Phase domain models

Phase and frequency:

Periodic function: $\overbrace{\underbrace{\cos(\theta)}_{\mathrm{sgn}(\cos(\theta))} \quad p(\theta)}^{\text{functions periodic with a period } 2\pi}$

$\cos(\theta)$: periodic with a period 2π

$$\cos(\theta + n \cdot 2\pi) = \cos(\theta)$$

$\theta = 2\pi f_0 t = \omega_0 t$

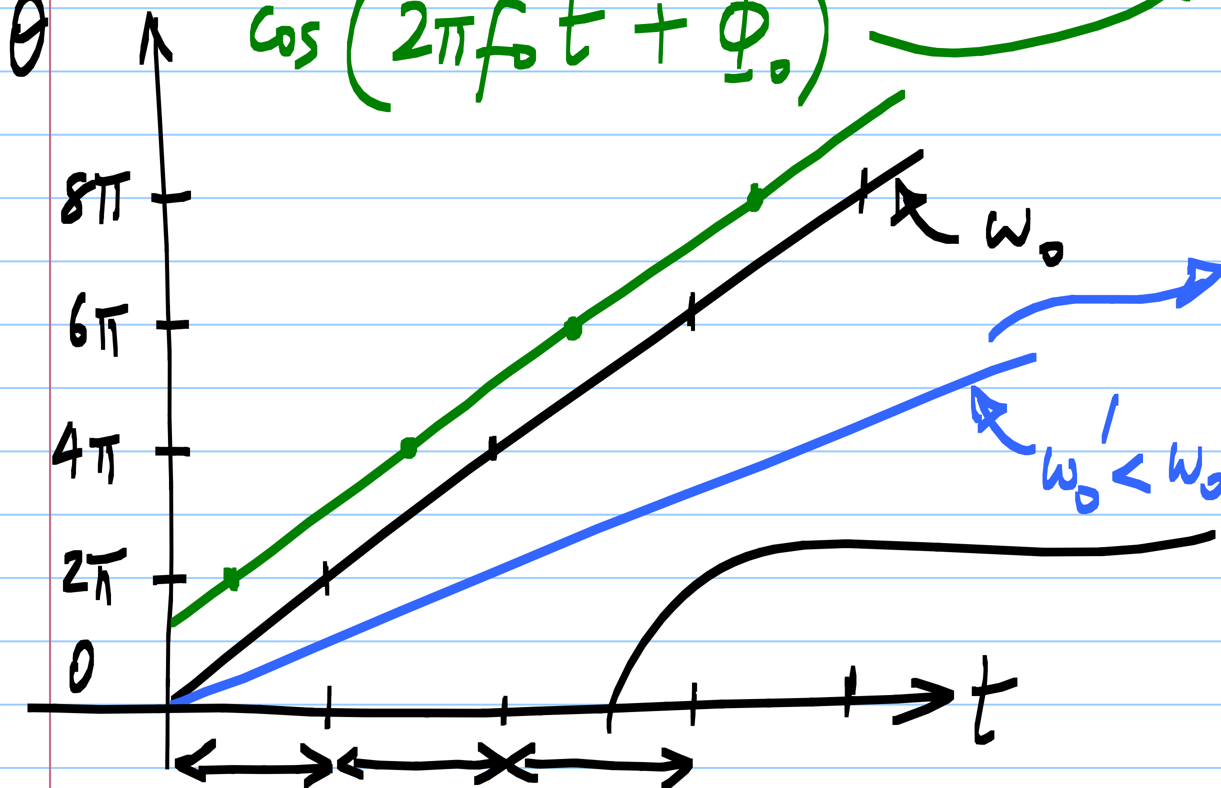
t: time

ω_0: constant

$\cos(\theta) = \cos(2\pi f_0 t)$ periodic in time

period $= \dfrac{2\pi}{\omega_0} = \dfrac{1}{f_0}$

$\cos(2\pi f_0 t + \Phi_0)$



longer time intervals for 2π phase interval

period $= \dfrac{2\pi}{\omega_0} = \dfrac{1}{f_0}$

$$\cos(\theta) = \cos(\underbrace{2\pi f_0}\, t + \Phi_0 + \phi(t))$$

phase noise

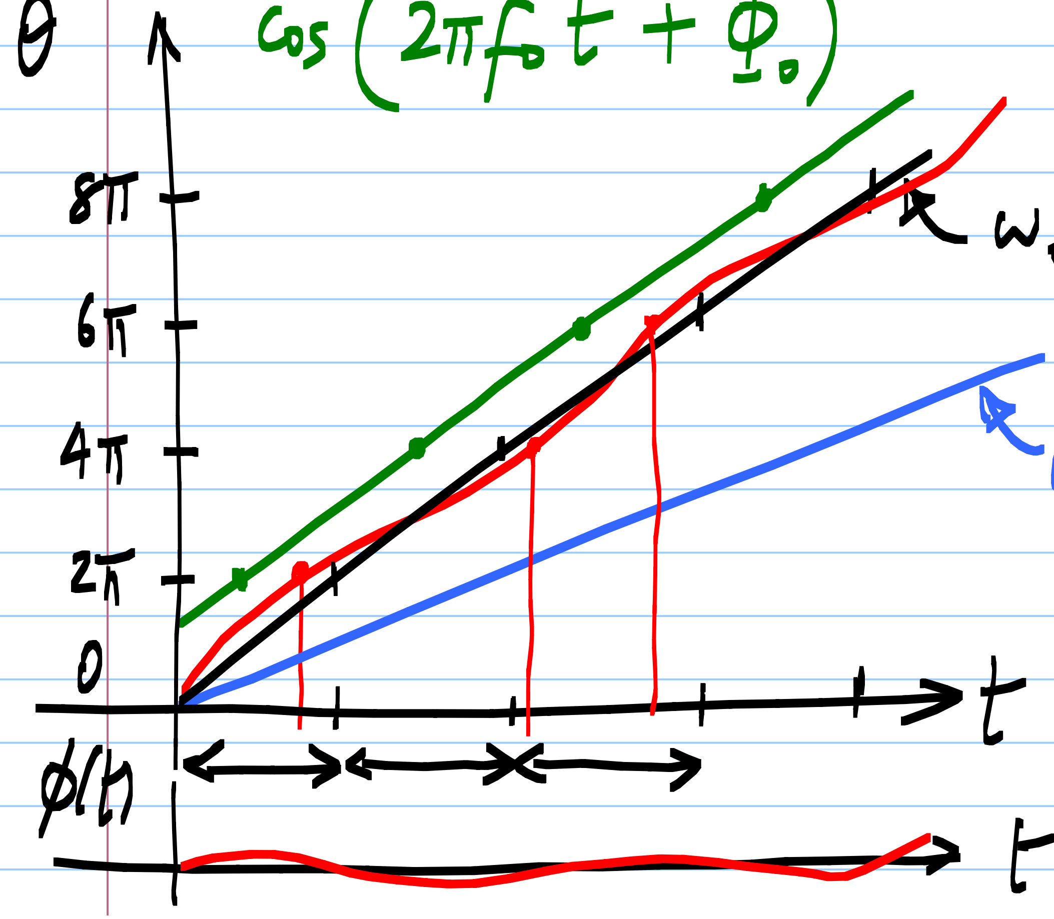
phase deviation

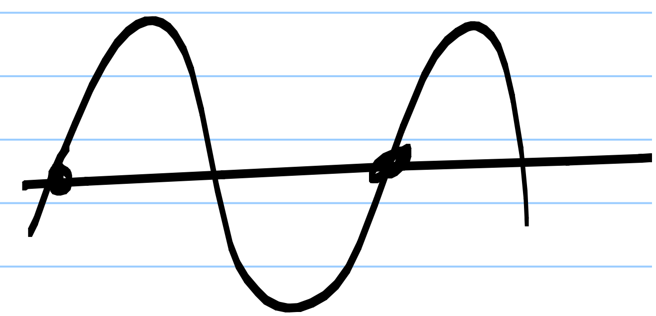
Zero mean

Not periodic in time.

$$\cos(\theta) = \cos(2\pi f_0 t)$$

$$\cos(2\pi f_0 t + \Phi_0)$$





Total phase $\Theta(t) = 2\pi f_0 t + \Phi_0 + \boxed{\phi(t)}$ — phase noise

$$\omega_0 t + \Phi_0 + \underbrace{\phi(t)}_{}$$

radian frequency

$$\frac{d\Theta}{dt} = \omega_0 + \frac{d\phi(t)}{dt} = \omega_0 \quad \text{if } \phi(t) = 0$$

frequency noise

$$\frac{1}{2\pi}\frac{d\Theta}{dt} = \underbrace{f_0 + \boxed{\frac{1}{2\pi}\frac{d\phi}{dt}}}_{\text{Instantaneous frequency}} = f_0 \quad \text{if } \phi(t) = 0$$

cyclic frequency

If $\text{avg}\left(d\phi/dt\right) = 0$; f_0 : average frequency

Measures of deviation from periodicity:

$$\theta(t) = 2\pi f_0 t + \Phi_0 + \phi(t)$$

$\phi(t)$: phase noise $\longrightarrow$ $\overline{\underline{\mathcal{L}(f)}}$

$\frac{1}{2\pi}\frac{d\phi}{dt}$: residual FM / freq. deviation

Period jitter: actual period − ideal period

Absolute jitter: Actual edge positions − ideal edge positions

Cycle-to-cycle jitter: current period − previous period

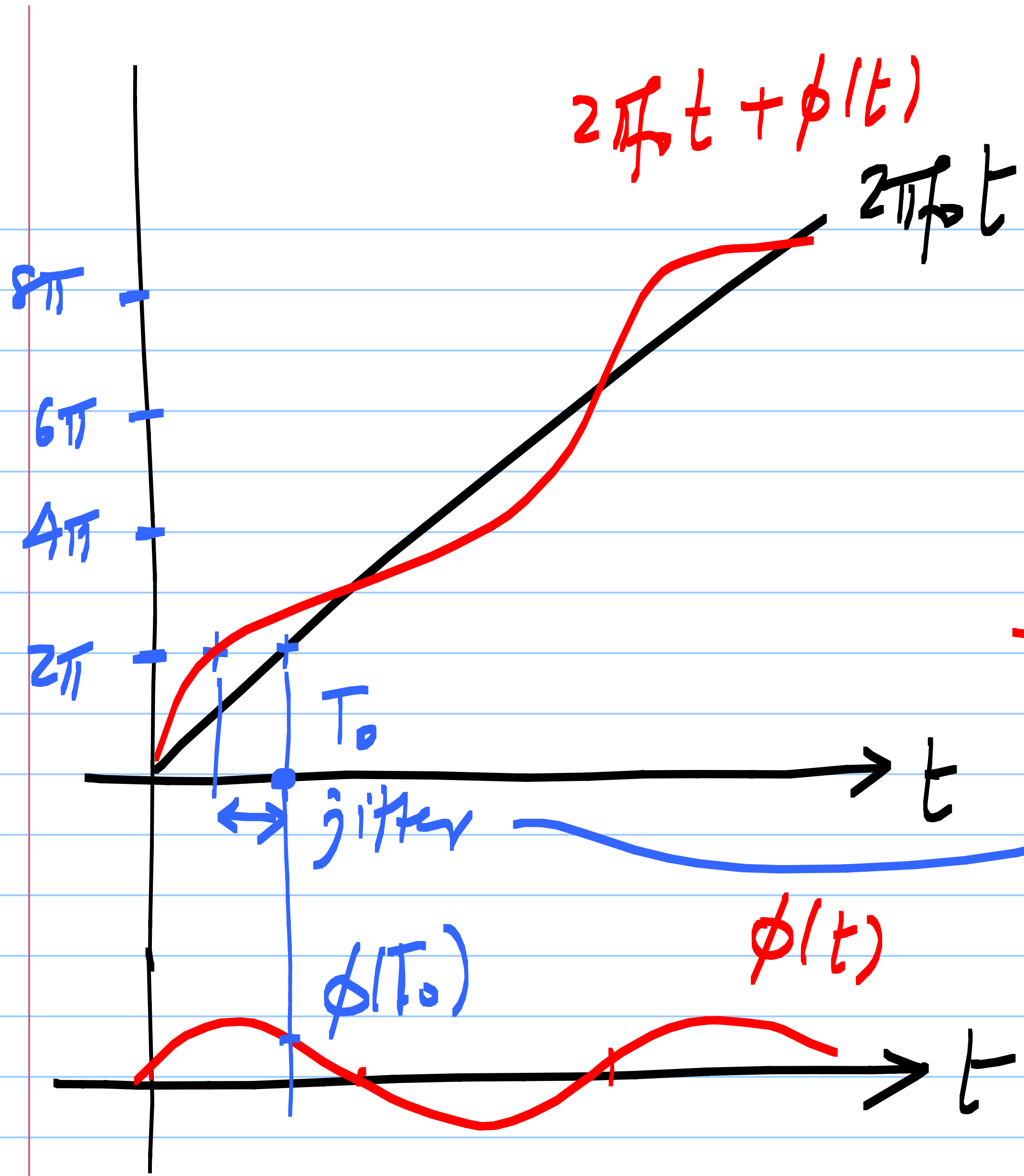
Phase: $\theta(t)$ or $\phi(t)$: commonly represented as continuous-time functions.

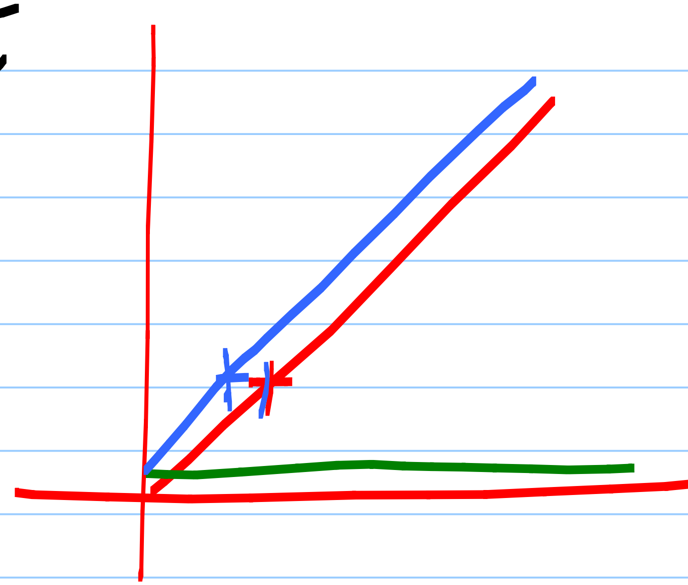
But, can be measured only at the edges — inherently sampled. $\sin(\theta_0(t))$

$$\theta_0(t) = \underbrace{2\pi f_0 t + \Phi_0}_{= k \cdot 2\pi} \quad \text{without jitter}$$

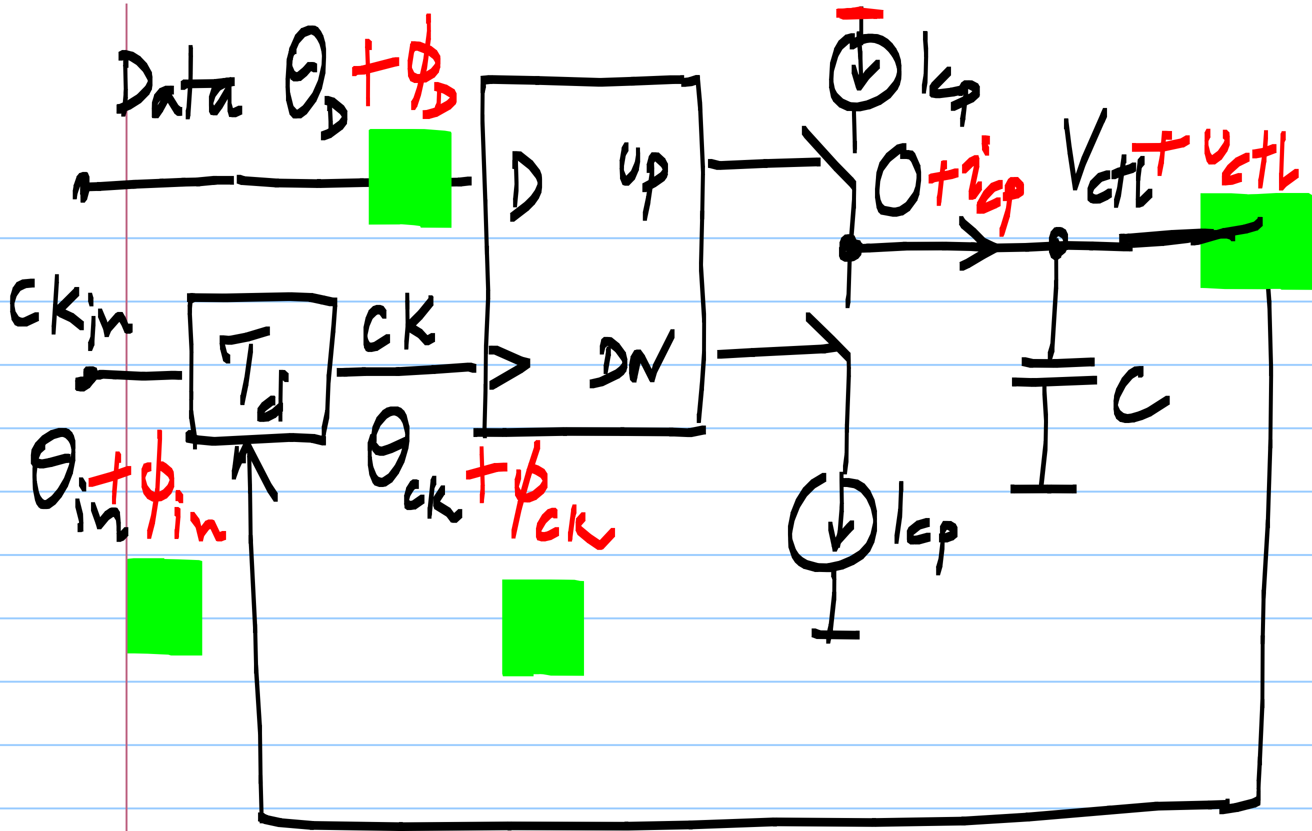
$$\theta(t) = 2\pi f_0 t + \Phi_0 + \phi(t)$$

Jitter $T[k] = \phi(kT_0 + T_{\tau_0}) / 2\pi f_0$

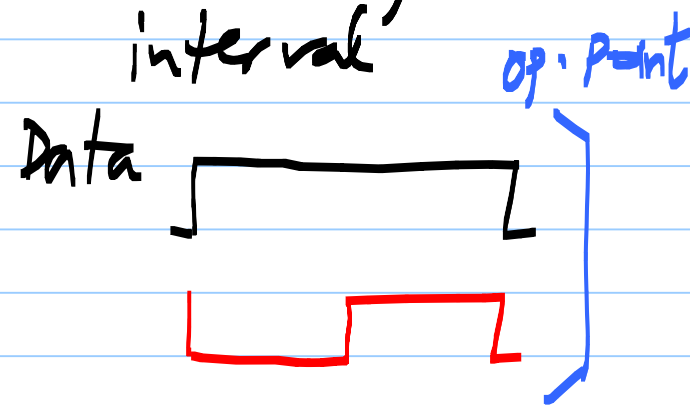




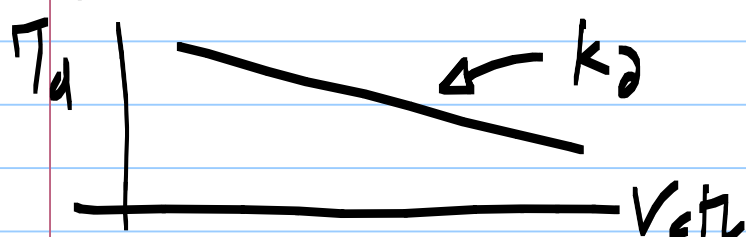
$$= \frac{\phi(T_0)}{2\pi f_0}$$

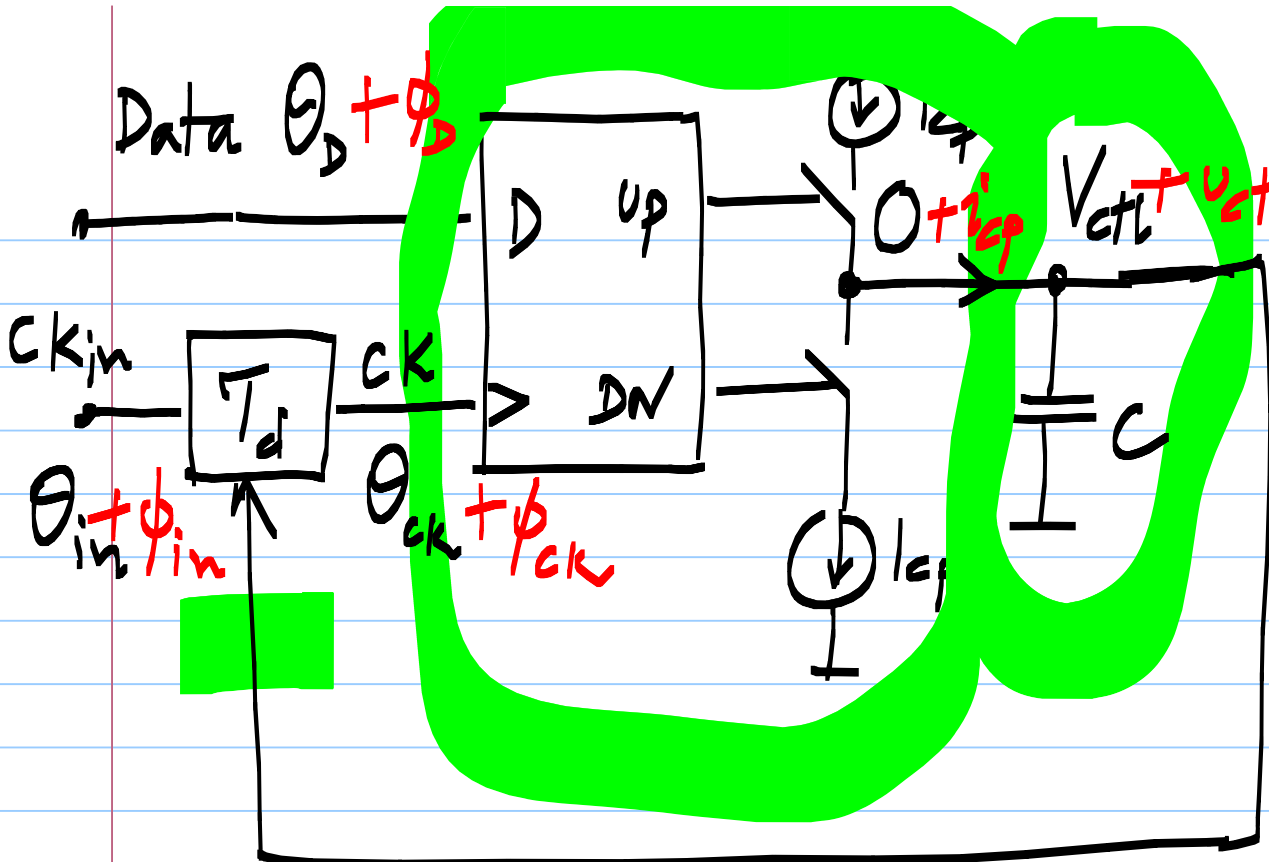


ideal: rising edges of CK in the middle of data symbol interval



$$T_d = T_0 - k_d \cdot V_{ctl}$$



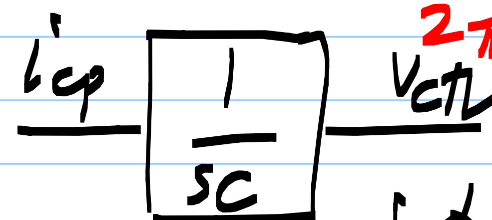


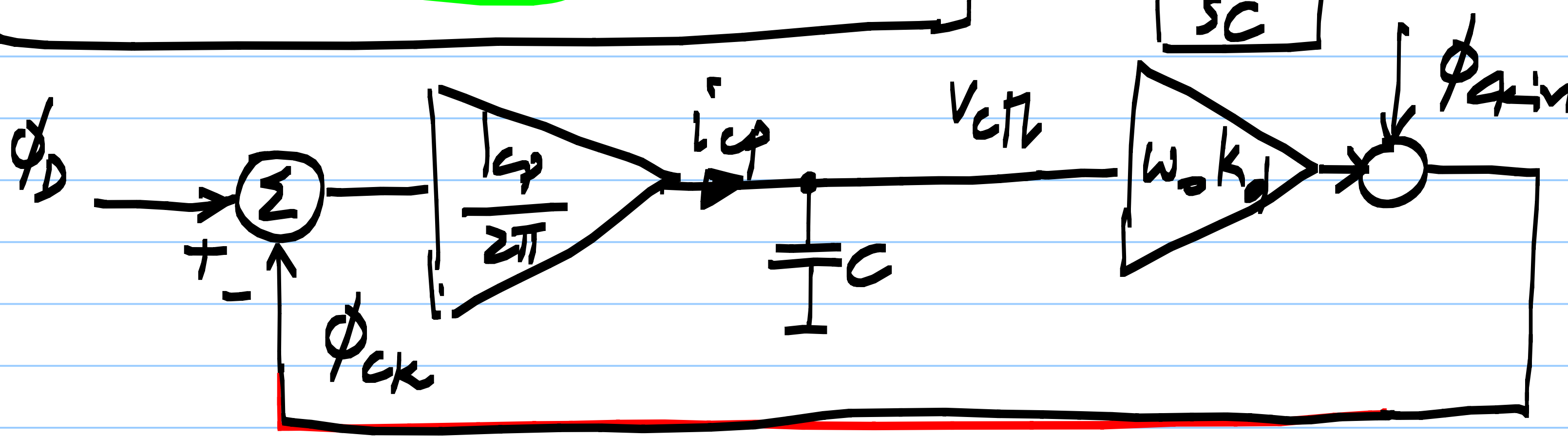
ϕ_D

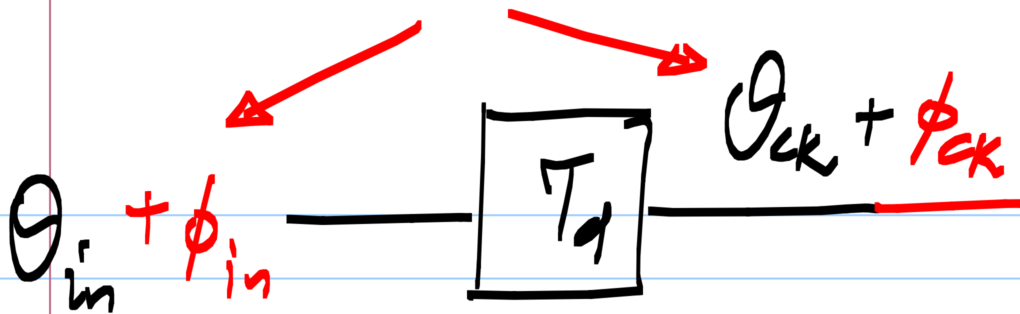
ϕ_{ck}

i_{cp}

$$i_{cp} = I_{cp} \cdot \frac{\phi_D - \phi_{ck}}{2\pi}$$







time delay T_d

phase delay $\omega_o T_d$

$$\theta_{ck} = \theta_{in} - \omega_o \cdot T_d = \theta_{in} - \omega_o\left(T_o - k_d V_{ctl}\right)$$

$$\theta_{ck} + \phi_{ck} = \theta_{in} + \phi_{in} - \omega_o\left(T_o - k_d \cdot V_{ctl} - k_d \cdot v_{ctl}\right)$$

$$\phi_{ck} = \phi_{in} + \omega_o \cdot k_d \cdot v_{ctl}$$