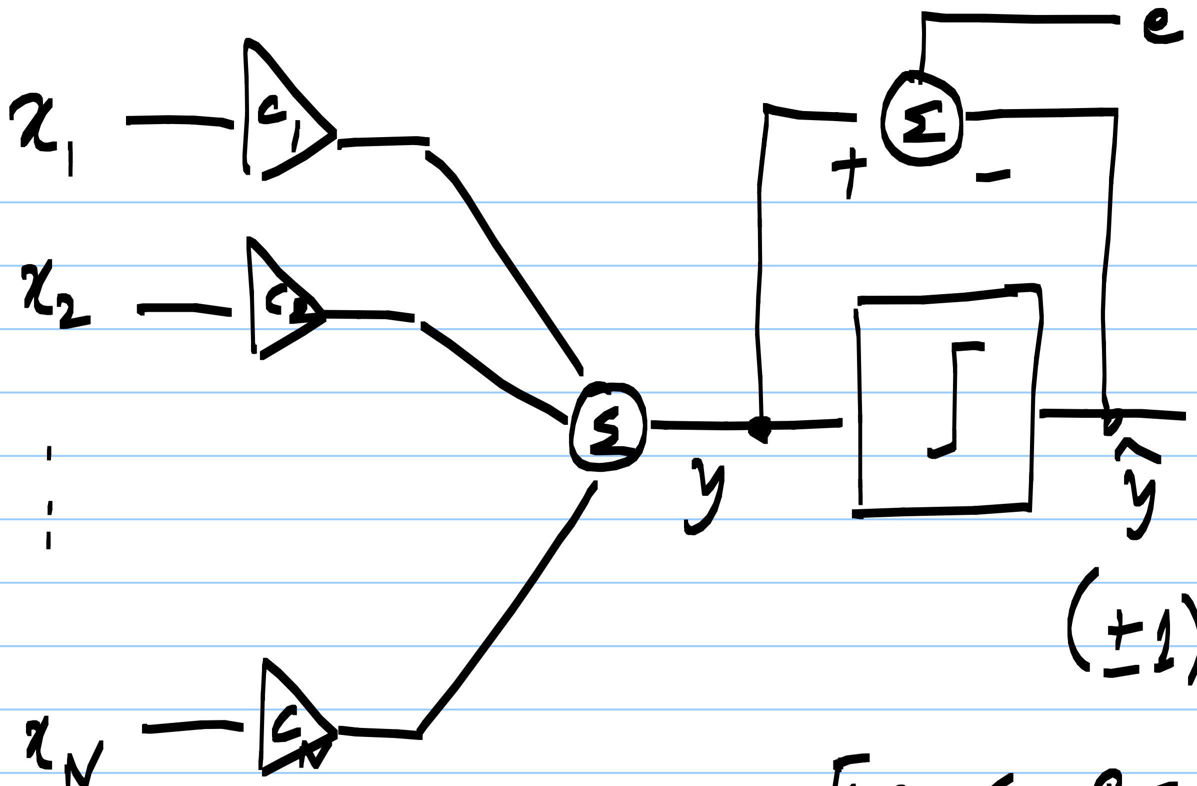


$$y = \sum_{k=1}^{N} c_k \cdot x_k$$

Error $e = y - \hat{y}$

$$\overline{e^2} = \overline{(y - \hat{y})^2}$$

Minimize e^2 wrt c_k $\qquad e = y - \hat{y}$

$$= \sum_k c_k x_k - \hat{y}$$

$$\frac{\partial}{\partial c_k} e^2 = 2e \cdot \boxed{\frac{\partial e}{\partial c_k}}$$

$$= 2 \cdot e \cdot x_k$$

<u>Gradient descent algorithm:</u>

$$c_k[l+1] = c_k[l] - \mu \cdot \frac{\partial}{\partial c_k} e^2$$

$$c_k[l+1] - c_k[l] = -\mu \, e \, x_k$$

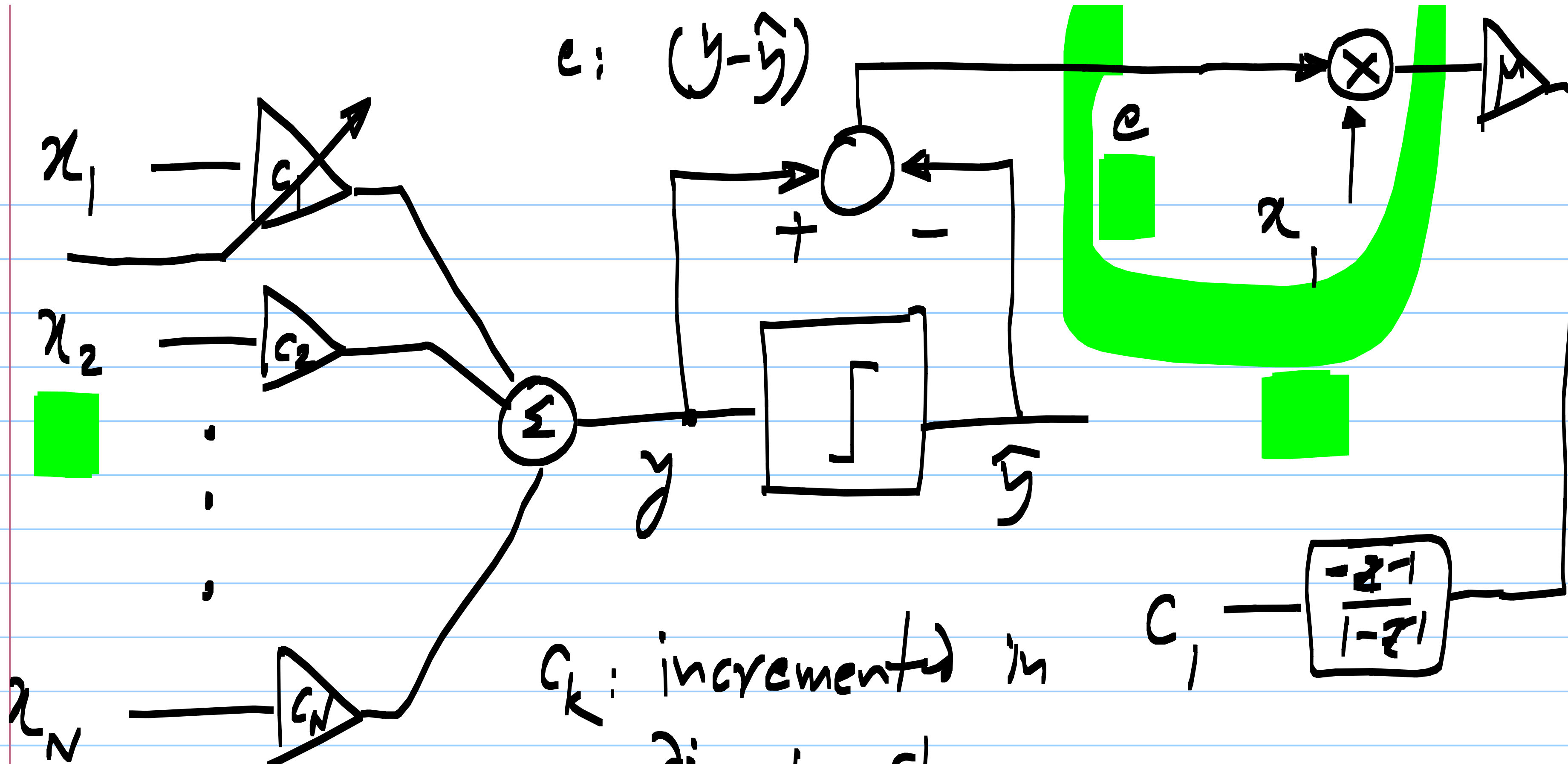
Gradient descent algorithm.

All coefficients c_k

$$\begin{cases} c_k[l+1] = c_k[l] - \hat{\mu} \cdot \overbrace{\frac{\partial}{\partial c_k} e^2}^{2 \cdot e \cdot x_k} \\ \qquad\quad\;\; = c_k[l] - \mu \cdot e \cdot x_k \end{cases}$$

c_k stops changing if $\frac{\partial}{\partial c_k} e^2 = 0$

μ: Controls adaptation rate



c_k: incremented in discrete steps

Difficult to compute $e \cdot x_k$ with a high precision

Gradient-descent LMS algorithm

Coeff. update equation

$$c_k[\ell+1] = c_k[\ell] - \mu \cdot e[\ell] \, x_k[\ell]$$

Sign-sign LMS

$$c_k[\ell+1] = c_k[\ell] - \mu \, \mathrm{sgn}(e) \cdot \mathrm{sgn}(x_k)$$

$$c_k[\ell+1] = c_k[\ell] - \mu \, \mathrm{sgn}(e) \cdot x_k$$

$$c_k[\ell+1] = c_k[\ell] - \mu \cdot e \cdot \mathrm{sgn}(x_k)$$

Sign-sign LMS $\quad c_k[\ell+1] = c_k[\ell] - \mu \, \mathrm{sgn}(e) \cdot \mathrm{sgn}(x_k)$

If $e > 0$ (y is larger than desired)

need to reduce $y = \sum c_k x_k$

$\Rightarrow$ If $x_k > 0$, reduce c_k

$x_k < 0$, increase c_k

$$c_k[l+1] = c_k[l] - \mu \cdot \mathrm{sgn}(e)\ \mathrm{sgn}(x_k)$$

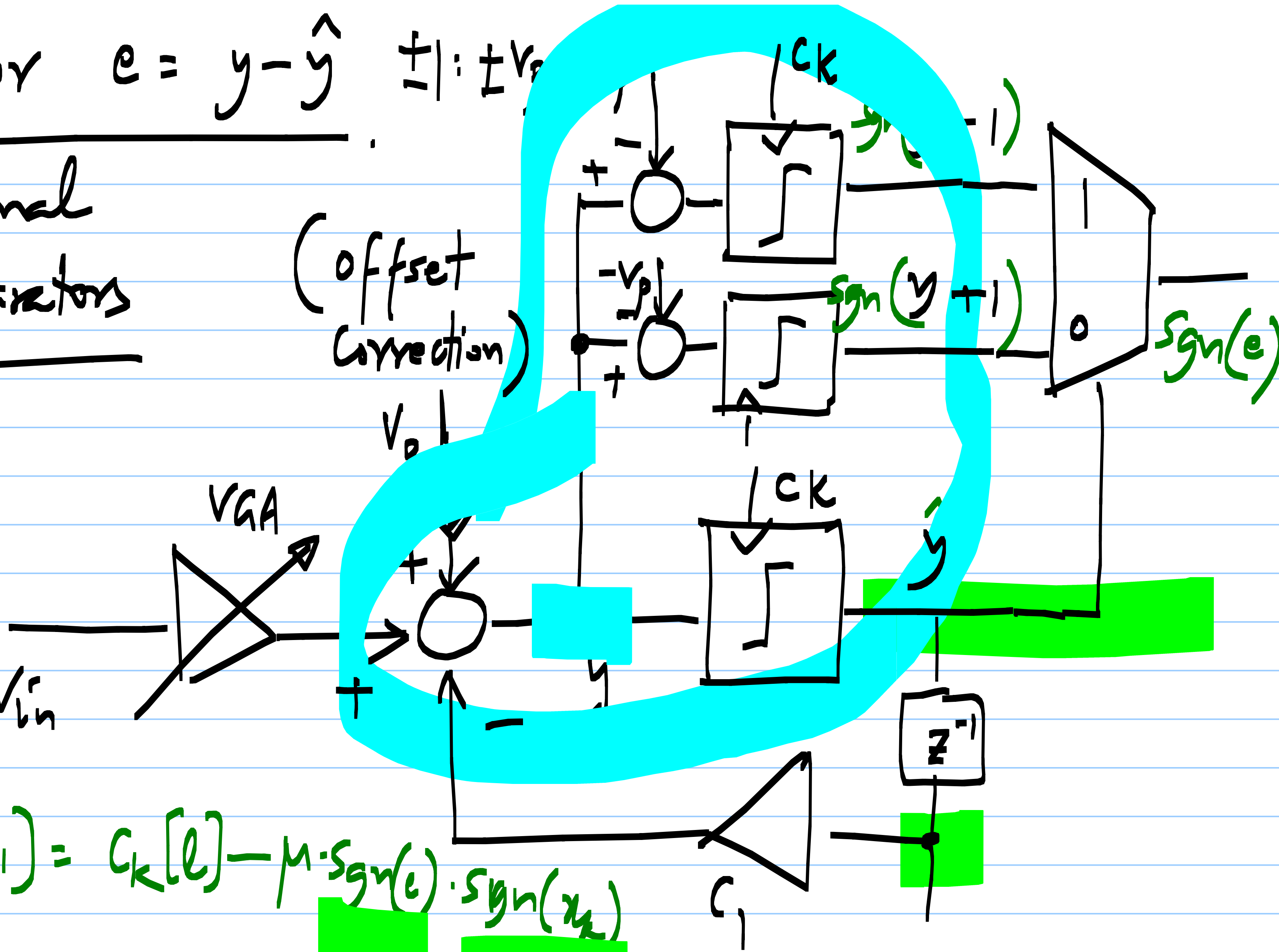
In practice, coefficients are updated once in many symbol intervals

- Coefficients cannot be changed rapidly
- Averge $\mathrm{sgn}(e) \cdot \mathrm{sgn}(x_k)$ over many cycles before update
- μ: likely to be constrained by the step size in c_k

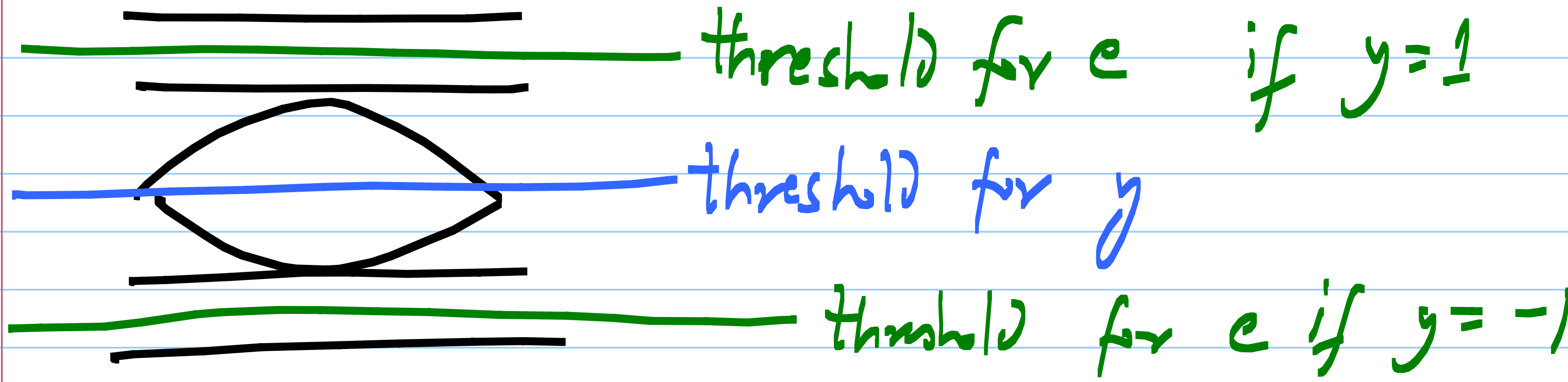
Error $e = y - \hat{y}$ $\pm 1 : \pm V_p$

Additional Comparators

(Offset Correction)



$C_k[l+1] = C_k[l] - \mu \cdot sgn(e) \cdot sgn(x_k)$



Gradients

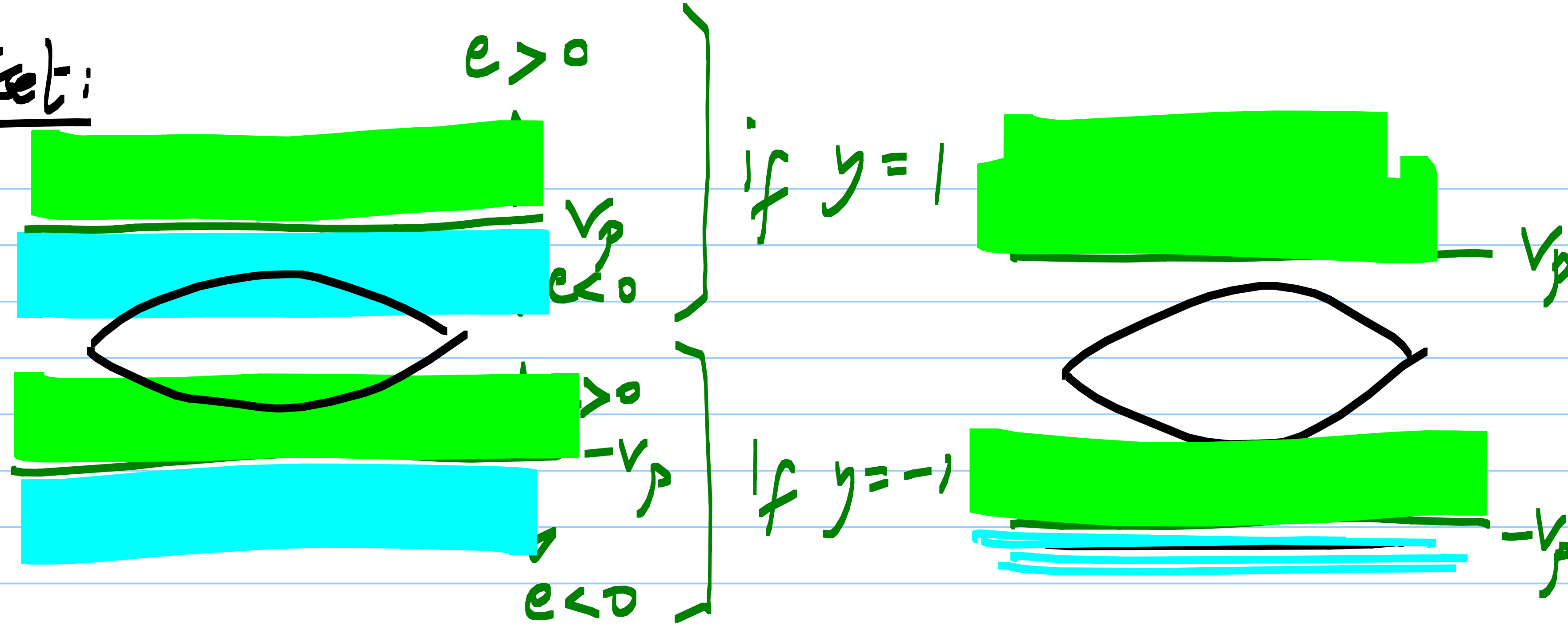
DFE taps: previous decision

Offset tap: 1

$$\boxed{c_{offset}[l+1] = c_{offset}[l] - \mu \cdot sgn(e)}$$

VGA: $sgn(v_{in})$ ~ current decision

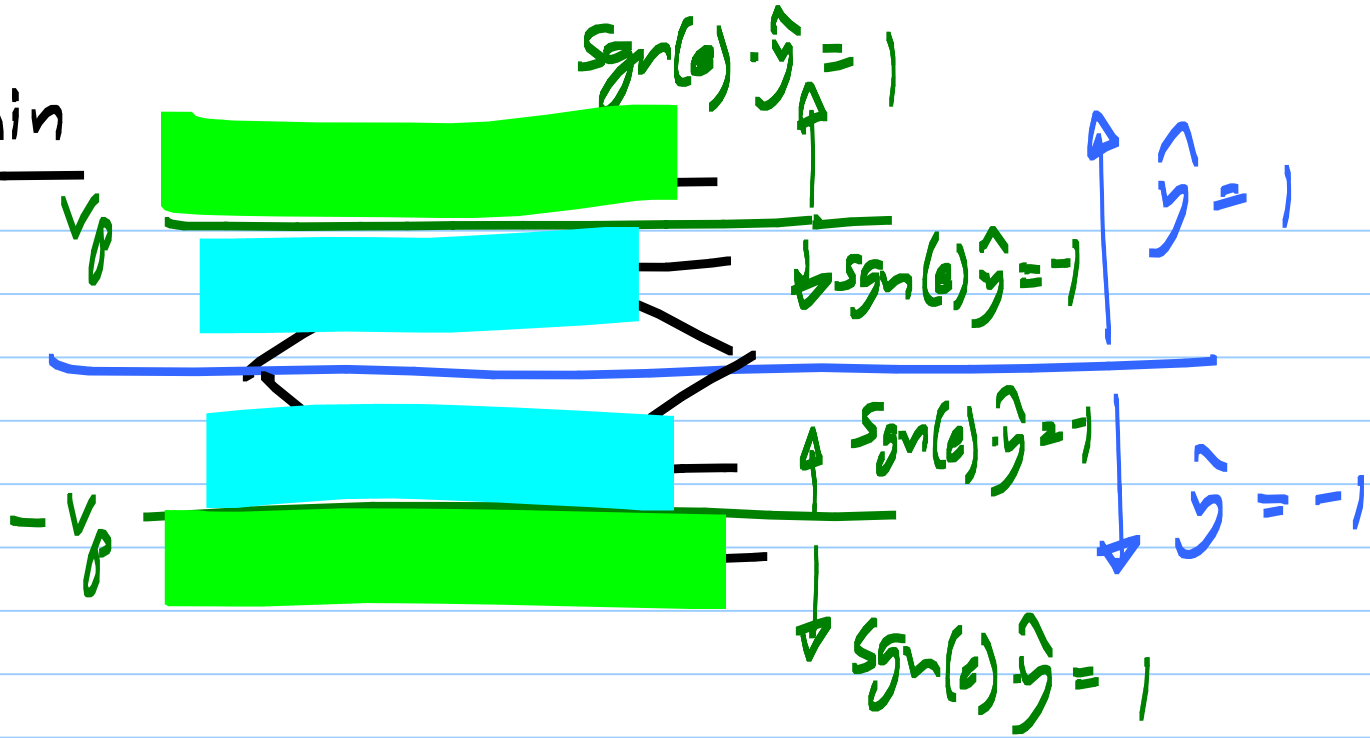
Offset:



$$c_{offset}[l+1] = c_{offset}[l] - \mu \cdot \operatorname{sgn}(e)$$

If average(sgn(e)) = 0, eye levels are centered around $\pm V_p$

Gain



$$c_{VGA}[\ell+1] = c_{VGA}[\ell] - \mu \,\mathrm{sgn}(e)\, \hat{y}$$