EE2015 — Inverse fourier transform — 6/11/2017

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_\omega(\omega)\exp(j\omega t)\,d\omega$$

$$x(t) = \int_{-\infty}^{\infty} X_f(f)\exp(j2\pi f t)\,df$$

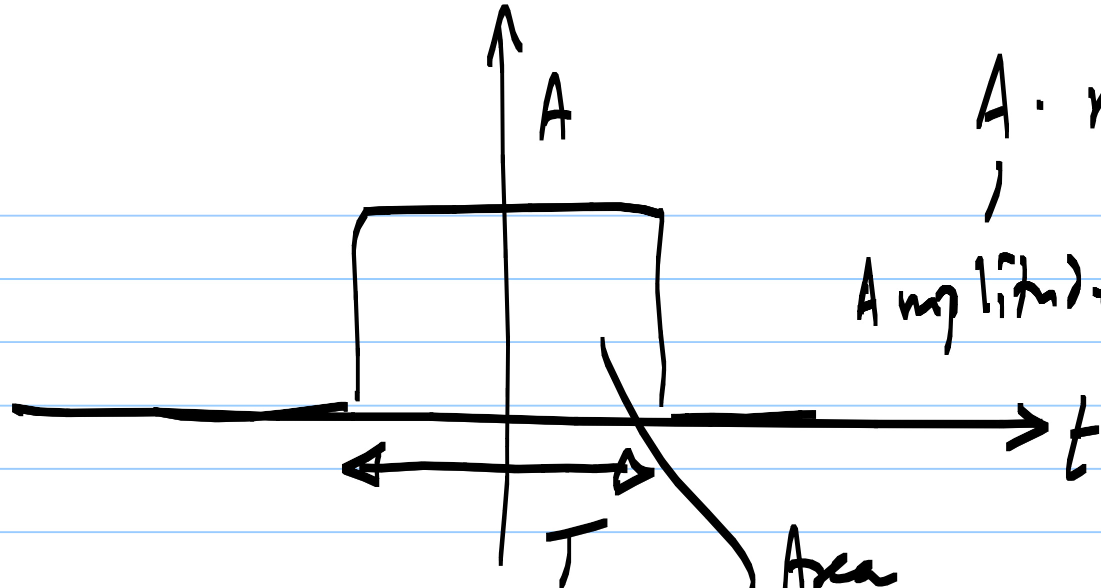
$$x(t) = \frac{1}{j2\pi}\int_{-\infty}^{\infty} X_{j\omega}(j\omega)\exp(j\omega t)\,d(j\omega)$$

$$X_{j\omega}(j\omega) = X_\omega(\omega) = \int_{-\infty}^{\infty} x(t)\, \exp(-j\omega t) \cdot dt$$

$$X_f(f) = \int_{-\infty}^{\infty} x(t)\, \exp(-j2\pi f t)\, dt$$

Fourier transform pairs

$x(t)$	$X_f(f)$	$X_{j\omega}(j\omega)$
1	$\delta(f)$	$2\pi\delta(\omega)$
$\delta(t)$	1	1
$\exp(j\omega_0 t)$	$\delta(f-f_0)$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)$		
$\sin(\omega_0 t)$		
$A\,\mathrm{rect}(t/T)$	$AT\,\mathrm{sinc}(fT)$	$AT\,\mathrm{sinc}\left(\omega T/2\pi\right)$



$$\underbrace{A}_{\text{Amplitude}} \cdot \mathrm{rect}\left(\frac{t}{\underbrace{T}_{\text{width}}}\right)$$

$$\underbrace{(A \cdot T)}_{\text{Area}} \, \mathrm{sinc}\left(f \underbrace{T}_{\text{width}}\right)$$

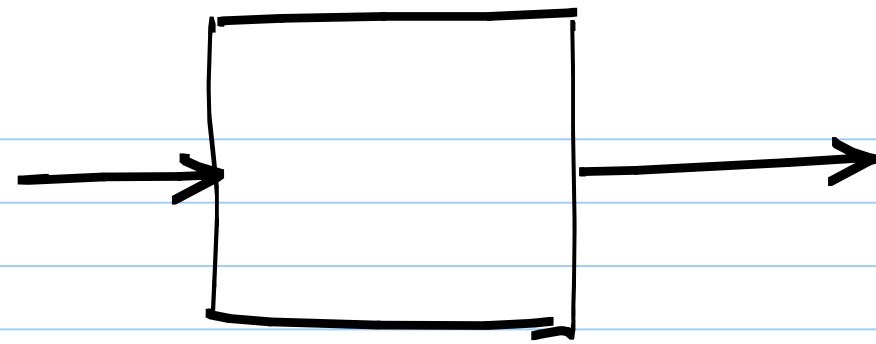
$$\omega T / 2\pi$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Inputs starting @ $t = 0$

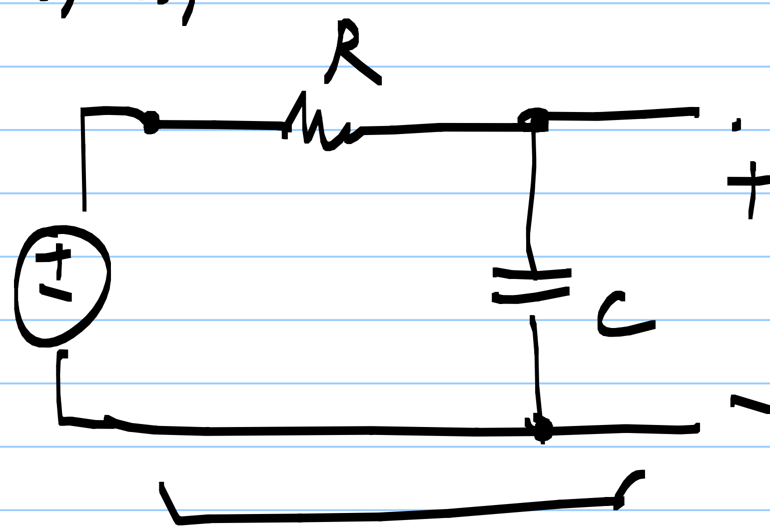
$x(t)$	$X_{j\omega}(j\omega)$
$u(t)$	$1/j\omega + \pi\delta(\omega)$ $2\pi\delta(\omega-\omega_0)$
$\exp(j\omega_0 t)\,u(t)$	$1/j(\omega-\omega_0) + \pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t)\ u(t)$	✓
$\sin(\omega_0 t)\ u(t)$	✓
✓ $\exp(-at)\ u(t)$	$1/(j\omega + a)$ $a > 0$

$\exp(-at)$



Forced

$\exp(-at)\,u(t)$



$$\left(\frac{1}{1-aRC}\right)\exp(-at)$$

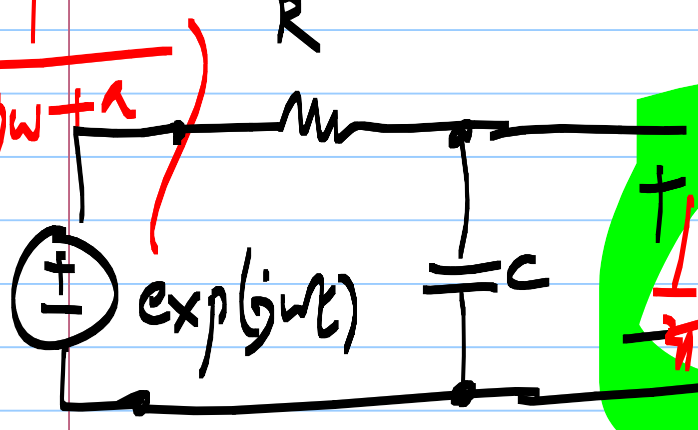
Natural =

$$(\quad)\exp(-t/RC)$$

$$\exp(-at)\,u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{j\omega + a}\exp(j\omega t)\cdot d\omega$$

I.F.T. of

$\frac{1}{2\pi}\frac{1}{j\omega+a}$



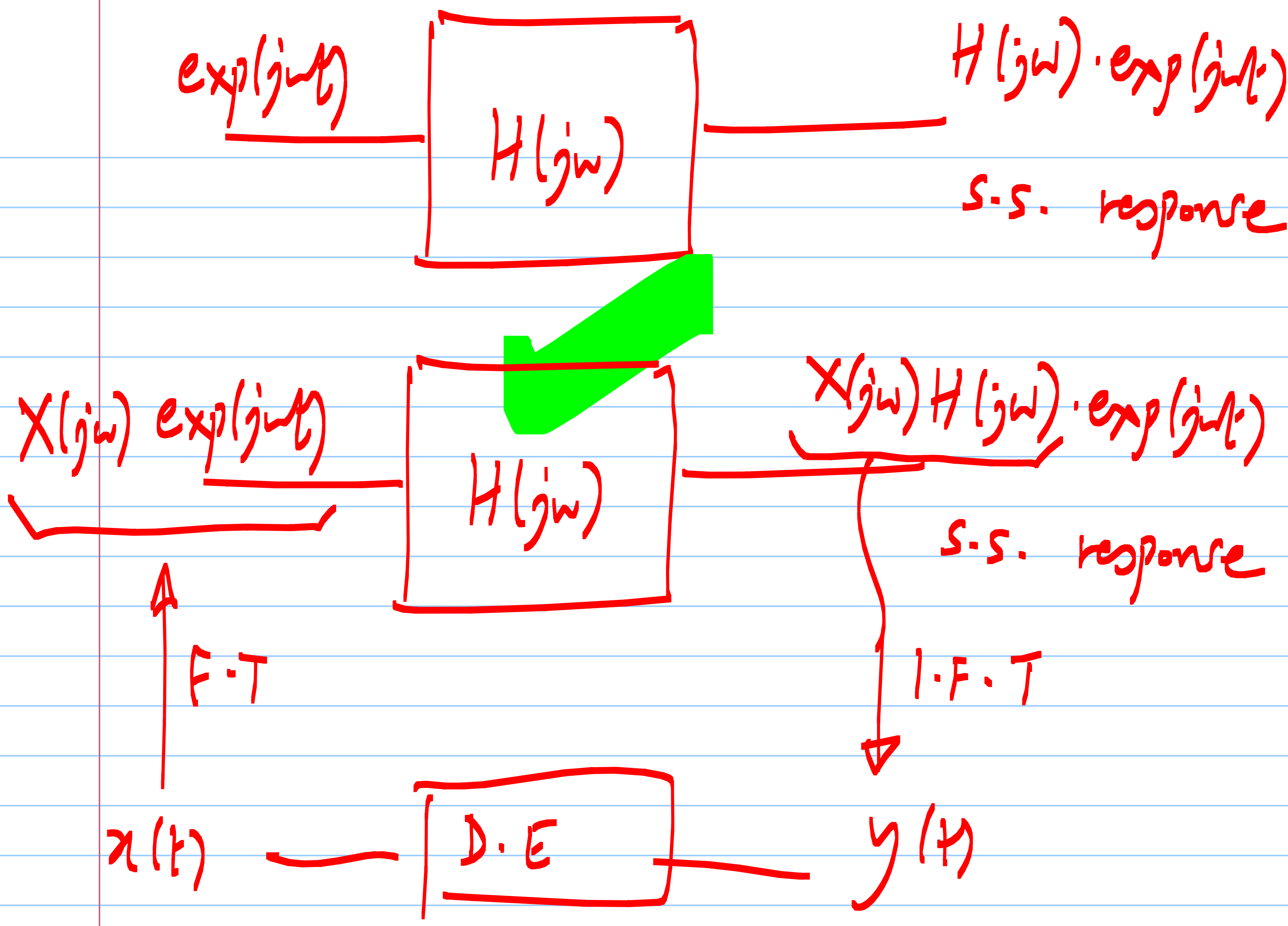
$\frac{1}{j\omega C}$

$+ \quad \frac{1}{2\pi}\frac{1}{j\omega+a}\cdot\frac{1}{1+j\omega RC}\cdot\exp(j\omega t) \quad -$

$Y(j\omega)$

(the o/p y(t)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\boxed{\frac{1}{j\omega+a}\cdot\frac{1}{1+j\omega RC}}\exp(j\omega t)\,d\omega$$


exp(jωt)
H(jω)
H(jω)·exp(jωt)
s.s. response
X(jω) exp(jωt)
H(jω)
X(jω) H(jω)·exp(jωt)
s.s. response
F.T
I.F.T
x(t)
D.E
y(t)

x

$$\exp(-at)\,u(t) \longrightarrow \boxed{RC\frac{dy}{dt} + y = x} \longrightarrow y(t)$$

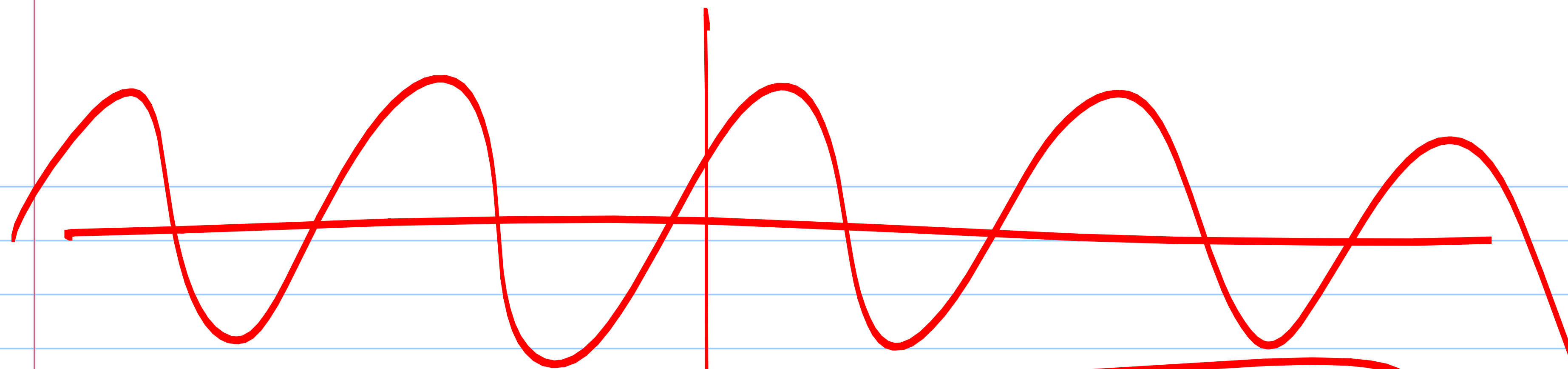
$$\frac{1}{(j\omega + a)} \longrightarrow \boxed{\frac{1}{1 + j\omega CR}} \longrightarrow \underbrace{\frac{1}{(j\omega + a)} \cdot \frac{1}{1 + j\omega CR}}$$

$$\left(\frac{1}{j\omega + a} - \frac{1}{j\omega + 1/CR}\right)\left(\frac{1}{1 - aRC}\right)$$

I.F.T

$$\left(\frac{1}{j\omega + a} - \frac{1}{j\omega + 1/_{CR}}\right)\left(\frac{1}{1 - aRC}\right)$$

$$\frac{1}{1 - aRC}\left(\exp(-at) - \exp(-t/_{RC})\right)u(t)$$



$\exp(j\omega t)$ $\quad -\infty \le t \le \infty$

$t \ge \infty$: s.s response