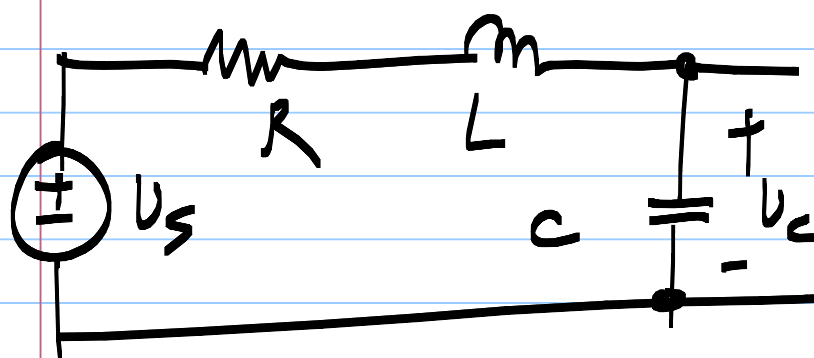
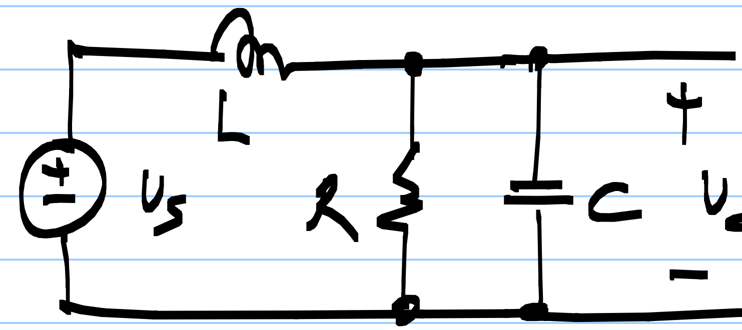
EE2015 **Second order circuits** 10/10/2017





$$LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = v_s$$

$$LC\frac{d^2v_c}{dt^2} + \frac{L}{R}\frac{dv_c}{dt} + v_c = v_s$$

Natural response:

$$LC \cdot \frac{d^2 v_c}{dt^2} + RC \frac{dv_c}{dt} + v_c = 0$$

$$v_c = \underbrace{A \cdot \exp(pt)}$$

$$\left(\underbrace{LC} \cdot p^2 + \underbrace{RC} \cdot p + 1 \right) = 0$$

$$p_{1,2} = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2 \cdot LC}$$

$$LC \cdot \frac{d^2 v_c}{dt^2} + \frac{L}{R} \cdot \frac{dv_c}{dt} + v_c = 0$$

$$LC \cdot p^2 + \frac{L}{R} \cdot p + 1 = 0$$

$$p_{1,2} = \frac{-\frac{L}{R} \pm \sqrt{\frac{L^2}{R^2} - 4LC}}{2 \cdot LC}$$

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$p_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Distinct real roots if:

$\overbrace{\exp(p_1 t),\ \exp(p_2 t)}$ Natural modes

$p_1, p_2 < 0$

stable

$$\frac{1}{R^2} \cdot \frac{L}{C} < \frac{1}{4} \qquad R^2 \cdot \frac{C}{L} < \frac{1}{4}$$

$\zeta > 1$, $Q < 1/2$

overdamped case

$\overbrace{\exp(p_1 t),\ t\exp(p_1 t)}$ Natural modes

Identical real roots if:

$$\frac{1}{R^2} \frac{L}{C} = \frac{1}{4} \qquad R^2 \cdot \frac{C}{L} = \frac{1}{4}$$

$\zeta = 1$, $Q = 1/2$

(critically damped)

$$p_{1,2} = -\frac{R}{2L} \qquad p_{1,2} = -\frac{1}{2RC}$$

Complex conjugate

$$\frac{1}{R^2} \cdot \frac{L}{C} > \frac{1}{4}$$

Series RLC

$$P_1, P_2 = -\text{[illegible]} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$P_1, P_2 = \sigma + j\omega$$

$$\underbrace{A \cdot \exp(\sigma + j\omega \cdot t) + A^* \exp(\sigma - j\omega \cdot t)}_{2 \, Re\left(A \exp(\sigma + j\omega \cdot t)\right)}$$

$$2 \cdot |A| \cdot \text{[illegible]} \cdot \cos(\omega t + \measuredangle A)$$

Parallel RLC

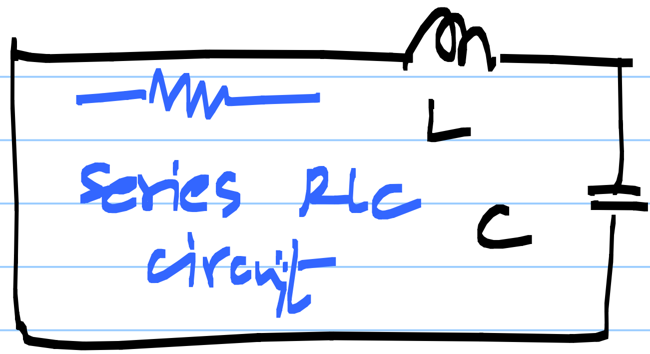
$$R^2 \cdot \frac{C}{L} > \frac{1}{4}$$

$$P_1, P_2 = -\frac{1}{2RC} \pm j\sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$\zeta < 1, \quad R > 1/2$$

underdamped

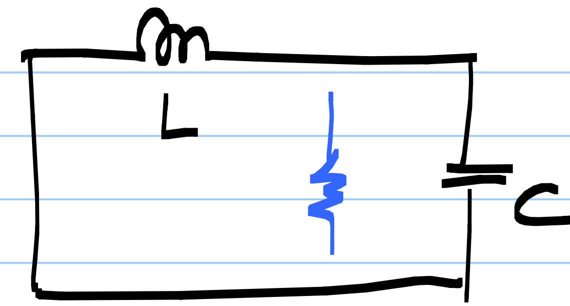
$\sigma = 0$ for $R = 0$



Lossless LC circuit
(Resonator)

Natural mode: $\sim \cos\left(\frac{t}{\sqrt{LC}} + \phi\right)$

$\alpha = 0$ for $R = \infty$



parallel RLC
circuit

$\cos\left(\frac{t}{\sqrt{LC}} + \phi\right)$

Series RLC ω_n, ζ, Q

char. equation:

$$LC \cdot p^2 + RC \cdot p + 1 = 0$$

$$\frac{p^2}{\omega_n^2} + [illegible] \cdot p + 1 = 0$$

$$\frac{p^2}{\omega_n^2} + [illegible] \cdot p + 1$$

$$p^2 + \frac{R}{L} \cdot p + \frac{1}{LC} = 0$$

$$p^2 + 2\zeta\omega_n \cdot p + \omega_n^2 = 0$$

$$p^2 + \frac{\omega_n}{Q} \cdot p + \omega_n^2 = 0$$

parallel RLC

$$LC \cdot p^2 + \frac{L}{R} \cdot p + 1 = 0$$

$$p^2 + \frac{1}{RC} \cdot p + \frac{1}{LC} = 0$$

Series RLC

Natural frequency

$$\omega_n = \frac{1}{\sqrt{LC}}$$

Quality factor $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Damping factor $\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$

Complex conj. roots:

$Q > \frac{1}{2}$; $\zeta < 1$

Parallel RLC

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$Q = \frac{1}{2\zeta}$

$Q = R\sqrt{\frac{C}{L}}$

$\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}}$

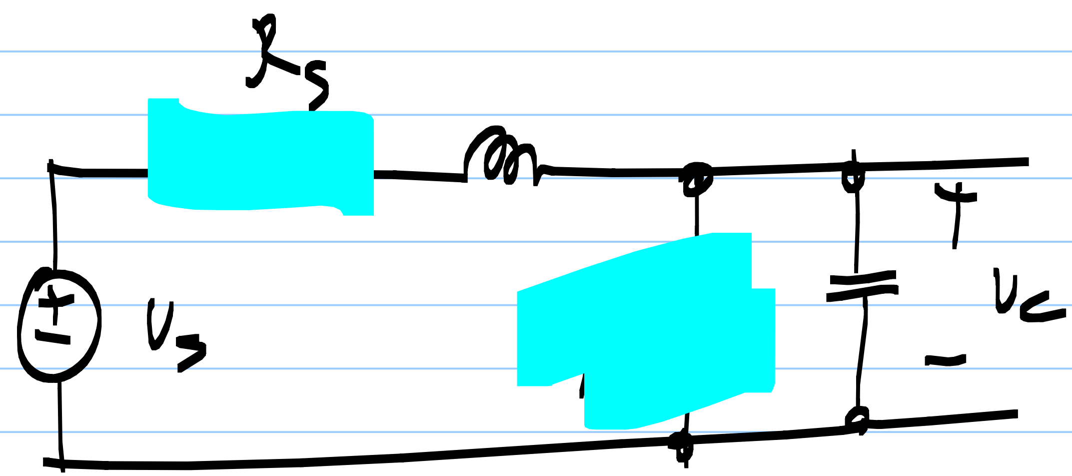
$\zeta = \frac{1}{2Q}$

Complex conj. roots:

$Q > \frac{1}{2}$; $\zeta < 1$

$Q = \infty, \ \zeta = 0$: undamped system

Ideal resonator



$Q < 1/2,\ \zeta > 1$

overdamped

$Q = 1/2,\ \zeta = 1$

crit. damped

$Q > 1/2,\ \zeta < 1$

underdamped

$Q = \infty,\ \zeta = 0$

undamped