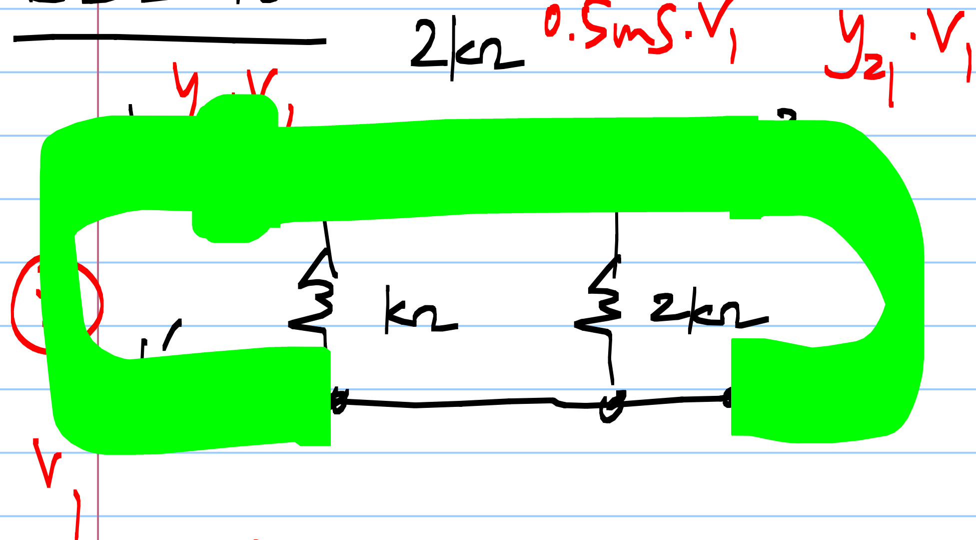
EE 2015

# Two-port networks w/ only resistors

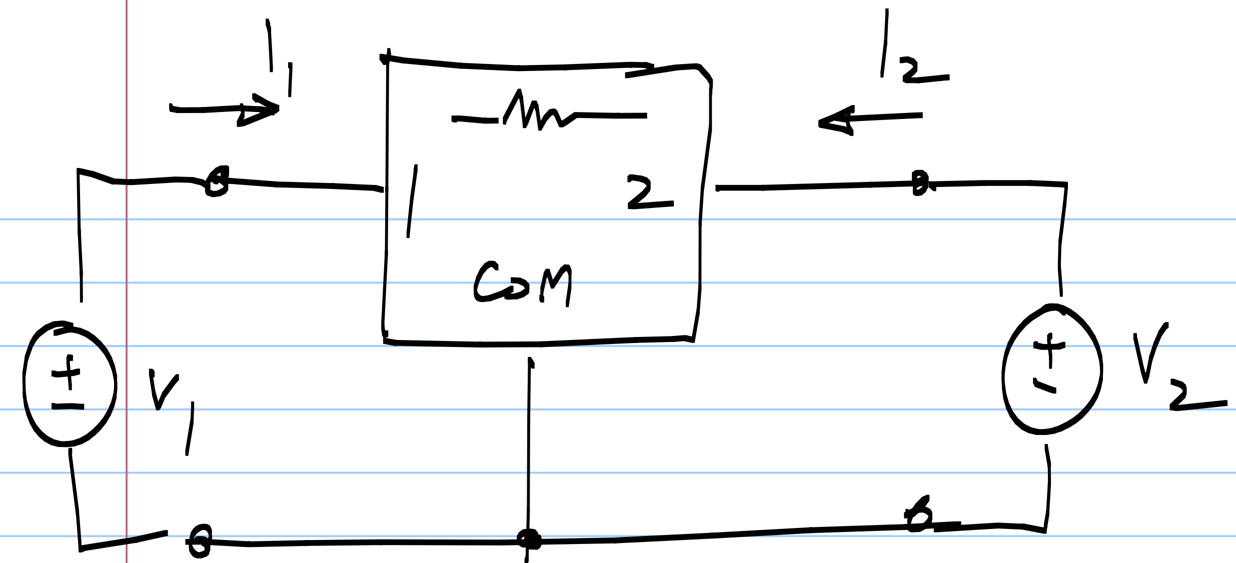
4/9/2017



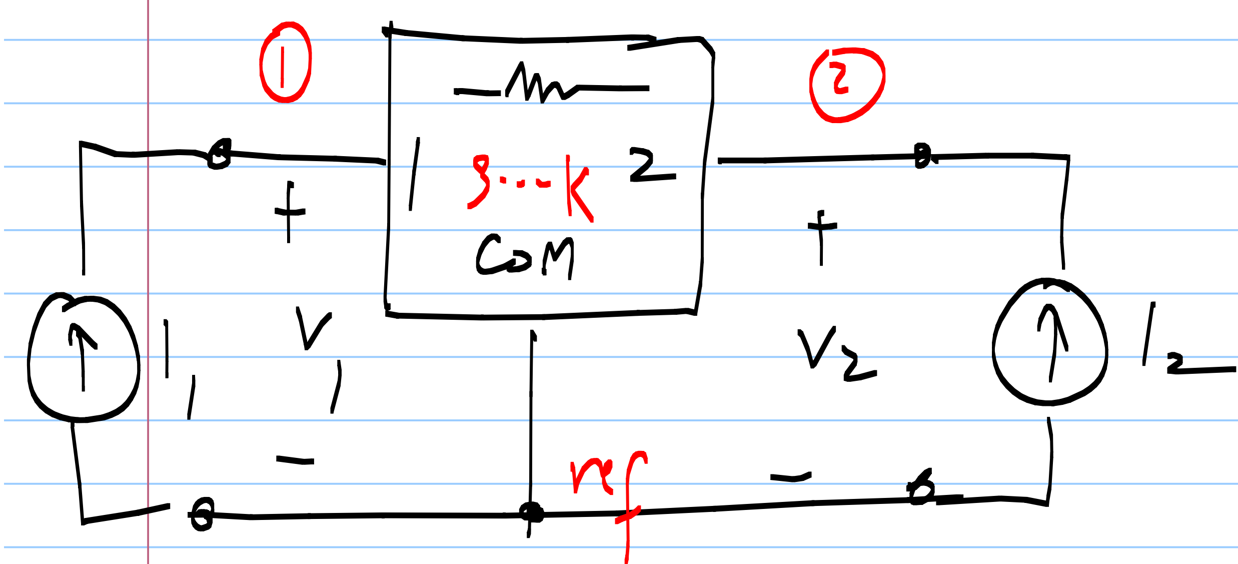

$$y = \begin{bmatrix} & -0.5 \\ -0.5 & \end{bmatrix} mS$$

$$\underbrace{(1mS + 0.5mS)}_{y_{11}} V_1$$

$$\underbrace{-0.5mS}_{y_{21}}$$




$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$




$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} G_A & G_B \\ 2\times 2 & 2\times k-2 \\ \hline & \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$k-2 \times 2$ $\qquad$ $k \times k$ $\qquad$ $k \times 1$

$a_{11} v_1 + a_{12} v_2$

$$\underbrace{\begin{bmatrix} G_A \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{2\times 1} + \underbrace{\begin{bmatrix} G_B \end{bmatrix}}_{2\times k-2} \underbrace{\begin{bmatrix} V_3 \\ \vdots \\ V_k \end{bmatrix}}_{k-2\times 1} = \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{2\times 1}$$

Solve for $(V_3 \ldots V_k)$

$$\underbrace{\begin{bmatrix} G_B^T \end{bmatrix}}_{k-2\times 2} \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{2\times 1} + \underbrace{\begin{bmatrix} G_D \end{bmatrix}}_{k-2\times k-2} \underbrace{\begin{bmatrix} V_3 \\ \vdots \\ V_k \end{bmatrix}}_{k-2\times 1} = \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}}_{k-2\times 1}$$

$$\begin{bmatrix} V_3 \\ \vdots \\ V_k \end{bmatrix} = -\begin{bmatrix} G_D^{-1} \end{bmatrix} \begin{bmatrix} G_B^T \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

resistive network

$$[G_A] \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} - [G_B][G_D^{-1}][G_B^T] \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\underbrace{\Big[ [G_A] - \underbrace{[G_B][G_D^{-1}][G_B^T]} \Big]}_{y\text{-parameter matrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

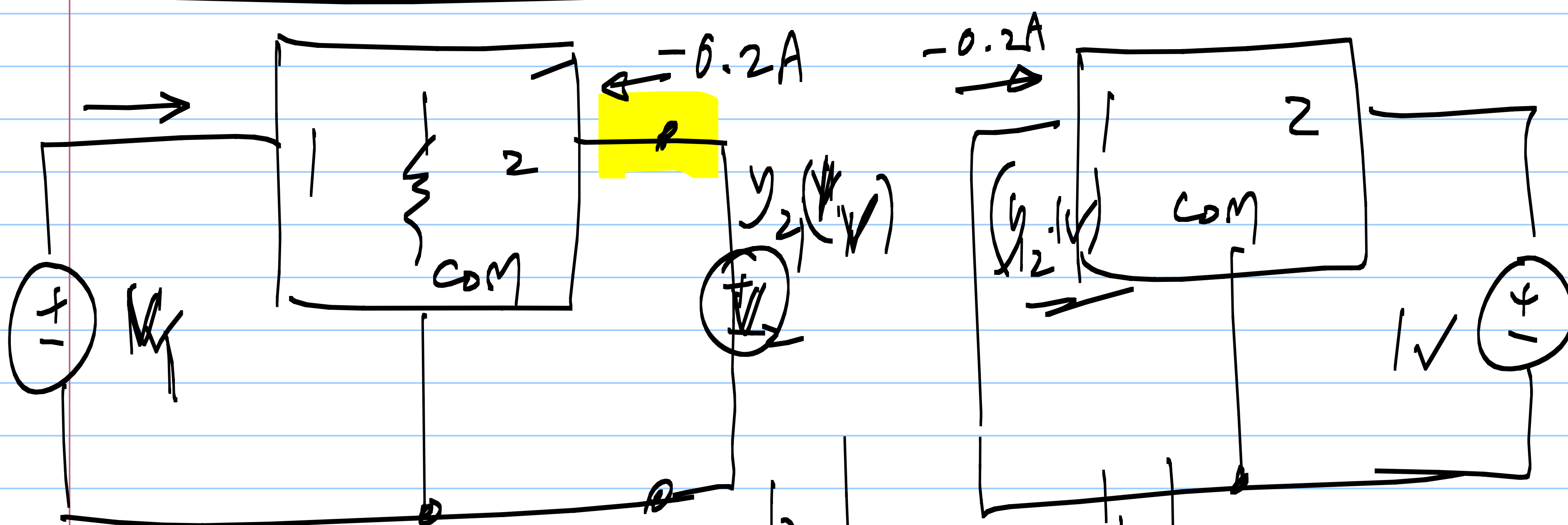
$2 \times 2$

$$\left( G_B \cdot G_D^{-1} \cdot G_B^T \right)^T$$

$$G_B \underbrace{\left( G_D \right)^{-1}} G_B^T$$

For a resistive 2-port network, $y_{12} = y_{21}$

(Reciprocity)




$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

[ For a resistive 2-port network, $\underline{\underline{y_{12} = y_{21}}}$

(Reciprocity)

<u>Reciprocity relationship.</u>

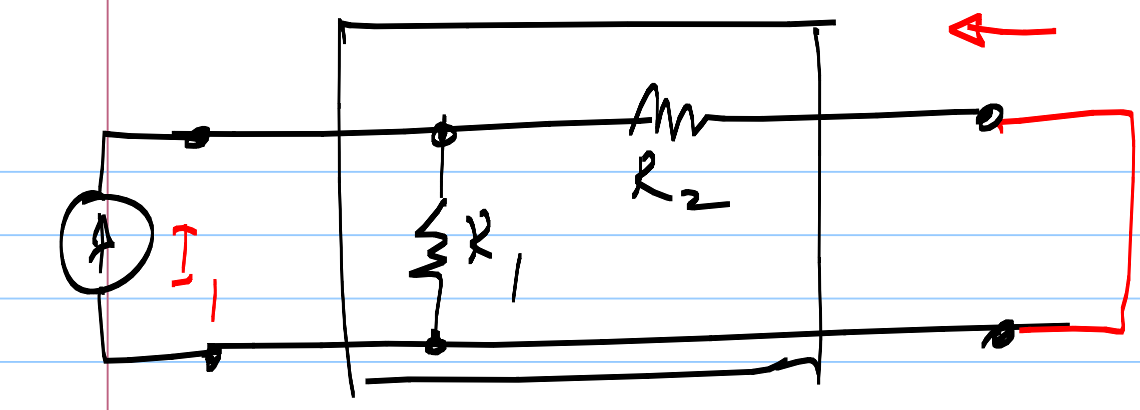
$$z_{12} = \frac{-y_{12}}{\Delta_y} \; ; \; z_{21} = \frac{-y_{21}}{\Delta_y}$$

$$y_{12} = y_{21}$$

$$z_{12} = z_{21}$$

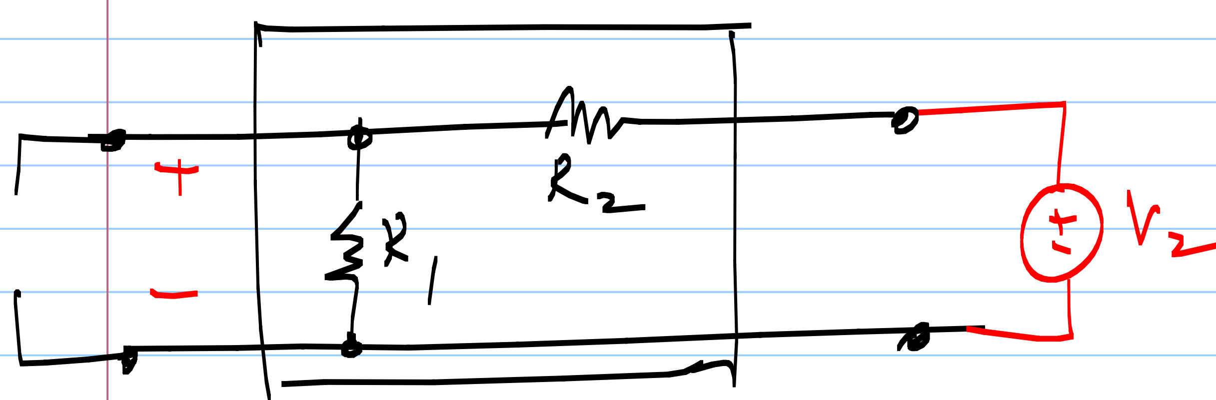
$$h_{12} = -h_{21}$$

$$g_{12} = -g_{21}$$




$\cdot I_1$

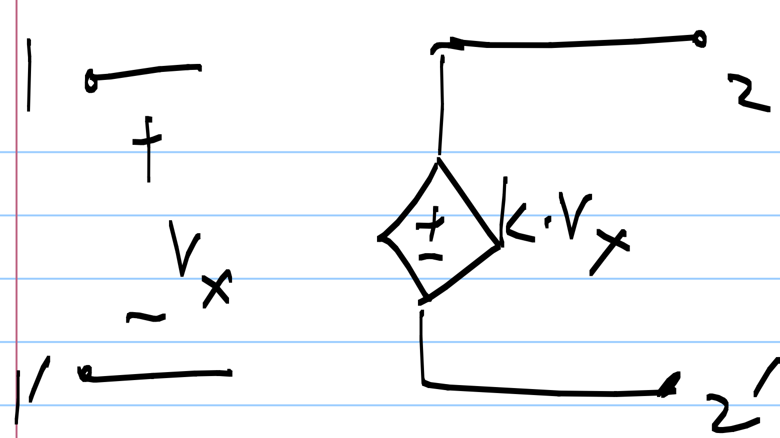
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

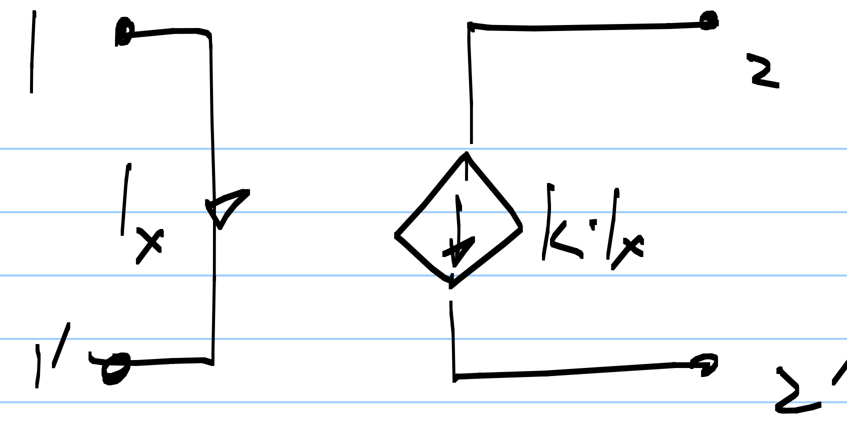



$= \frac{R_1}{R_1 + R_2}$

$= -\frac{R_1}{R_1 + R_2}$

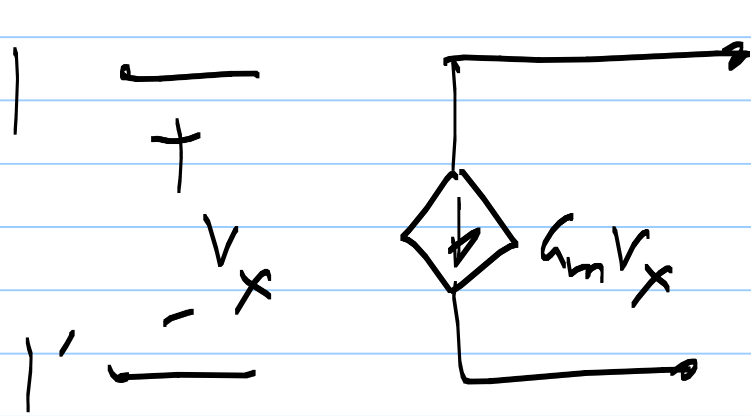
$\frac{R_1}{R_1 + R_2} \cdot V_2$



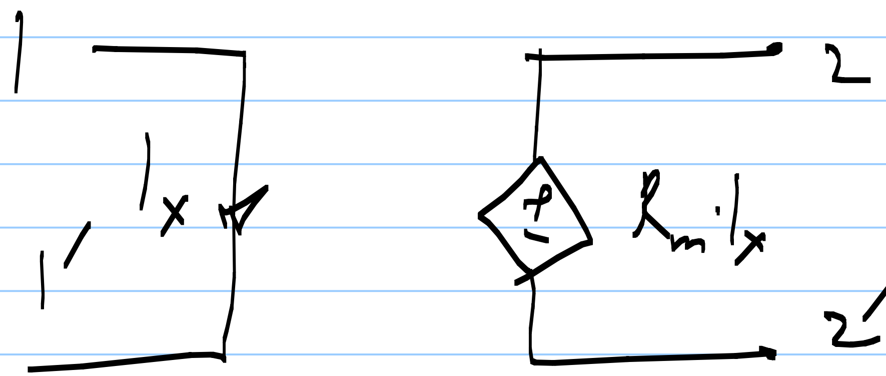




VCVS

y, z, h, g parameters




VCCS




CCVS