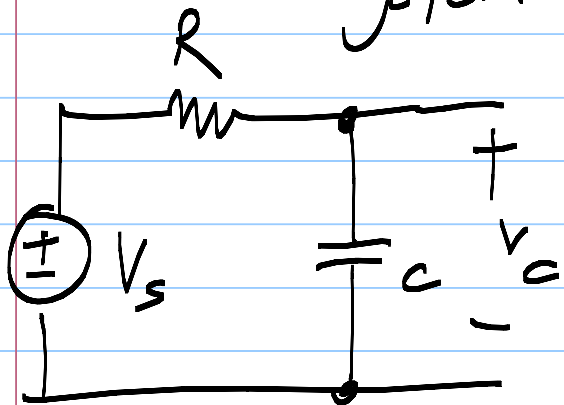
EE2015 Response of a first order system to sinusoids 26/9/2017



$$\underbrace{RC\frac{dV_c}{dt} + V_c}_{\exp(st)\ \rightarrow\ \frac{\exp(st)}{1+sCR}} = V_p\cos(\omega t)$$

$$\frac{V_p}{2}\left(\exp(j\omega t) + \right.$$

$$\frac{V_p}{2}\exp(j\omega t) \longrightarrow \frac{V_p}{2}\exp(j\omega t)\,\frac{1}{1+j\omega CR}$$

$$\frac{V_p}{2}\exp(-j\omega t) \longrightarrow \frac{V_p}{2}\exp(j\omega t)\,\frac{1}{1-j\omega CR}$$

Forced response: $\frac{V_p}{2}\left[\frac{\exp(j\omega t)}{1+j\omega CR}+\frac{\exp(-j\omega t)}{1-j\omega CR}\right]$

$$= \frac{V_p}{\cancel{2}}\;\frac{\overbrace{\exp(j\omega t)+\exp(-j\omega t)}^{\cancel{2}\cos\omega t}-j\omega CR\left(\exp(+j\omega t)-\exp(-j\omega t)\right)}{1+(\omega CR)^2}$$

$j: \sqrt{-1}$

$$= V_p \cdot \frac{\overbrace{\cos\omega t + \omega CR\cdot\sin\omega t}^{(\ )\cos(\omega t+\psi)}}{1+(\omega CR)^2}$$

$\sqrt{1+(\omega CR)^2}\,\cos(\omega t - \tan^{-1}\omega CR)$

$$= V_p\,\frac{\cos(\omega t - \tan^{-1}(\omega CR))}{\sqrt{1+(\omega CR)^2}}$$

$$RC\frac{dV_c}{dt} + V_c = V_p\cos(\omega t) \qquad V_c = \alpha\cos\omega t + \beta\sin\omega t$$

$$\frac{V_p}{\sqrt{1+(\omega CR)^2}}\cdot\cos\left(\omega t - \tan^{-1}(\omega CR)\right)$$

$$RC\left(-\alpha\omega\sin(\omega t) + \beta\omega\cos\omega t\right) = V_p\cos\omega t$$

$$\beta\sin\omega t + \alpha\cos\omega t$$

$$\left.\begin{aligned} \alpha + \beta\omega CR &= V_p \\ -\alpha\omega CR + \beta &= 0 \end{aligned}\right]$$

$$\alpha = \frac{V_p}{1+(\omega CR)^2}\cdot\cos\omega t$$

$$\beta = \frac{V_p\cdot\omega CR}{1+(\omega CR)^2}\cdot\sin\omega t$$

Natural response: $\alpha_N \cdot \underbrace{\exp(-t/RC)}$

$$V_c(t) = \frac{V_p \cos\left(\omega t - \tan^{-1}(\omega CR)\right)}{\sqrt{1+(\omega CR)^2}} + \alpha_N \exp(-t/RC)$$

$$V_c(0) = \frac{V_p \cos\left(-\tan^{-1}(\omega CR)\right)}{\sqrt{1+(\omega CR)^2}} + \alpha_N$$

$$\alpha_N = V_c(0) - \frac{V_p \cos\left(-\tan^{-1}(\omega CR)\right)}{\sqrt{1+(\omega CR)^2}}$$

$\cos(\omega t)$ — Lin. system — $y_{\cos}(t)$ Forced response to $\cos(\omega t)$

Real components

$\sin(\omega t)$ — Lin. system — $y_{\sin}(t)$ Forced response to $\sin(\omega t)$

Each of these is real

$\cos \omega t + j \sin \omega t$ — Lin system — $y_{\cos}(t) + j\, y_{\sin}(t)$ Forced response to $\exp(j\omega t)$

Forced response to $\cos(\omega t)$: $\mathrm{Re}\left[\text{Forced response to } \exp(j\omega t)\right]$

[differential eq. with real coefficients]

$$RC\frac{dv_c}{dt} + v_c = v_s$$

$$\exp(st) \longrightarrow \exp(st)\left[\frac{1}{1+sCR}\right]$$

$H(s)$

$$\mathrm{Re}\left\{\exp(j\omega t)\left[\frac{1}{1+j\omega CR}\right]\right\}$$

$$\frac{1}{1+j\omega CR} = \frac{1-j\omega CR}{1+(\omega CR)^2}$$

$\exp(j\omega t) \cdot [H(j\omega)]$

Complex number: $\alpha + j\beta = A \cdot \exp(j\theta)$

$$H(j\omega) = \frac{1}{1 + j\omega CR} = \frac{1}{\sqrt{1 + (\omega CR)^2}} \exp(-j \tan^{-1}(\omega CR))$$

$$H(j\omega) \cdot \exp(j\omega t) = \frac{1}{\sqrt{1+(\omega CR)^2}} \cdot \underbrace{\exp(-j \tan^{-1}(\omega CR)) \cdot \exp(j\omega t)}_{\exp(j\omega t - j\tan^{-1}(\omega CR))}$$

$\downarrow$

Re( )

$$\longrightarrow \frac{1}{\sqrt{1+(\omega CR)^2}} \cdot \underline{\cos(\omega t - \tan^{-1} \omega CR)}$$

Forced response of a linear system : $H(s) \cdot V_p \exp(st)$

to $V_p \exp(st)$

$$H(j\omega) = |H(j\omega)| \cdot \exp(j \angle H(j\omega))$$

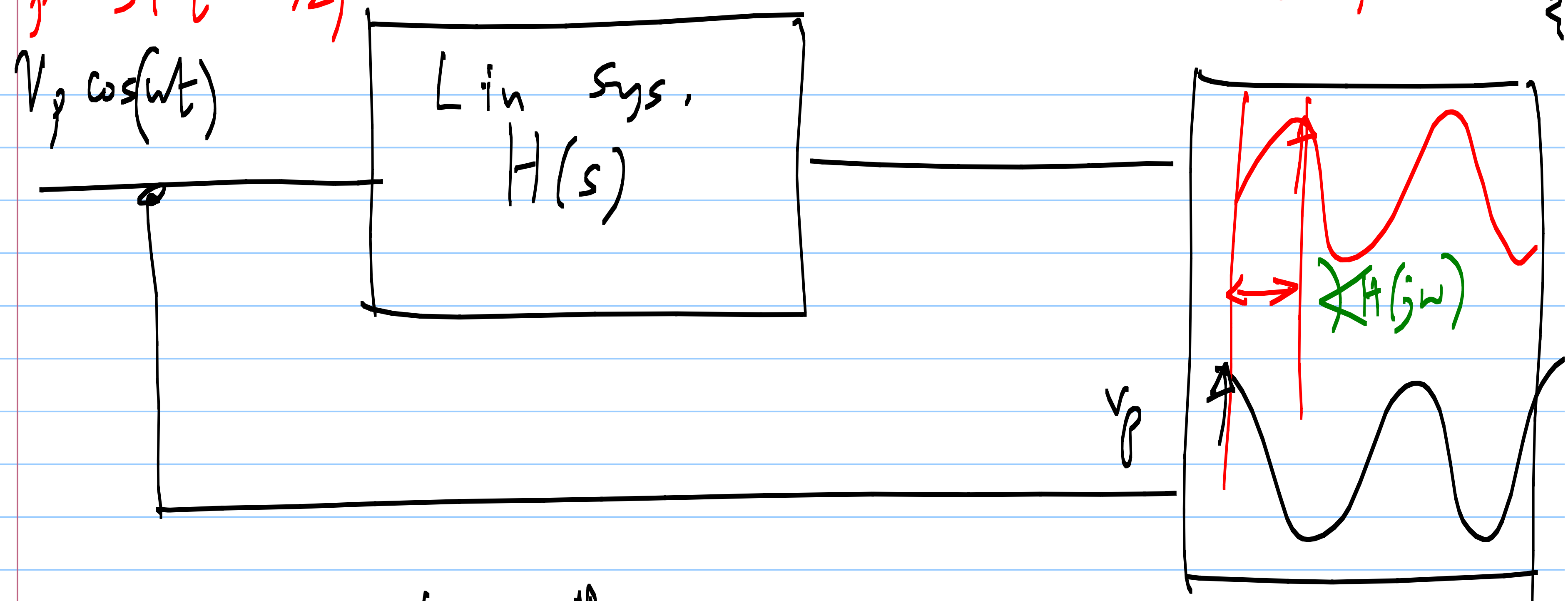
" $V_p \exp(j\omega t)$: $H(j\omega) \cdot V_p \exp(j\omega t)$

" [illegible] $\cos(\omega t)$: $\mathrm{Re}\left[H(j\omega)\, V_p \exp(j\omega t)\right]$

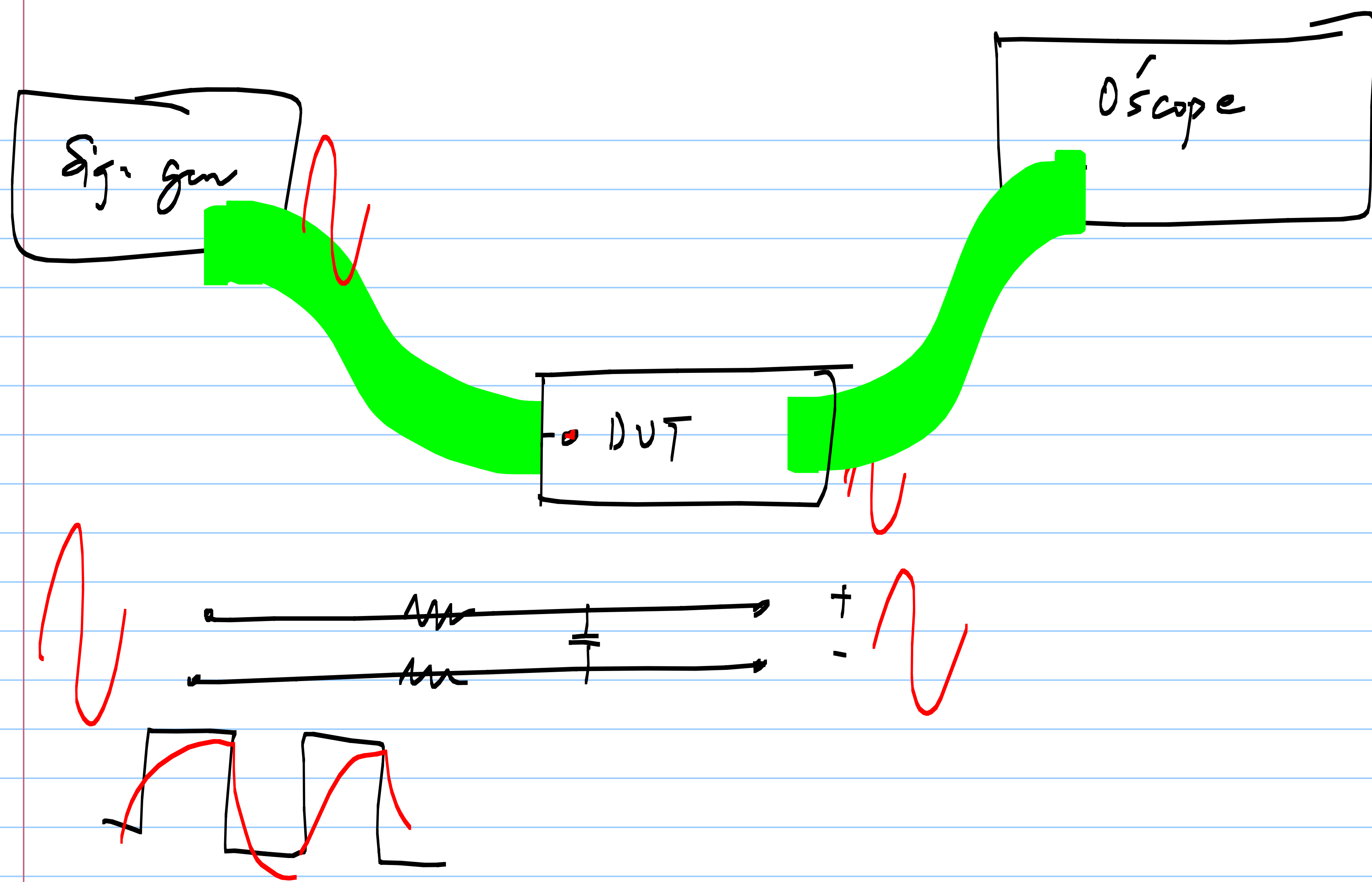
$$\mathrm{Re}\left[V_p \, |H(j\omega)| \cdot \underbrace{\exp(j\omega t + j \angle H(j\omega))}\right]$$

[illegible] $\cos(\omega t +$ [illegible] $)$

$v_p \cos(\omega t + \phi)$
$v_p \cos(\omega t - \pi/2)$



$$V_p \cdot |H(j\omega)| \cdot \cos\left(\omega t + \sphericalangle H(j\omega) + \phi\right)$$


Sig. gen
DUT
O'scope
+
-