EE 2015

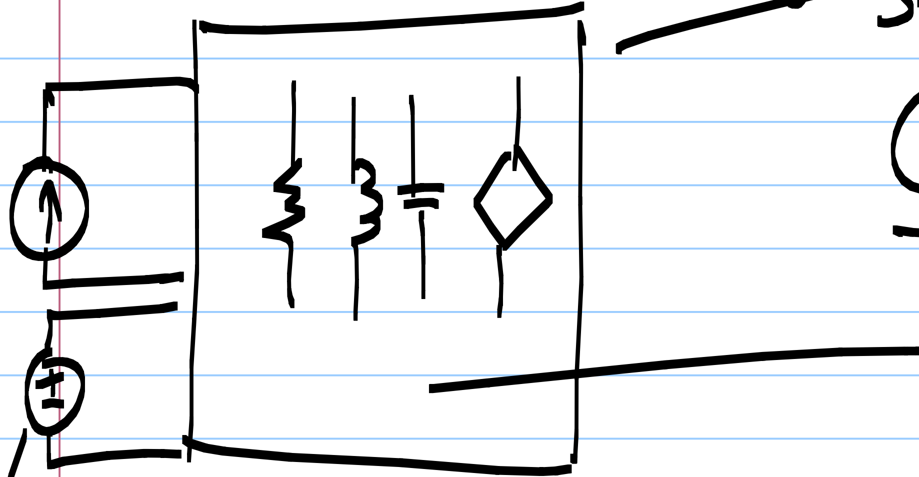
3/10/2017

Phasor:

~~Complex number multiplying $\exp(j\omega t)$~~

Sinusoidal steady state analysis

@ frequency ω



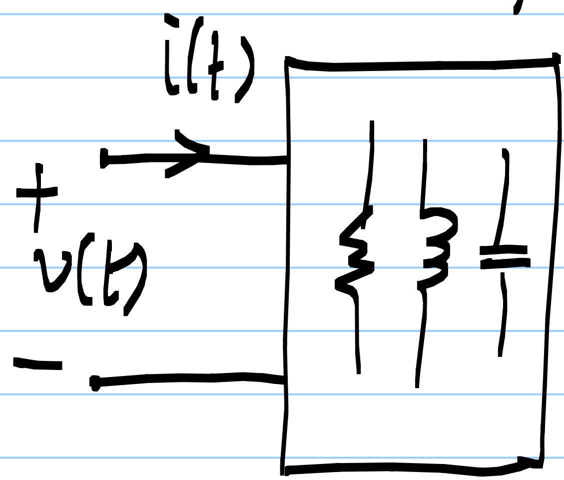
$v_k =$ [illegible] $\exp(j\omega t)$

$i_k =$ [illegible] $\exp(j\omega t)$

$I_p \cos(\omega t + \phi_1)$ → [illegible] $\exp(j\omega t)$

$V_p \sin(\omega t + \phi_2)$ [illegible] $\exp(j\omega t)$

with sinusoidal excitation, in steady state, $(\exp(j\omega t))$

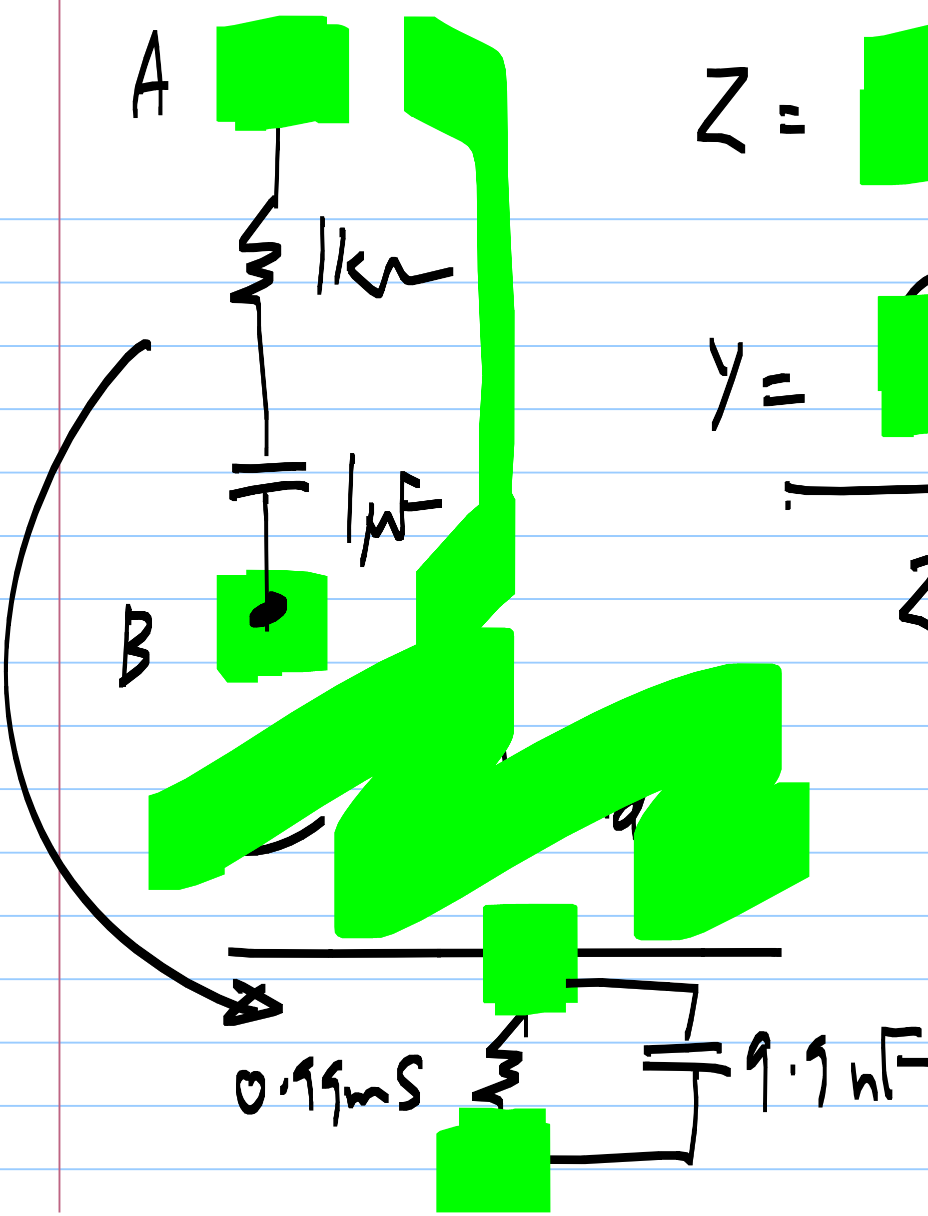


$$\frac{1}{Z} = \underbrace{Y}_{\text{Admittance}} = \underbrace{\overbrace{G}^{\text{Conductance}} + j\overbrace{B}^{\text{Susceptance}}}_{} \quad S$$

dependent on ω

Impedance Ω

$$\frac{v(t)}{i(t)} = \frac{\bar{V}\cdot\exp(j\omega t)}{\bar{I}\cdot\exp(j\omega t)} = \frac{\bar{V}}{\bar{I}} = \overbrace{\underbrace{R}_{\text{Resistance}} + j\underbrace{X}_{\text{Reactance}}}^{\text{Impedance}} = Z$$

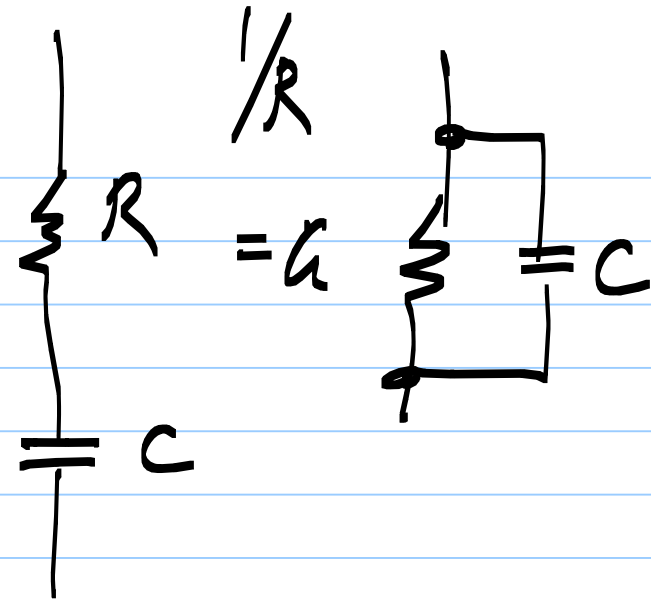


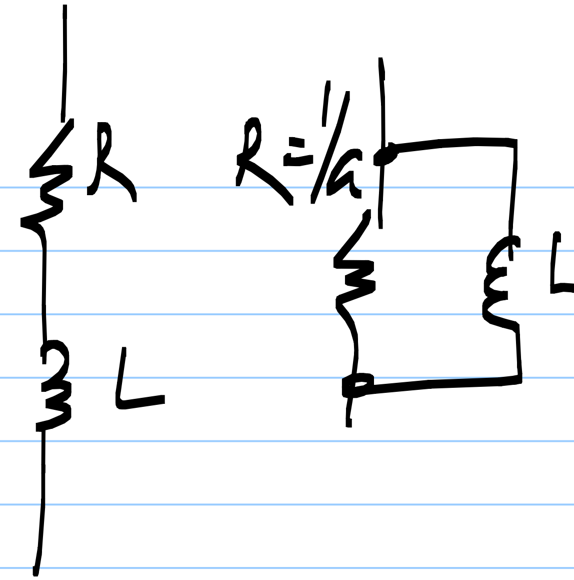
$Z = \quad - \quad \Omega = \dfrac{1 - j0.1\ \text{k}\Omega}{j\omega C}$

$Y = \quad + \quad \text{mS}$

$Z = R + jX$

$= \sqrt{R^2 + X^2} \cdot \exp\left(j \tan^{-1} \frac{X}{R}\right)$





$$Z = R - \frac{j}{\omega C}$$

negative reactance

$$Y = G + j\omega C$$

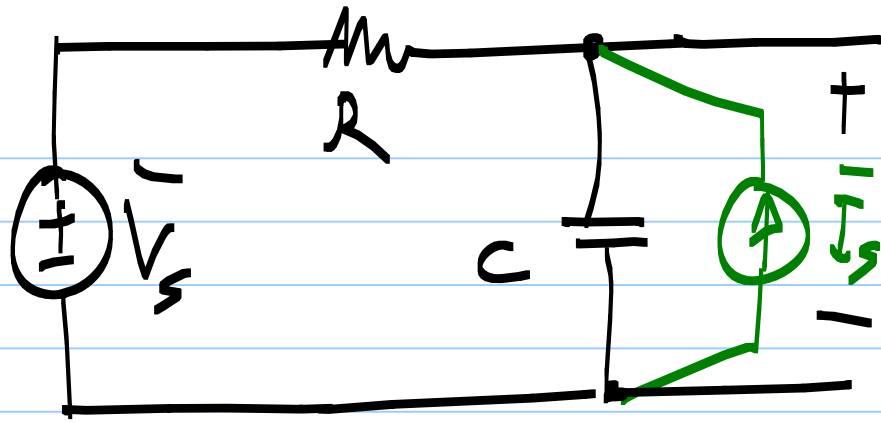
+ve
susceptance

$$Z = R + j\omega L$$

+ve
reactance

$$Y = G - \frac{j}{\omega L}$$

-ve
susceptance



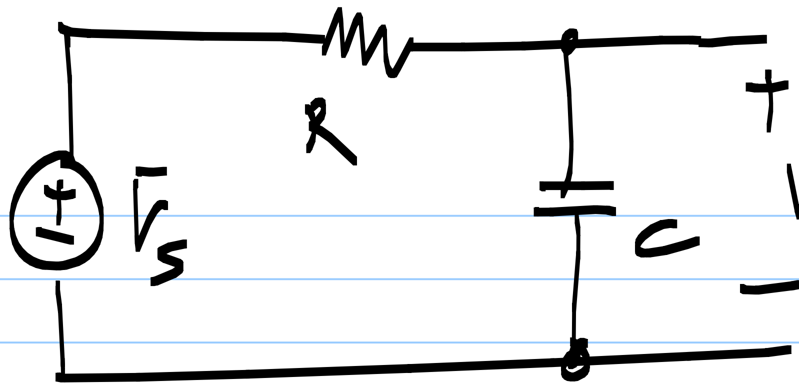
$$\bar{V}_s \cdot \frac{1}{1 + j\omega CR}$$

$$\bar{I}_s \cdot \frac{R}{1 + j\omega CR}$$

$$\mathrm{Re}\left[\bar{V}_s \exp(j\omega t)\right] \qquad \mathrm{Re}\left[\bar{I}_s \exp(j\omega t)\right]$$

$$\bar{V}_c$$

$$v_c(t) = \mathrm{Re}\left[\bar{V}_c \cdot \exp(j\omega t)\right]$$



$$H(j\omega) = \frac{\bar{V}_c}{\bar{V}_s} = \frac{1}{1 + j\omega CR}$$

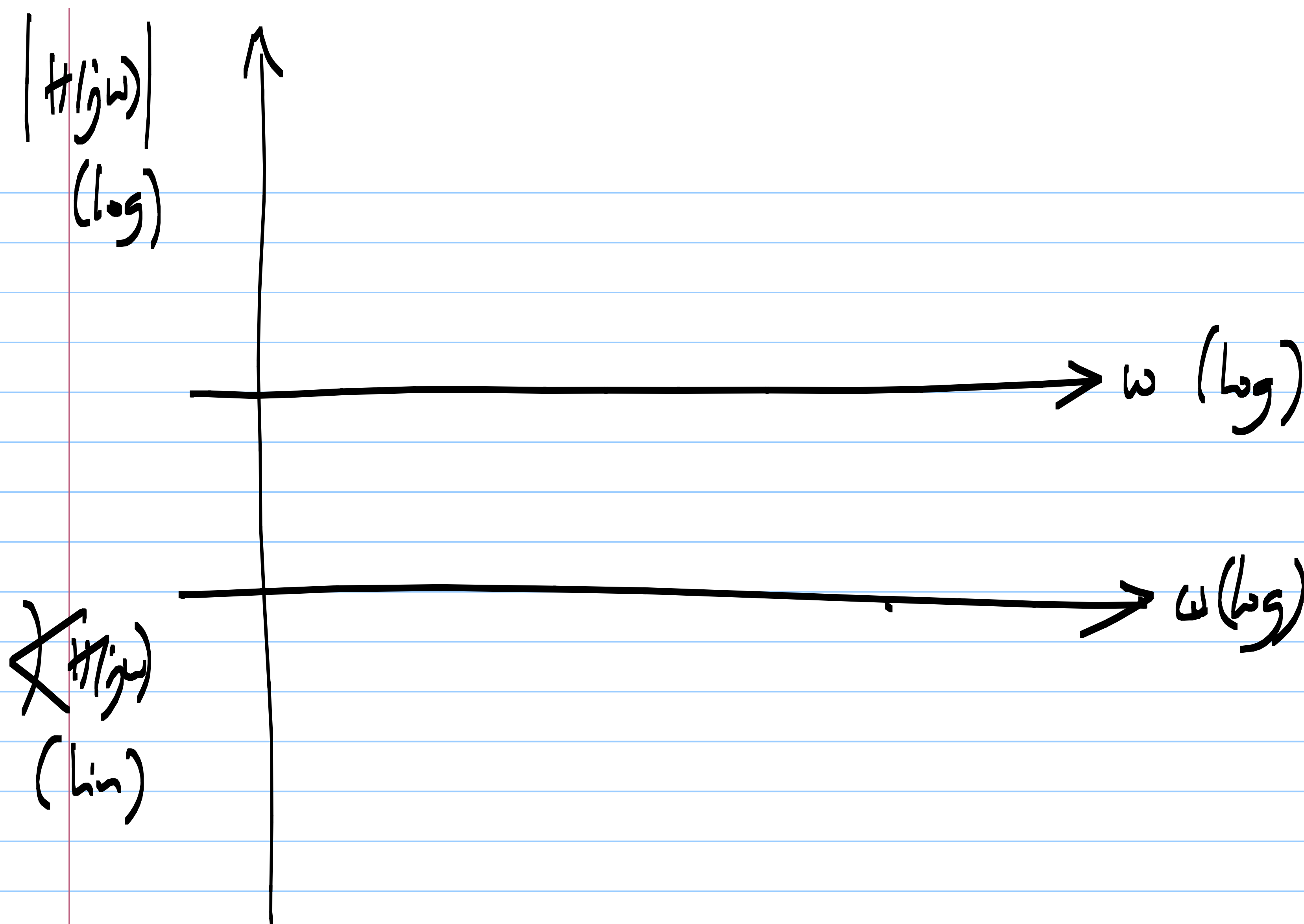
$$\left|\frac{\bar{V}_c}{\bar{V}_s}\right| \quad \& \quad \measuredangle \frac{\bar{V}_c}{\bar{V}_s}$$

log. scale

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

$$\measuredangle H(j\omega) = -\tan^{-1}(\omega CR)$$

vs. ω
(log scale)



|H(jω)|
(log)
ω (log)
∠H(jω)
(lin)
ω (log)

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

$$\underbrace{\log |H(j\omega)| = -\frac{1}{2}\log\left(1+(\omega CR)^2\right)}$$

$\omega \ll 1/CR$ $\quad |H(j\omega)| \simeq 1$

$$\log |H(j\omega)| = 0$$

$\omega \gg 1/CR$ $\quad |H(j\omega)| \simeq \frac{1}{\omega CR}$

$\omega CR \ll 1$

low freq.

high freq.

$$\log |H(j\omega)| = -\log(\omega CR)$$