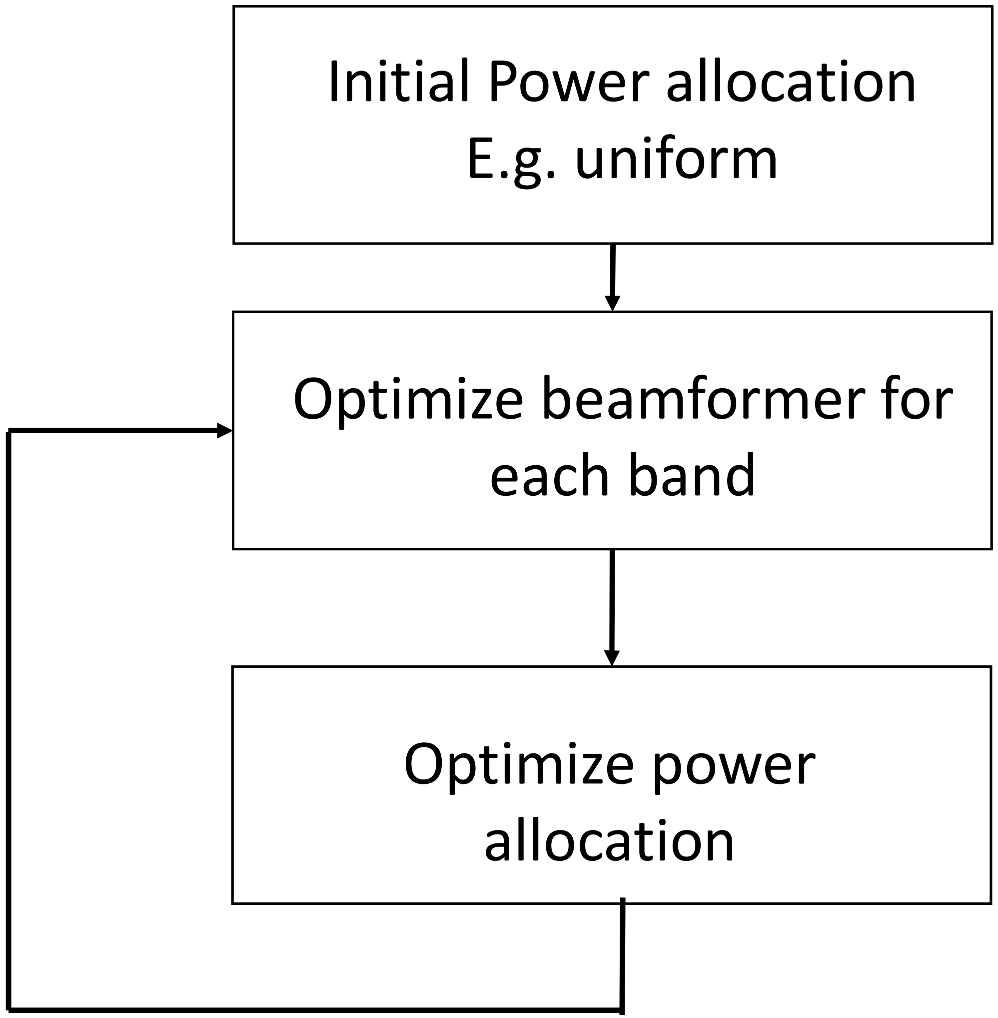
(R_1, R_2) is Pareto optimal

implies

(R_{1k}, R_{2k}) is Pareto optimal in each band k.

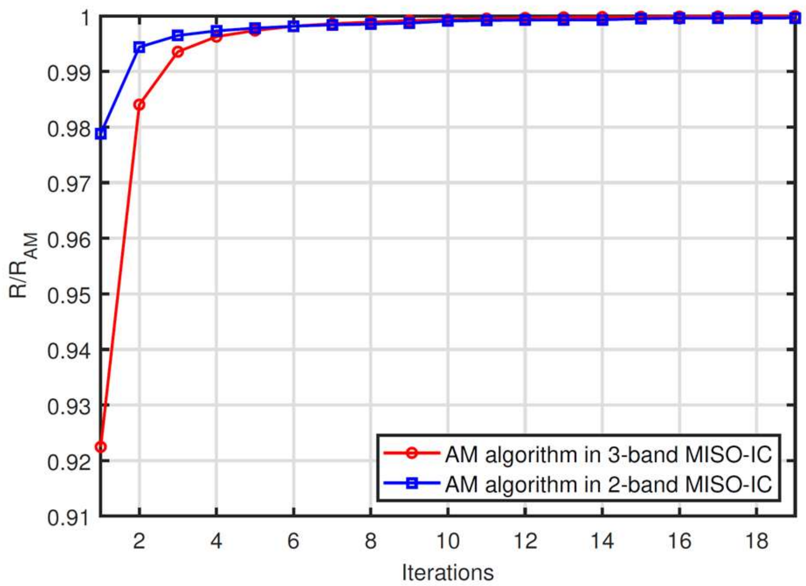
- For a given power allocation, overall multi-band problem reduces to K single-band problems, one for each band

Alternating Maximization



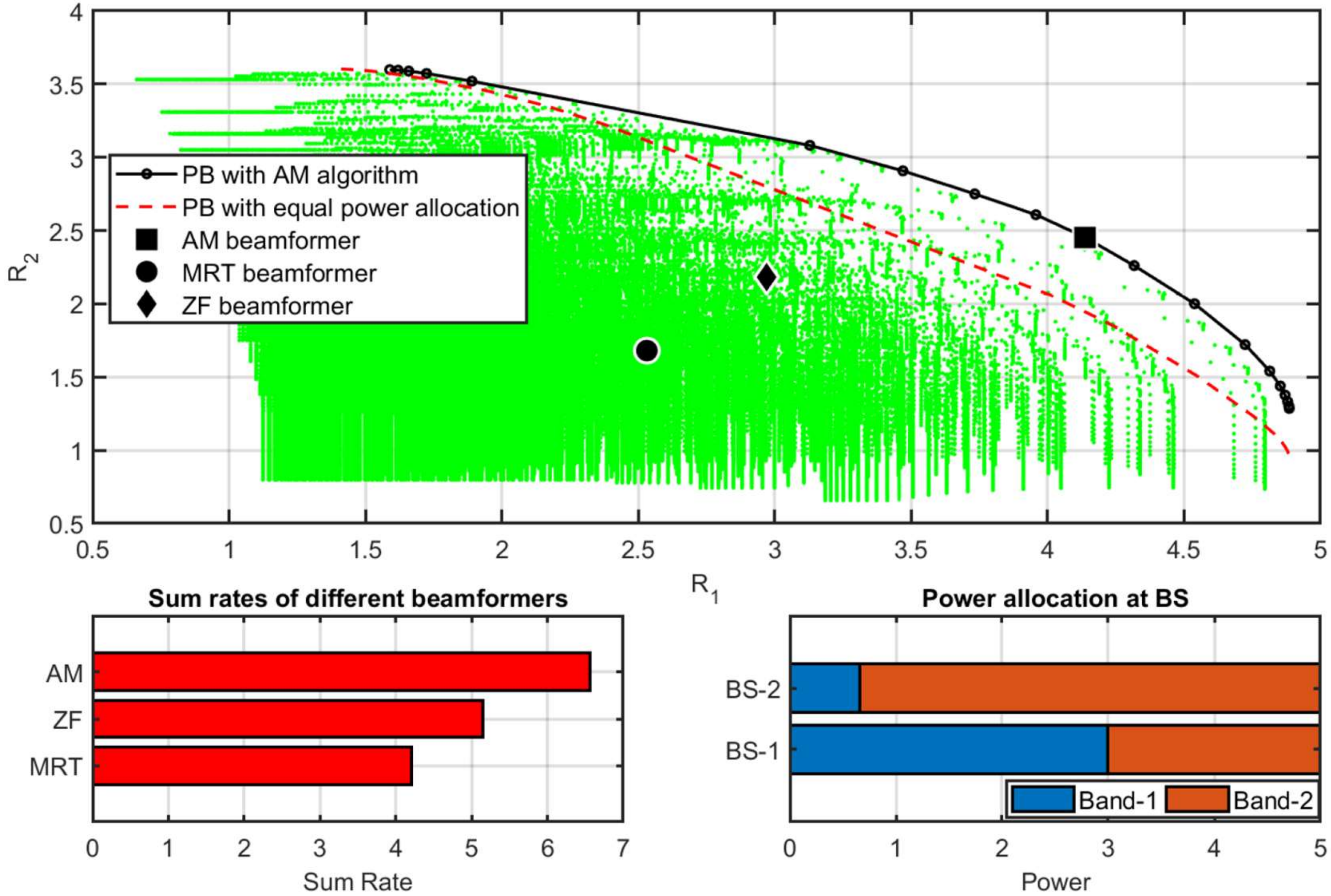
Alternating Maximization

- Power allocation step
  - Bisection method
  - Ellipsoid method

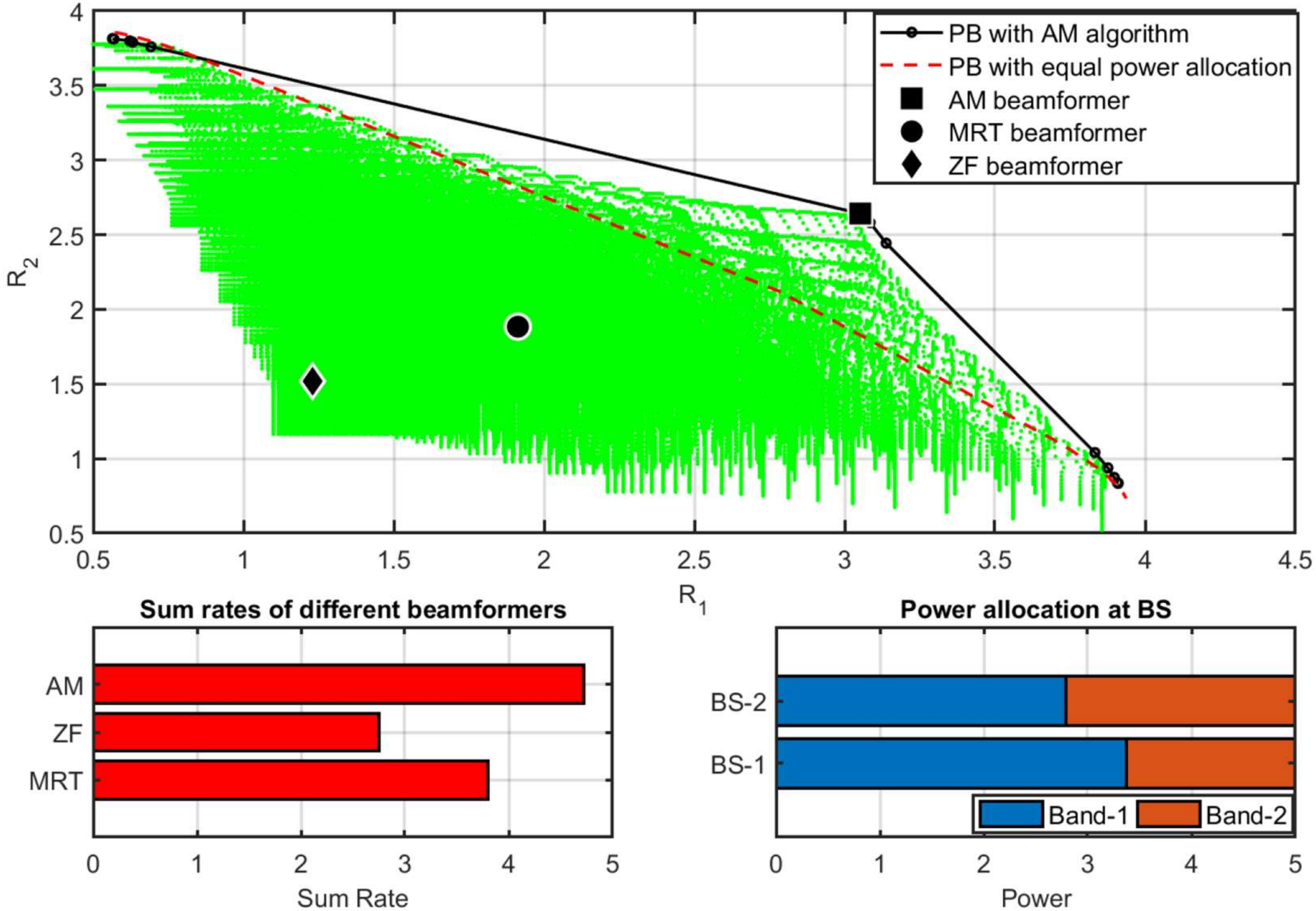


- Each iteration non-decreasing in objective
- Convergence to local maxima possible
- Try multiple initializations and choose the best

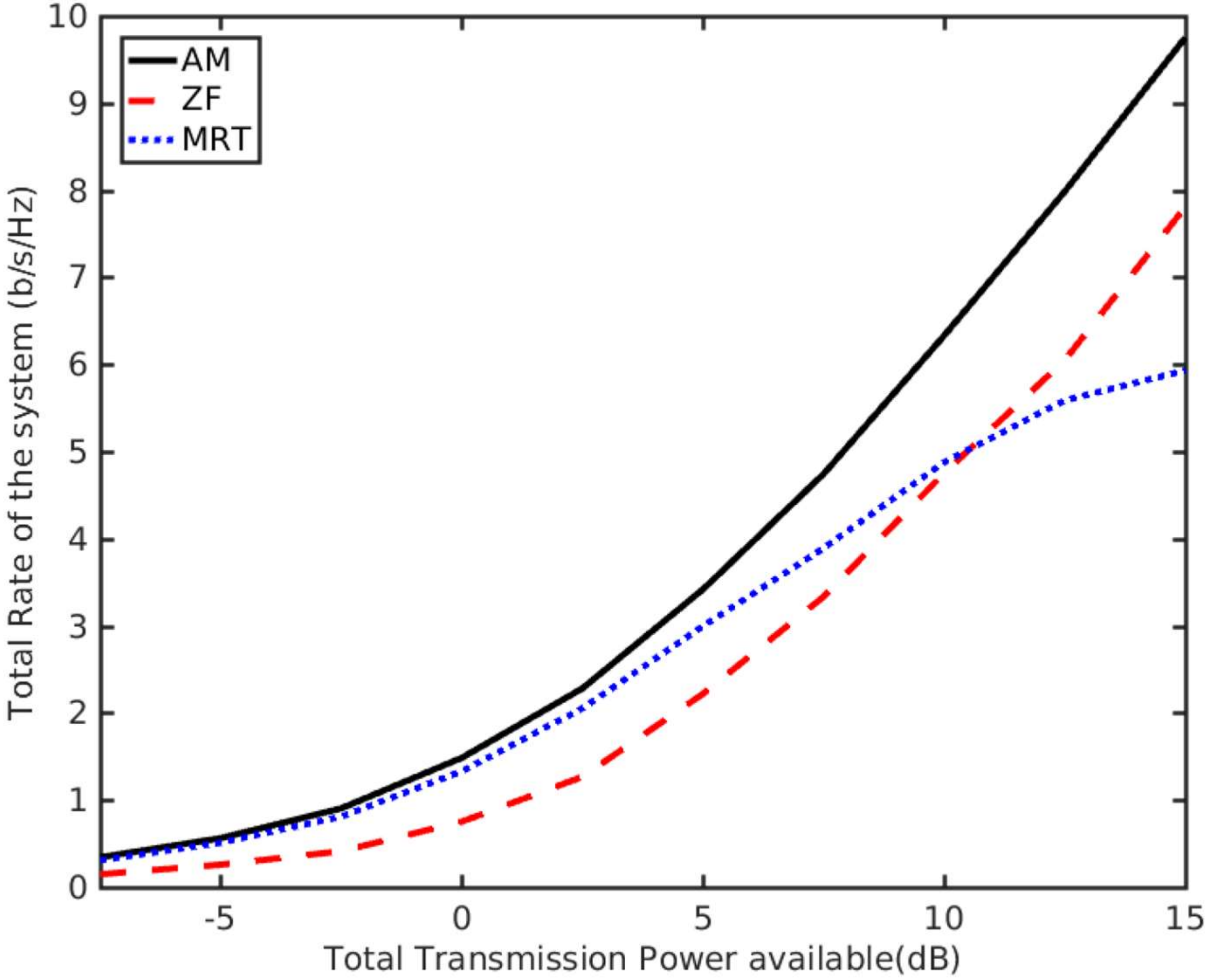
Simulation Results: 2-band



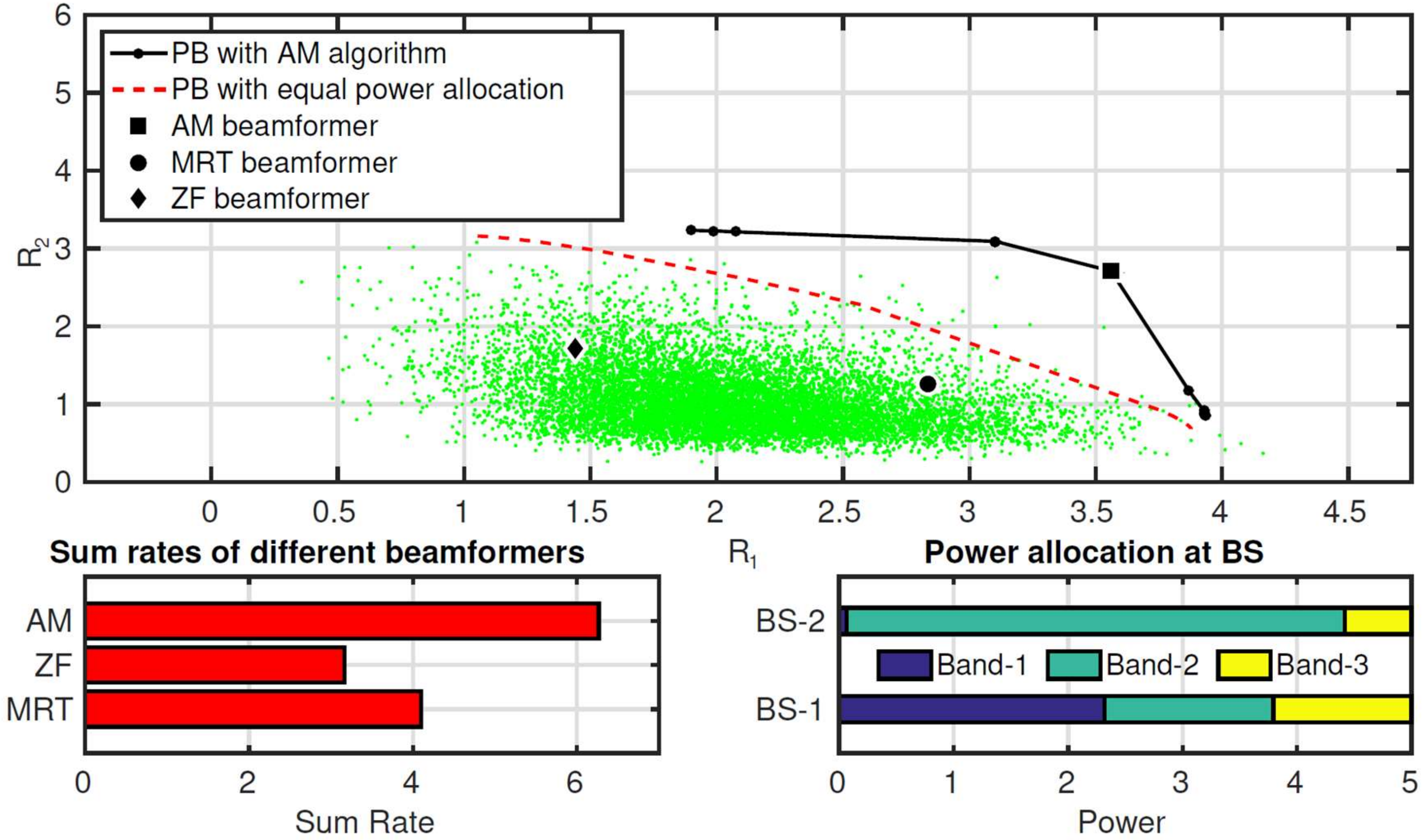
Simulation Results: 2-band



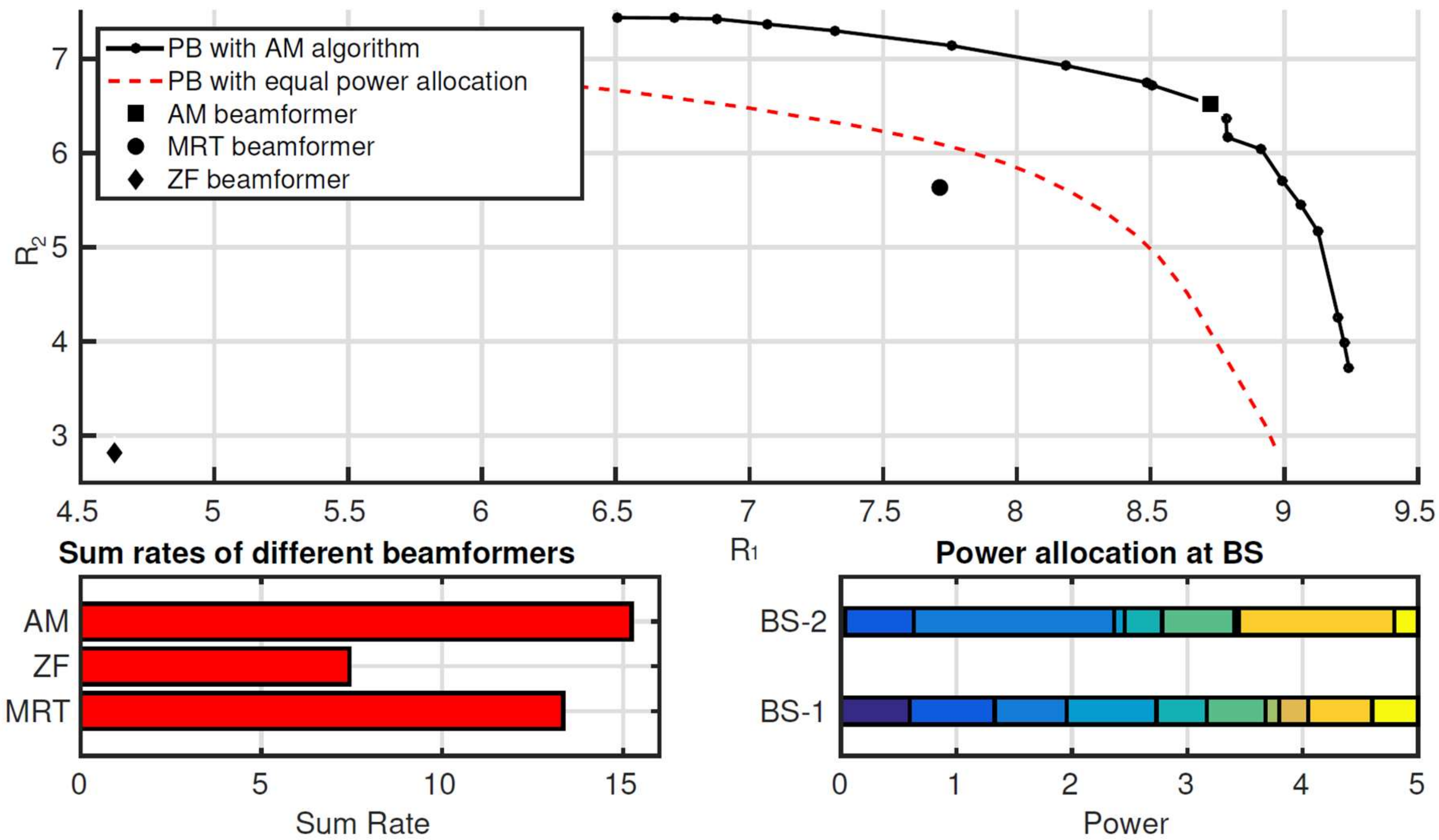
Simulation Results: 2-band



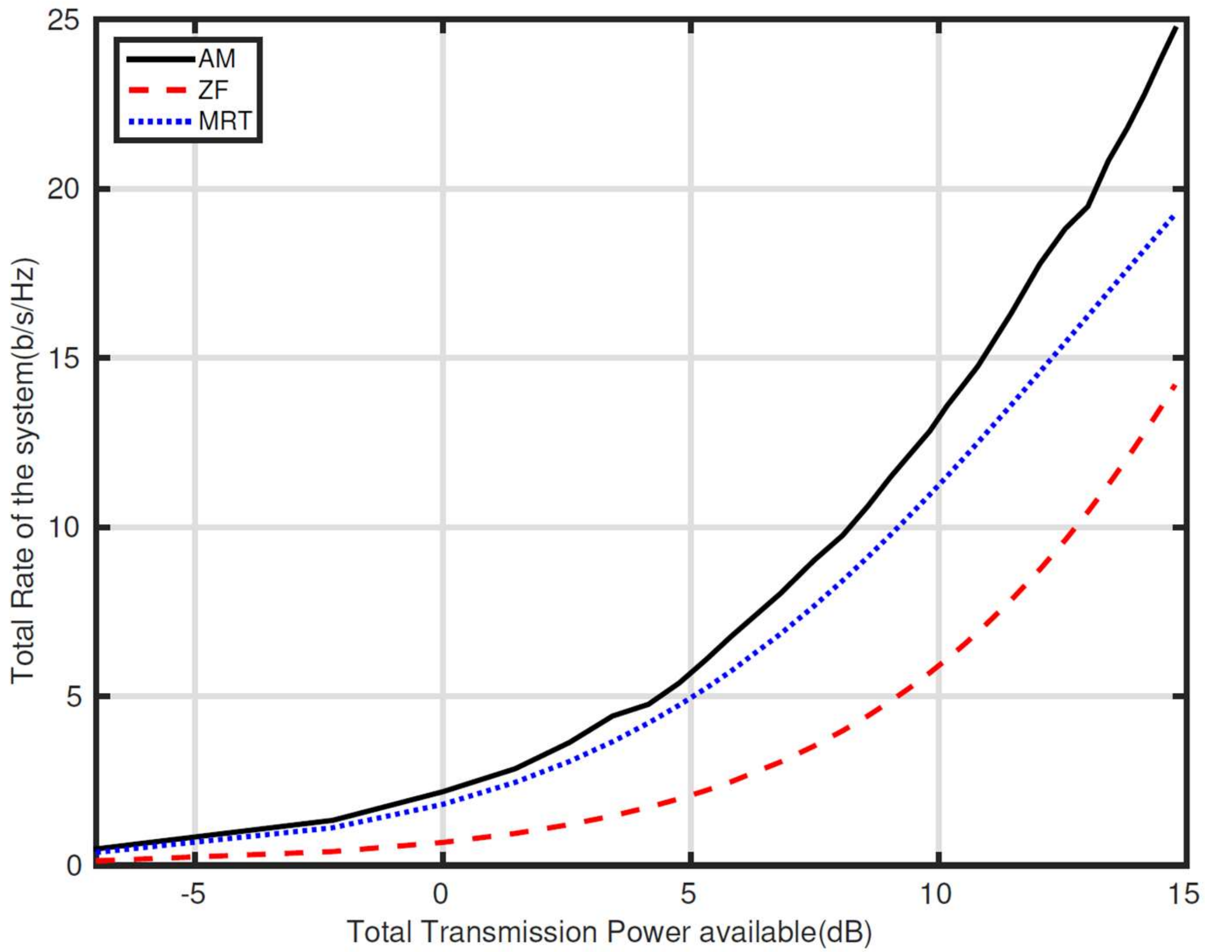
Simulation Results: 3-band



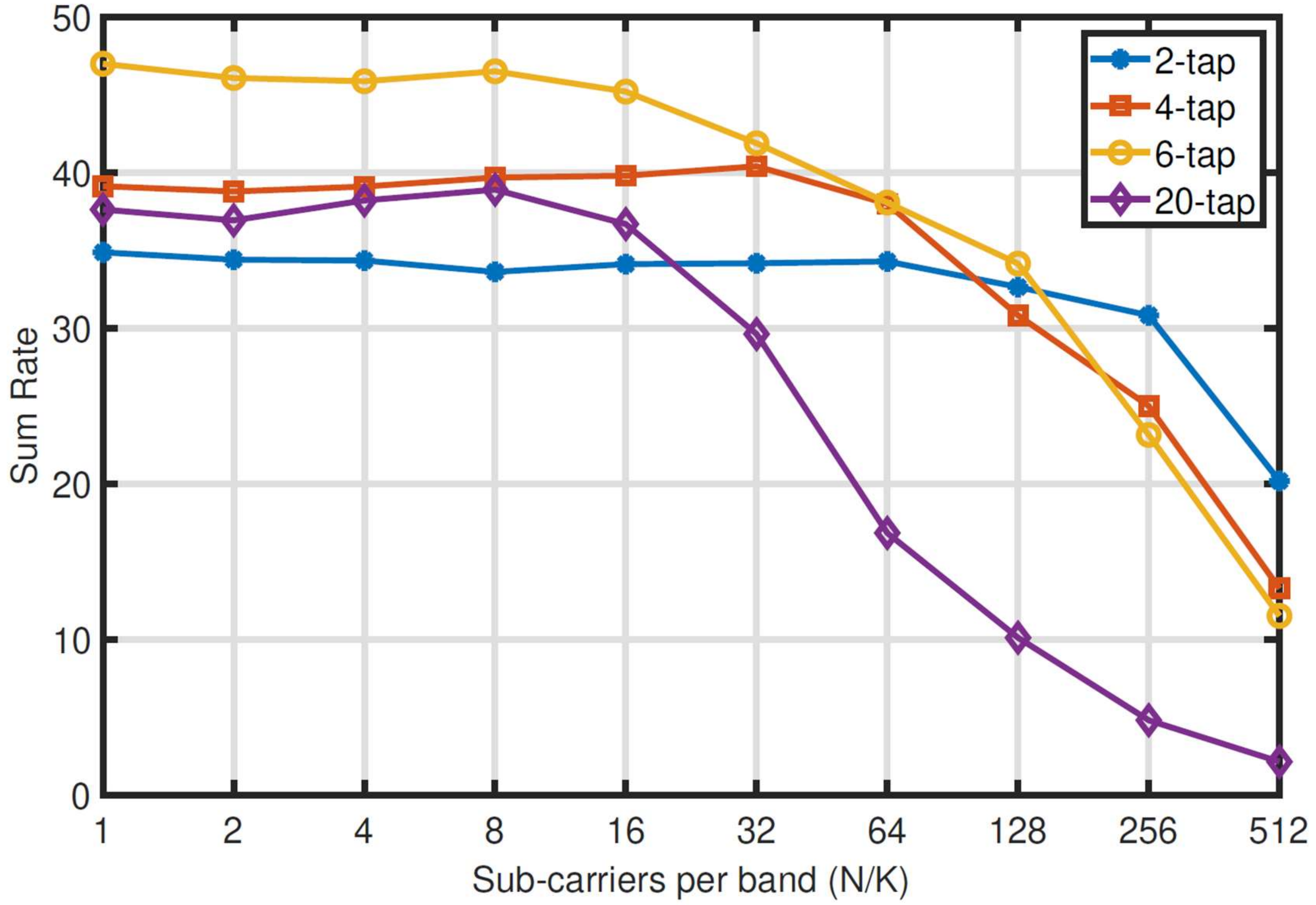
Simulation Results: 10-band



Simulation Results: 10-band



Simulation Results: Frequency selective channel



Summary

- Beamforming for the multicell downlink
  - Single-user detection and Gaussian codebooks

- Distributed solution with limited coordination
  - Single band case:
    - Closed form solution for given interference constraints
    - Gradient ascent for weighted sum rate maximization
  - Multiple band case:
    - Alternating maximization: Power allocation and beamforming
    - Significant gain over equal power allocation, MRT, ZF

- Ongoing: 3-cell coordination closed form solution

https://www.ee.iitm.ac.in/~skrishna/