Analog Filters Design/Synthesis

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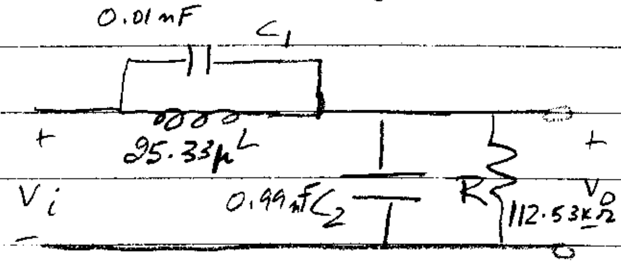


$$H(s) = \frac{(C_2 \| R)}{(C_2 \| R) + (C_1 \| L)} = \frac{s^2 L C_1 + 1}{s^2 L (C_1 + C_2) + s L/R + 1}$$

$$|H(s)|_{\omega = 1\times10^6 \times 2\pi} = -20\log \left| \frac{1 - \left[\frac{2\pi\times10^6}{2\times10^7}\right]^2}{1 + j\,\frac{2\pi\times10^6 \times 2\sqrt{2}\pi}{4\pi^2\times10^3\times10^3} - \left(\frac{2\pi\times10^6}{2\pi\times10^6}\right)^2} \right|$$

$$= -20\log \frac{|1 - 1/100|}{|\sqrt{2}j|}$$

$$\approx -20\log \frac{1}{\sqrt{2}} = 3\,dB$$

$\therefore A_p = 3\,dB$

2 $\quad \omega_z = 2\pi\times10^7$ rad/s $\quad \Omega_z = 10$ rad/s

Use frequency scaling

$$s \to s/\beta$$

$2\pi\times10^7 \leftrightarrow 10/\beta \Rightarrow \beta = \frac{1}{2\pi\times10^6}$ ≡ frequency axis contracts

$R \leftrightarrow R$ $\quad R' = 112.5\ \Omega$

$sL \to s(L/\beta)$ $\quad \therefore L' = \frac{L}{2\pi\times10^6} \equiv 159\ H$

$sC \to s(C/\beta)$

$C_1' = 62.8\mu F \quad C_2' = 62\mu F$

Now do impedance Scaling

$R \leftrightarrow \alpha R$ $\qquad$ $1 \to \alpha(112.5)$

$L \leftrightarrow \alpha L$ $\qquad$ $L = \frac{159.24}{112.5} \approx 1.414\,H$

$C \leftrightarrow C/\alpha$

$C_1 = 7.072\ mF$

$C_2 = 698.2\ mF$

$$H(S) = \frac{1 + \left(\frac{S}{\Omega_z}\right)^2}{1 + \frac{S}{Q\Omega_p} + \frac{S^2}{\Omega_p^2}} \qquad \left[\Omega_p \approx 1\ rad/sec\right]$$

$$= \frac{1 + \left(\frac{S}{10}\right)^2}{1 + \sqrt{2}S + S^2}$$

$\Omega_s = ?$

$|H(s)| = -20dB \approx 0.1$

$$\left| \frac{1 - \frac{\Omega^2}{100}}{1 + \sqrt{2}j\Omega - \Omega^2} \right| = 0.1$$

$\Omega \approx 3\ rad/sec$

3/

Low pass to High Pass Transform

$s \cdot S = \omega_p \cdot \Omega_p$ [invert the frequency axis]

$\omega_p = 10M \quad \Omega_p = 1\,rad/sec$

$$H(S) = \frac{1+\left(S/10\right)^2}{1+\sqrt{2}\,S+S^2} \quad \therefore \quad H(s) = \frac{1+\left(\frac{\Omega_p\omega_p}{10s}\right)^2}{1+\sqrt{2}\frac{\Omega_p\omega_p}{s}+\left(\frac{\Omega_p\omega_p}{s}\right)^2}$$

$$H(s) = K\frac{\left(1+s/\omega_z\right)^2}{1+\frac{s}{\omega_p Q}+\left(s/\omega_p\right)^2} = \frac{1+\left(\frac{\omega_p}{10s}\right)^2}{1+\sqrt{2}\frac{\omega_p}{s}+\left(\frac{\omega_p}{s}\right)^2}$$

$\omega_z = \frac{\omega_p}{10} = 1M\,rad/s$ = frequency of the notch

$K = 1/100$

$L = 1.414H \equiv C = \frac{1}{\Omega_p\omega_p L} = 70.7\,nF$

$C_1 = 7.07\,mF \equiv L_1 = \frac{1}{C_1\Omega_p\omega_p} = 14.28\,mH$

$C_2 = 691.11\,mF \equiv L_2 = 0.14\,mH$

$R = 1\Omega$

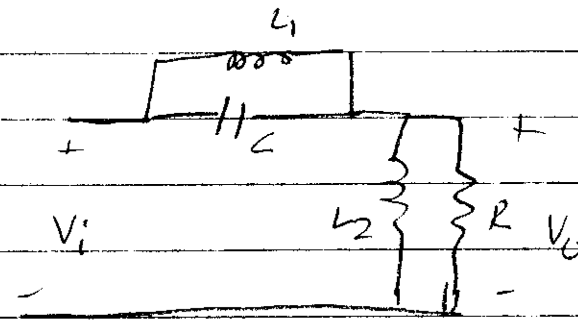
$\therefore$ Impedance Scaling

$R \leftrightarrow \alpha R \qquad 1 \leftrightarrow \alpha(10^4) \qquad \alpha = 10^{4}$

$C' = 7.07\,pF$

$L_1' = 142\,mH$

$L_2' = 1.428\,mH$

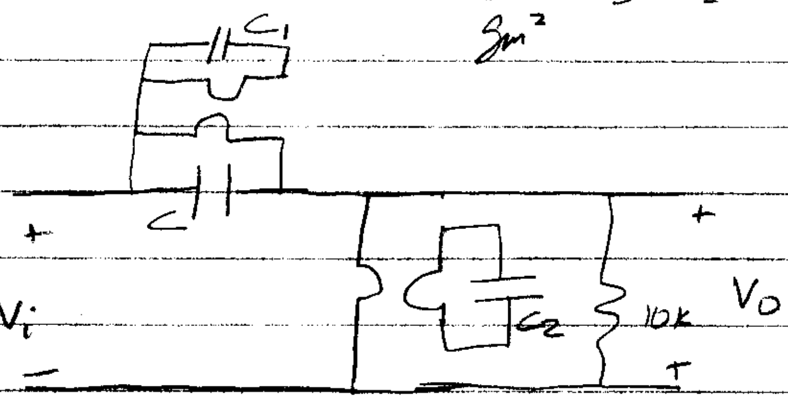


replace L_1', L_2' by gyrators.

$g_m = 10^{-4}$

$L_1 = \frac{C_1}{g_m^2} \Rightarrow C_1 = 1.41 nF$

$L_2 = 1.428 m = \frac{C_2}{g_m^2} \Rightarrow C_2 = 14.28 pF$



4.

LP ↔ BP

$$S/\Omega_z = \frac{\Omega_p}{\Omega_z \omega_b} \frac{s^2 + \omega_0^2}{s}$$

$\omega_0 = \sqrt{\omega_{p1}\omega_{p2}} = 110 M rad/s$

$\omega_b = \omega_{p2} - \omega_{p1} = 91 M rad/s$

$L \equiv (L' + C')$ $\quad L' = \frac{L\Omega_p}{\omega_b} = 67.29 nH$

$C' = \frac{\omega_b}{\omega_0^2 \Omega_p L} = 1.23 nF$

$C_1 \equiv (L_1' \parallel C_1')$ $\quad L_1' = \dfrac{\omega_b}{\omega_0^2 \Omega_p C_1} = 0.248\,\mu H$

$C_1' = \dfrac{C_1 \Omega_p}{\omega_b} = 0.33\,nF$

$C_2 \equiv (L_2' \parallel C_2')$ $\quad L_2' = 2.5\,nH \quad C_2' = 32.9\,nF$

Impedance Scaling

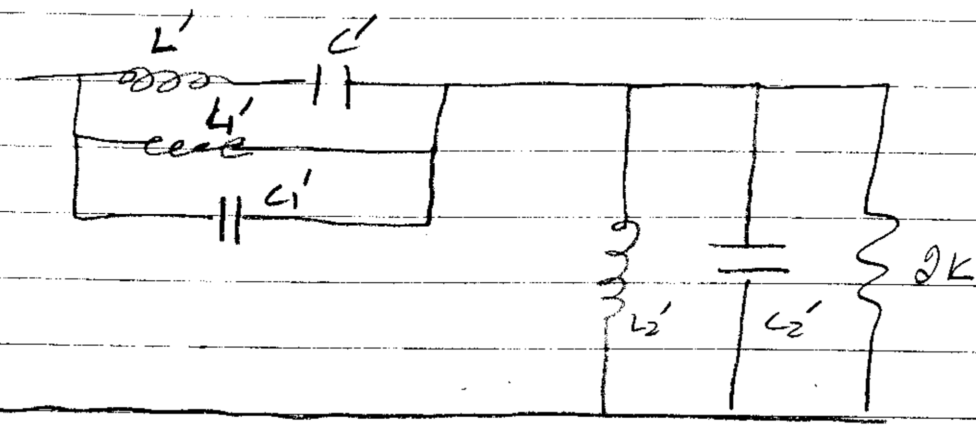
$R \leftrightarrow \alpha R \qquad 1 \leftrightarrow \alpha(2\times10^3) \qquad \alpha = \dfrac{1}{2\times10^3}$

$\therefore L' = 134.58\,\mu H$

$C' = 0.615\,pF$

$L_1' = 0.496\,mH \qquad L_2' = 5.02\,\mu H$

$C_1' = 0.165\,p \qquad C_2' = 16.455\,pF$



Stopband edges $\quad \Omega_s = \dfrac{\Omega_p}{\omega_b} \dfrac{\omega_0^2 - \omega_s^2}{\omega_s} \qquad 3 = \dfrac{1}{81M} \dfrac{(\omega_s^2 - (110M)^2)}{\omega_s}$

solve to get $\omega_{s1} = 145.9\,M \quad \omega_{s2} = 82.9\,M\,rad/s$

Gain @ 110 Mrad/s $\approx 1 \approx 0\,dB$ $(\omega_0 = 110\,M\,rad/s)$

$\omega_{z1}\,\omega_{z2} = \omega_0^2 \quad \omega_{z2} = \dfrac{(110M)^2}{47M} = 257.45\,Mrad/s$

5/ LP – BR transform

$$\frac{s}{\Omega_p} \leftrightarrow \frac{\omega_b \cdot s}{\omega_0^2 + s^2}$$

$$\omega_0 = \sqrt{\omega_{s1}\omega_{s2}} = \sqrt{8.1M \times 10M} = 9M \text{ rad/s}$$

$$\frac{\Omega_s}{\Omega_p} = \frac{\omega_b \omega_s}{\omega_0^2 - \omega_s^2} = \frac{\omega_b \omega_{s1}}{(\omega_{s1}\omega_{s2} - \omega_{s1}^2)} = \frac{\omega_b}{\omega_{s2} - \omega_{s1}}$$

$$\therefore \omega_b = 3 \times 1.9M \text{ rad/s} = 5.7M \text{ rad/s}$$

$$L \equiv (L' \parallel C') \quad L' = \frac{\omega_b \Omega_p}{\omega_0^2} L = 99.5 \text{ nH}$$

$$C' = \frac{1}{\Omega_p \omega_b L} = 194.2 \text{ nF}$$

$$C_1 \equiv (L_1' + C_1') \quad L_1' = \frac{1}{\Omega_p \omega_b C_1} = 24.8 \mu H$$

$$C' = \frac{\Omega_p \omega_b}{\omega_0^2} C_1 = 497.6 pF$$

$$C_2 \equiv (C_2' + L_2') \quad L_2' = 253.8 \text{ nH}$$

$$C_2' = 48.7 \text{ nF}$$

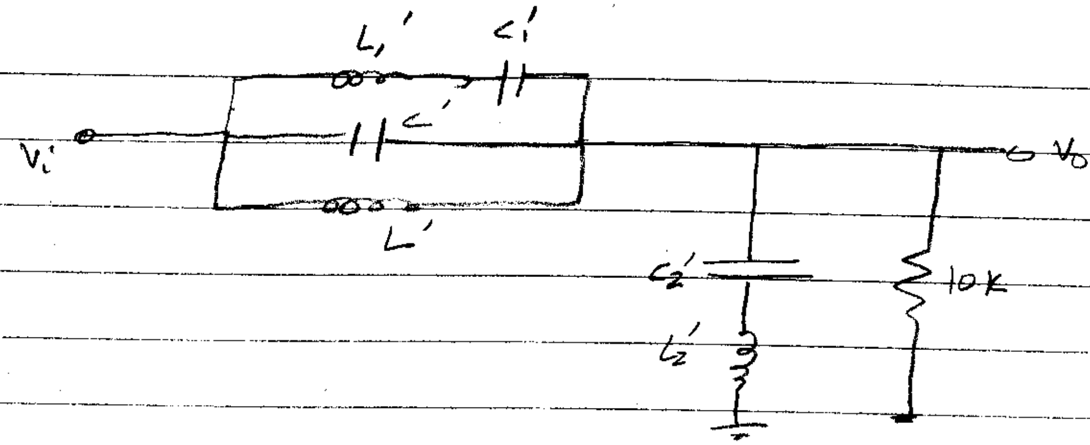
<u>Impedance Scaling</u>

$$R \leftrightarrow \alpha R \qquad 1 \leftrightarrow \alpha (10^4) \qquad \alpha = 10^{-4}$$

$$L' = (99.5n \times 10^4) = 994 \mu H$$

$$C' = 19.416 pF \qquad L_1' = 250.6 \text{ mH}$$

$$C_1' = 49.26 \text{ ff} \quad L_2' = 2.54 \text{ mH} \qquad C_2' = 4.86 pF$$



Passband edges

$$\omega_{p1} - \omega_{p2} = \omega_b = 5.7M$$

$$\sqrt{\omega_{p1}\omega_{p2}} = \omega_o = 9M \text{ rad/s}$$

solve to get $\omega_{p1} = 6.59M$ rad/s $\omega_{p2} = 10.89M$ rad/s

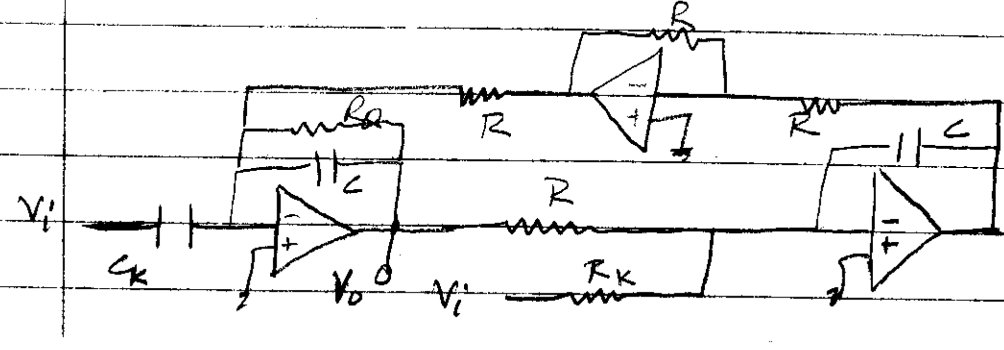
@ 9M rad/sec $|H(j\omega_o)| = |H_{prototype}(j\omega)| = -40\text{ dB}$

<u>6</u>

$$H(s) = \left(\frac{\omega_z}{\omega_p}\right)^2 \frac{1 + \left(\frac{s}{\omega_z}\right)^2}{1 + \frac{s}{\omega_p Q} + \left(\frac{s}{\omega_p}\right)^2}$$

Use V_{i3}, R $\leftarrow$ $= \dfrac{\left(\frac{\omega_z}{\omega_p}\right)^2}{D(s)} + \dfrac{\left(\frac{s}{\omega_p}\right)^2}{D(s)} \rightarrow$ Use V_{i1}, C

Take output at output of first opamp



$$\omega_p = \frac{1}{RC} = 10M \text{ rad/s}$$

$C = 100pF \quad R = 1K\Omega \quad R_Q = \frac{1}{\sqrt{2}} k\Omega$

$$\frac{R_K}{R} = \left(\frac{\omega_p}{\omega_z}\right) \quad \therefore R_K = 100R = 1M\Omega$$

7 Frequency Scaling [from filter in (4)]

$s \leftrightarrow s/\beta$

$100 M \leftrightarrow 10M \times 2\pi/\beta \quad \beta = \frac{2\pi}{10} = \frac{\pi}{5}$

$L' = \frac{13\mu}{\beta} = 0.01 mH \quad C' = .615/\beta \, pF = 0.97pF$

similarly $L_1' = 0.79 mH \quad C_1' = 0.26pF$

$L_2' = 7.99 \mu H \quad C_2' = 26 pF$