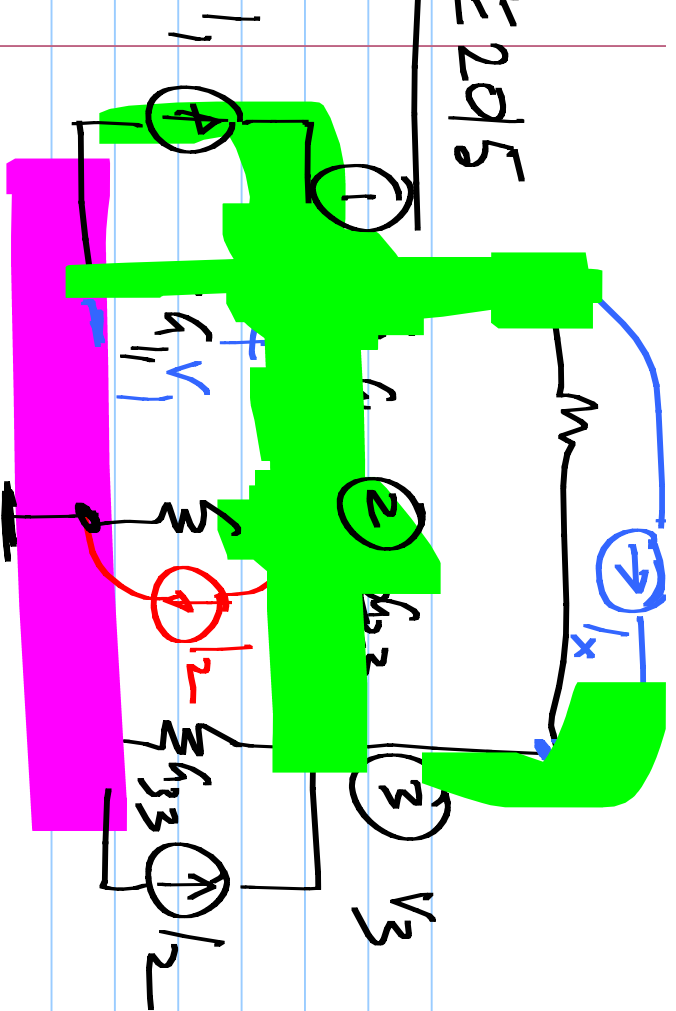


EE 2015

11/8/2017



Nodal analysis

KCL

$$\textcircled{1} \quad (g_{11} + g_{12} + g_{13}) V_1 - g_{12} V_2 - g_{13} V_3 = I_1 - I_x$$

$$\textcircled{2} \quad -g_{12} V_1 + (g_{12} + g_{22} + g_{23}) V_2 - g_{23} V_3 = -I_2$$

$$\textcircled{3} \quad -g_{13} V_1 - g_{23} V_2 + (g_{13} + g_{23} + g_{33}) V_3 = I_2 + I_x$$

$$\begin{bmatrix}
 G_{11} + G_{12} + G_{13} & -G_{12} & -G_{13} \\
 -G_{12} + G_{22} + G_{23} & -G_{23} & \\
 -G_{13} & G_{13} + G_{23} + G_{33} &
 \end{bmatrix}
 \begin{bmatrix}
 \\ \\
 \end{bmatrix}
 =
 \begin{bmatrix}
 \\ \\
 \end{bmatrix}$$

Symmetric Conductance matrix

Diagonal: $\sum_i G_i$

$[G]_{ij}$: - (conductance [G] between node i & node j)

$$\underline{V} = \underline{I}$$

node voltage vector

Source vector

$$\underline{V} = [G]^{-1} \cdot \underline{I}$$

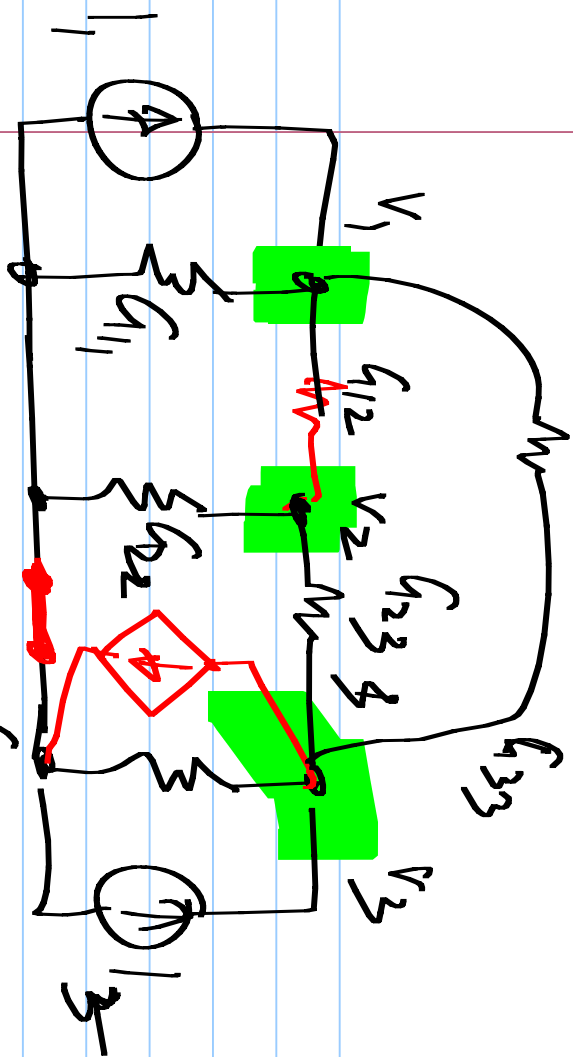
By solving

$$[G]v = I$$

we get v_1, v_2, v_3

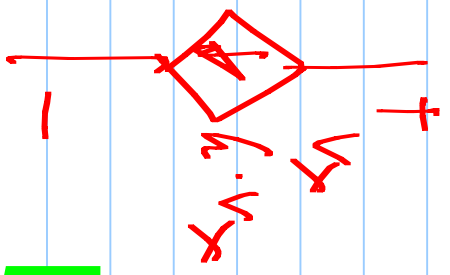
Current out of node 3:

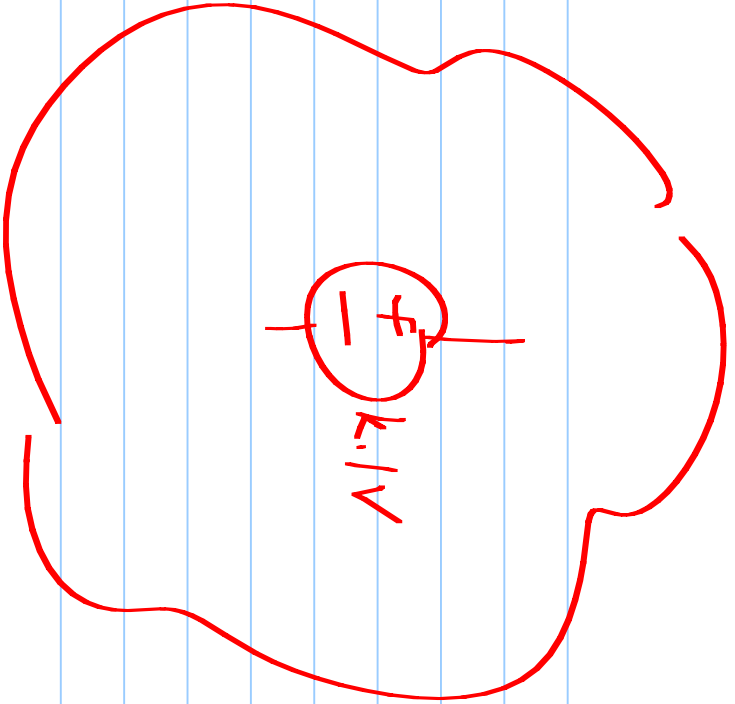
$$+g_m(V_1 - V_2)$$



$$\begin{bmatrix}
 -g_{12} & g_{12} + g_{22} + g_{23} & -g_{23} \\
 g_{12} + g_{11} + g_{13} & -g_{12} & -g_{13} \\
 g_{13} + g_{23} + g_{33} & -g_{23} & -g_{33}
 \end{bmatrix}
 \begin{bmatrix}
 V_1 \\
 V_2 \\
 V_3
 \end{bmatrix}
 =
 \begin{bmatrix}
 I_1 \\
 0 \\
 I_3
 \end{bmatrix}$$

matrix node voltages

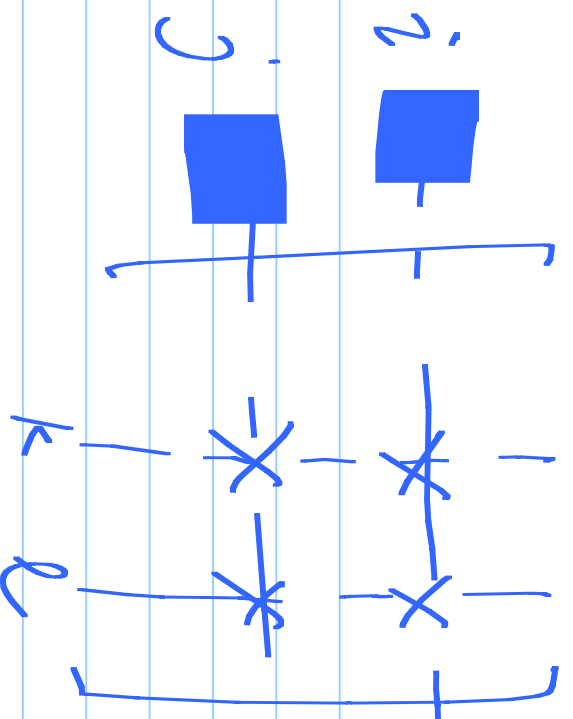
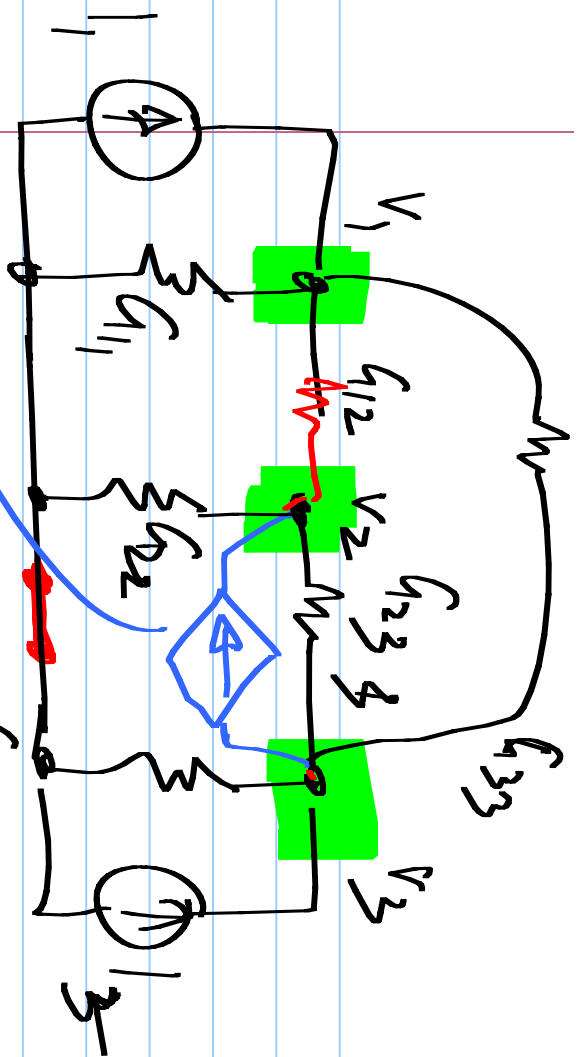




0

0

—
+
0
✓



Handwritten nodal analysis equations:

$$G_{11}(V_1 - V_2) + G_{11}V_1 = I_1$$

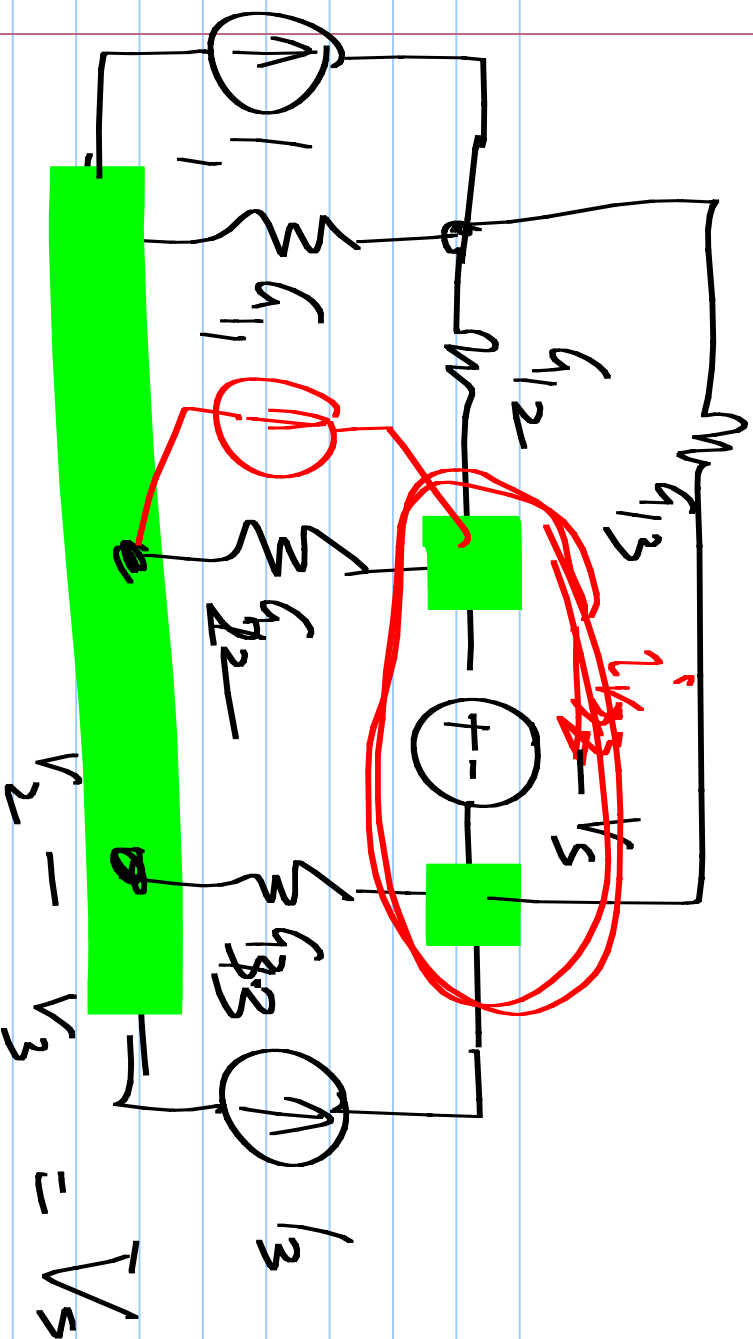
$$-G_{12}(V_1 - V_2) + G_{22}V_2 + G_{23}(V_2 - V_3) = I_2$$

$$-G_{23}(V_2 - V_3) + G_{33}V_3 = 0$$

The matrix form is written as:

$$\begin{bmatrix} G_{11} + G_{12} + G_{13} & -G_{12} & -G_{13} \\ -G_{12} - G_m & G_{12} + G_{22} + G_{23} & -G_{23} \\ +G_m & -G_{23} & G_{23} + G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ 0 \end{bmatrix}$$

The matrix is labeled "matrix" and the voltages are labeled "node voltages".



$$V_2 - V_3 = V_s$$

$$(g_{11} + g_{12} + g_{13}) V_1 - g_{12} \cdot V_2 - g_{13} \cdot V_3 = I_1$$

$$-g_{22} \cdot V_1 + (g_{12} + g_{22}) \cdot V_2 + \overset{?}{V_{23}} = \overset{?}{\phi}$$

$$-g_{13} \cdot V_1 - \overset{?}{V_{23}} + (g_{13} + g_{33}) V_3 = I_3$$