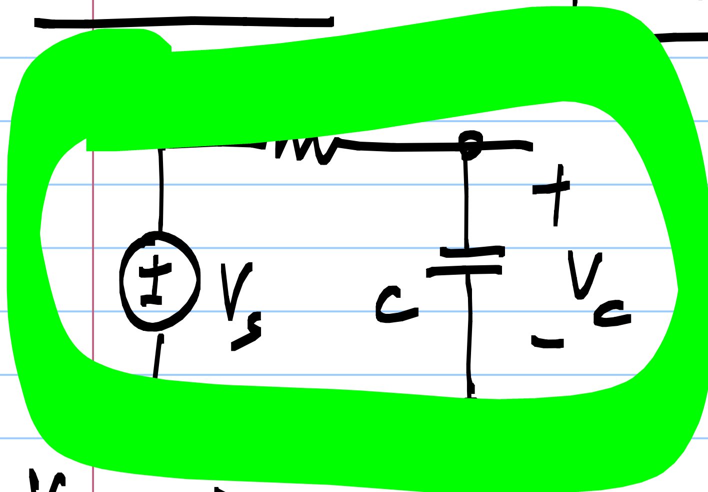
EE 2015 — **Circuits with L, C (R)** — 27/9/2017

→ Differential equations



$$RC \cdot \frac{dV_c}{dt} + V_c = V_s$$

$$a_N \frac{d^N v_c}{dt^N} + a_{N-1} \frac{d^{N-1} v_c}{dt^{N-1}} + \dots a_1 \frac{dv_c}{dt} + a_0 v_c = V_s$$

$V_s = 0 \Rightarrow$ homogenous eq. (Natural response)

Forced / steady-state response — Natural response

V_s: constant

$$V_s \quad + \quad \left(v_c(0) - V_s\right) \exp\left(-t/RC\right)$$

$H \cdot V_s \cdot \exp(st)$

Depends on initial conditions & input

Zero-state — Zero-input response

$$V_s\left(1 - \exp\left(-t/RC\right)\right) \quad + \quad v_c(0) \exp\left(-t/RC\right)$$

Input: $V_s = V_p$

$$RC \cdot \frac{dV_c}{dt} + V_c = V_s$$

$$RC \cdot H \cdot s V_p \exp(st) + H V_p \exp(st) = V_p \cdot \exp(st)$$

$$H = \frac{1}{1 + sCR}$$

$$V_c = H \cdot \exp(st)$$

Forced response to $V_p \exp(st)$ is $H V_p \exp(st)$

$$a_N \cdot \frac{d^N V_c}{dt^N} + a_{N-1} \frac{d^{N-1} V_c}{dt^{N-1}} + \dots + a_1 \frac{dV_c}{dt} + a_0 V_c$$

$$(a_N \cdot s^N + a_{N-1} s^{N-1} + \dots + a_1 s + a_0) H = 1 = V_s$$

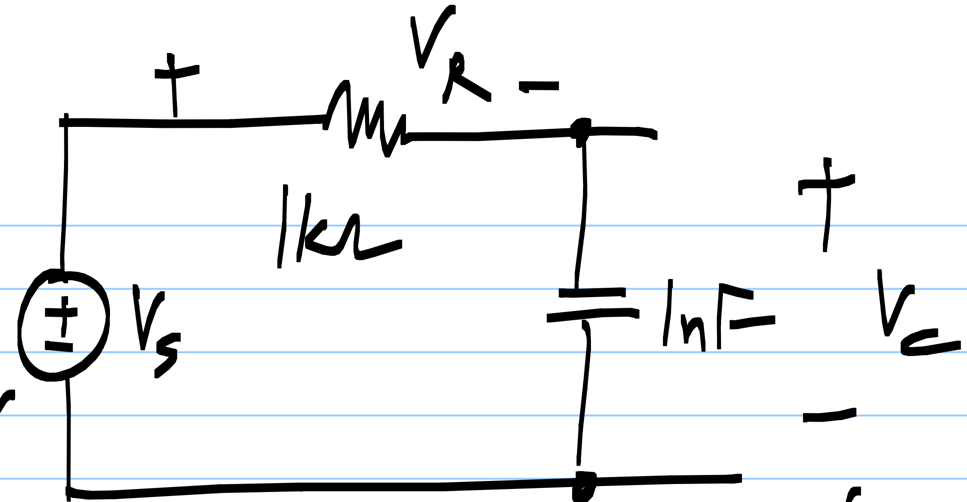
Response to $\cos(\omega t + \phi)$ = Real part of [response to $\exp(j(\omega t + \phi))$]

$$|H(j\omega)| \cdot \cos(\omega t + \phi + \angle H(j\omega))$$

Steady-state response to $\exp(st)$: $H(s) \cdot \exp(st)$

" $\exp(j(\omega t + \phi))$: $H(j\omega)\exp(j(\omega t + \phi))$

$\exp(j\phi) \cdot \exp(j\omega t)$ $\quad |H(j\omega)| \exp(j(\omega t + \phi + \angle H(j\omega)))$



$CR = 10^{-6}\,s$

Find $\underline{\underline{V_C}}$ & $\underline{\underline{V_R}}$

$$H_C(j\omega) = \frac{1}{1 + j\omega CR}$$

$$2V\cos(\omega_0 t) = 2V\cos(2\pi f_0 t)$$

$\omega_0 : 2\cdot 10^6\ rad/s$

$f_0 = \omega_0/2\pi$

$$= \frac{1}{1 + j\,2\cdot 10^6 \cdot 10^{-6}} = \frac{1}{1+j2} = \frac{1}{\sqrt{5}} \cdot \angle{-63^\circ}$$

$$2V \cos(2\cdot 10^6 t) \longrightarrow 0.89V \cdot \cos(2\cdot 10^6 t - 63^\circ)$$

1.1 rad

$$V_R = V_s - V_C; \quad RC \cdot \frac{dV_C}{dt}$$

$$RC\frac{dV_C}{dt} + V_C = V_s$$

$$V_R = V_s - V_C$$

$$V_C = V_s - V_R$$

$$RC\frac{dV_s}{dt} - RC\frac{dV_R}{dt} + V_s - V_R = V_s$$

$$RC \cdot \frac{dV_R}{dt} + V_R = R \cdot C\frac{dV_s}{dt}$$

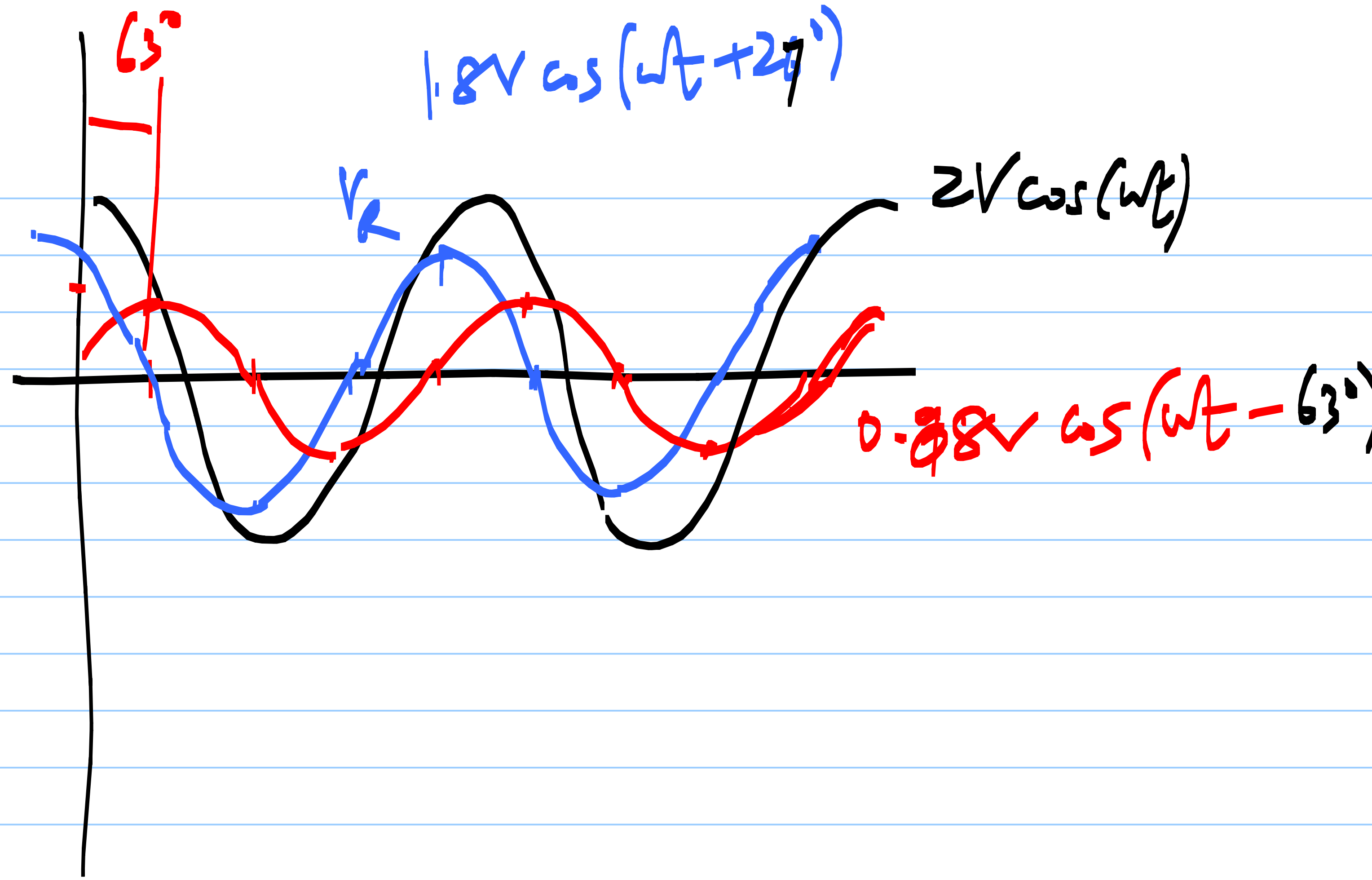
$$RC \cdot \frac{dV_R}{dt} + V_R = RC \frac{dV_s}{dt}$$

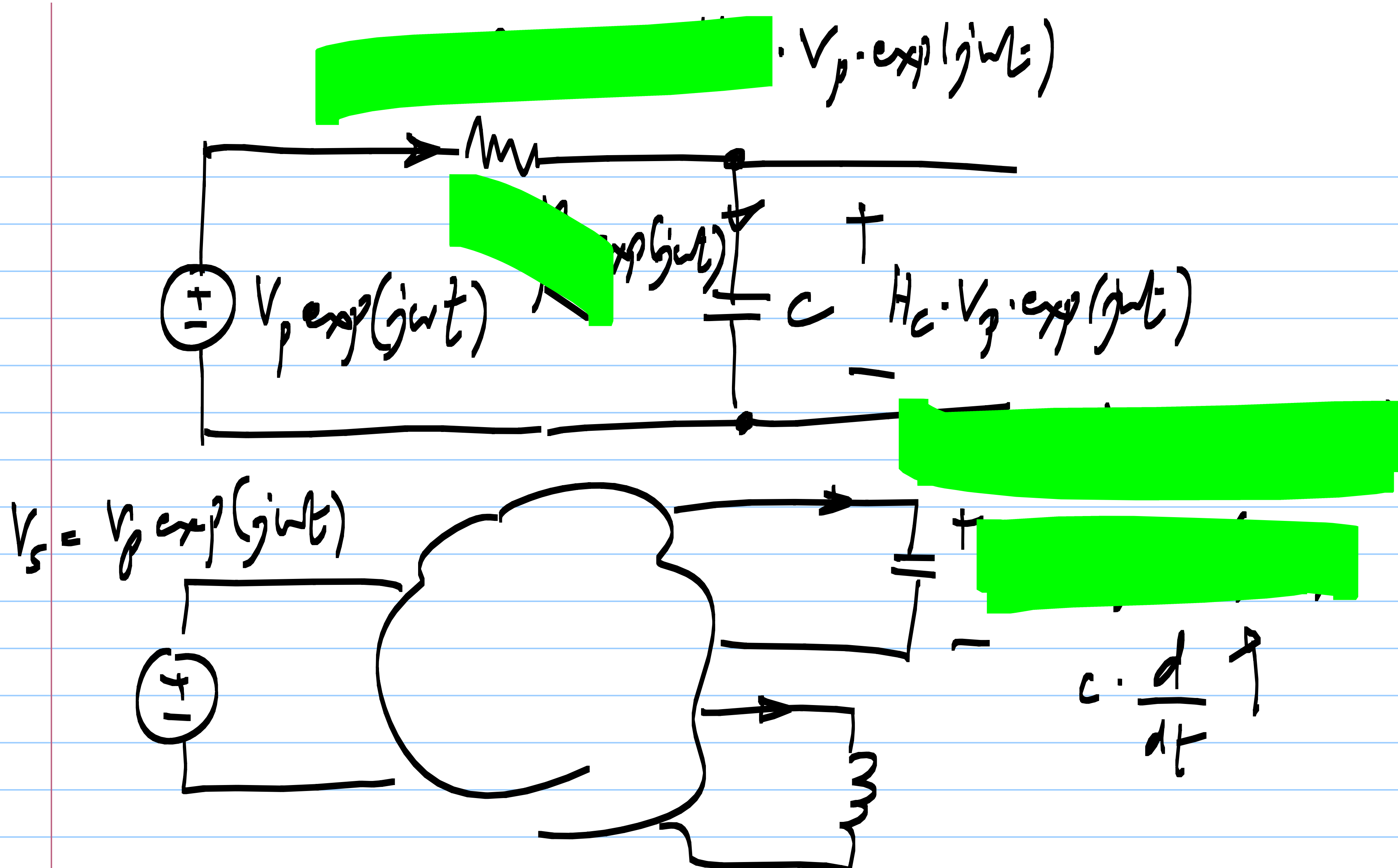
$V_s : \exp(j\omega t)$; $V_R : H_R \exp(j\omega t)$

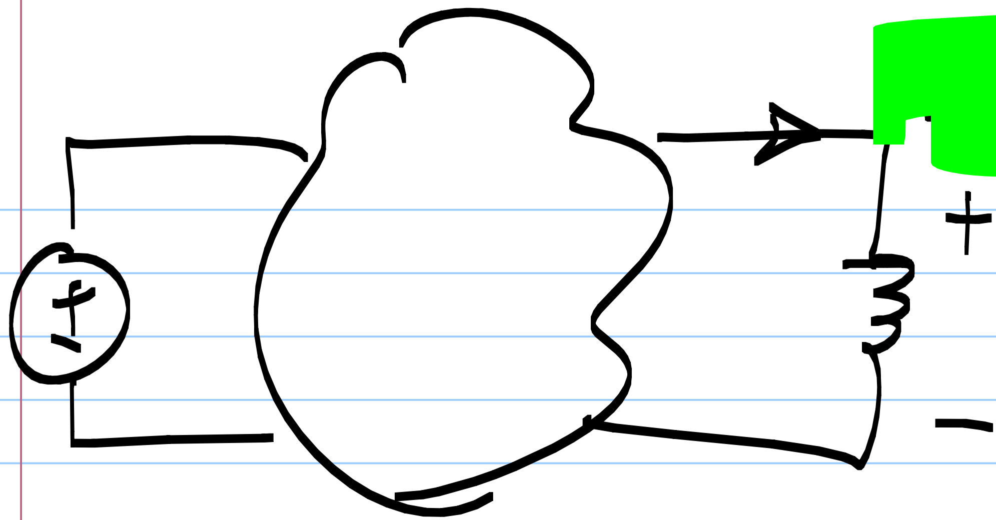
$$RC \cdot j\omega \cdot H_R \cdot \exp(j\omega t) + H_R \exp(j\omega t) = RC \cdot j\omega \cdot \exp(j\omega t)$$

$$H_R = \frac{j\omega CR}{1 + j\omega CR} = \frac{j2}{1 + j2}$$

$$= \frac{2}{\sqrt{5}} \cdot \angle 27^\circ \qquad 1.8V \cos(\omega t + 27^\circ)$$


63°
1.8V cos(ωt+27°)
v_R
2V cos(ωt)
0.88V cos(ωt−63°)


[redacted] $\cdot V_p \cdot \exp(j\omega t)$
$V_p \exp(j\omega t)$
[redacted] $\exp(j\omega t)$
C
$+$
$-$
$H_c \cdot V_p \cdot \exp(j\omega t)$
[redacted]
$V_s = V_p \exp(j\omega t)$
[redacted]
$c \cdot \frac{d}{dt}$



$L \cdot \frac{d}{dt}($