HW8 Solutions

1

$$H_c(s) = \frac{2}{1 + s/\omega_p}$$

(a) Bilinear Transform $\qquad s = 2f_s \cdot \frac{1-z^{-1}}{1+z^{-1}}$

$$H_d(z) = \frac{2}{\left[1 + \frac{2}{T_s\omega_p}\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{2(1+z^{-1})}{\left(1+\frac{2}{T_s\cdot\omega_p}\right)+\left(1-\frac{2}{T_s\cdot\omega_p}\right)z^{-1}}$$

$$= \frac{2(1+z^{-1})}{16.9 - 14.9z^{-1}}$$ for $\omega_p = 2\pi \times 20K$ rad
pole $\approx .125$

$$= \frac{2(1+z^{-1})}{2.592 - 0.59z^{-1}}$$ pole $= 1.12$

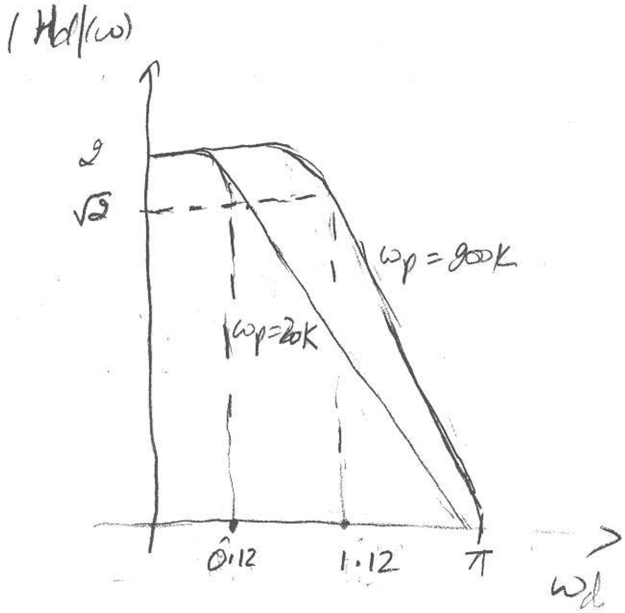
$$H_d(e^{j\omega_d}) = \frac{2 + 2e^{-j\omega_d}}{\left(1+\frac{2}{T_s\omega_p}\right)+\left(1-\frac{2}{T_s\omega_p}\right)e^{-j\omega_d}} = \frac{2 + 2\cos\omega_d - 2j\sin\omega_d}{1+\frac{2}{T_s\omega_p} - \left(1-\frac{2}{T\omega_p}\right)\cos\omega_d + j\left(1-\frac{2}{T\omega_p}\right)\sin\omega_d}$$

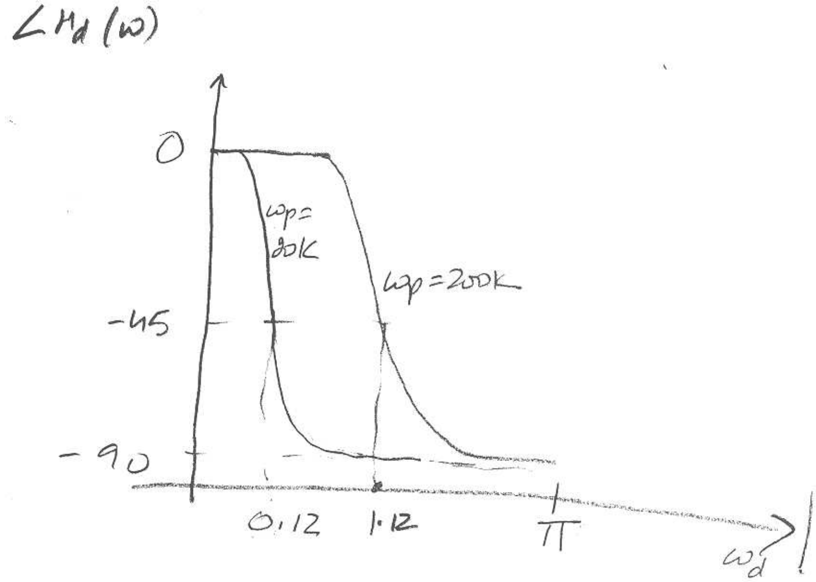
$\omega_p = 2\pi \times 20K$
$$|H_d(e^{j\omega_d})|^2 = \frac{(2+2\cos\omega_d)^2 + (2\sin\omega_d)^2}{(16.9 - 14.9\cos\omega_d)^2 + (14.9\sin\omega_d)^2} = \frac{2(1+\cos\omega_d)}{197 - 126\cos\omega_d}$$

$$\measuredangle H_d(e^{j\omega}) = \tan^{-1}\left(-\frac{\sin\omega_d}{1+\cos\omega_d}\right) - \tan^{-1}\left(\frac{7.46\sin\omega_d}{8.46 - 7.46\cos\omega_d}\right)$$

$\omega_p = 2\pi \times 200K$

$$H_d(e^{j\omega}) = \frac{2(1+\cos\omega_d)}{2.25 - 0.59z^{-1}}$$





(b) LDI Transform

$$H_d(z) = \frac{2}{1+\frac{1}{T_s\omega_p}\left(z^{1/2}-z^{-1/2}\right)}$$

$\omega_p = 2\pi \times 20kHz$

$\omega_p = 2\pi \times 200kHz$

$$H_d(e^{j\omega_d}) = \frac{2}{1+j\,15.95 \sin\frac{\omega_d}{2}}$$

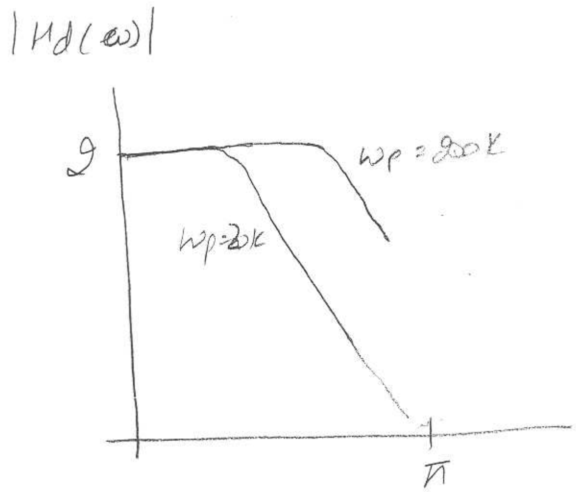
$$H(d)(e^{j\omega_d}) = \frac{2}{1+1.6 \sin\frac{\omega_d}{2}}$$

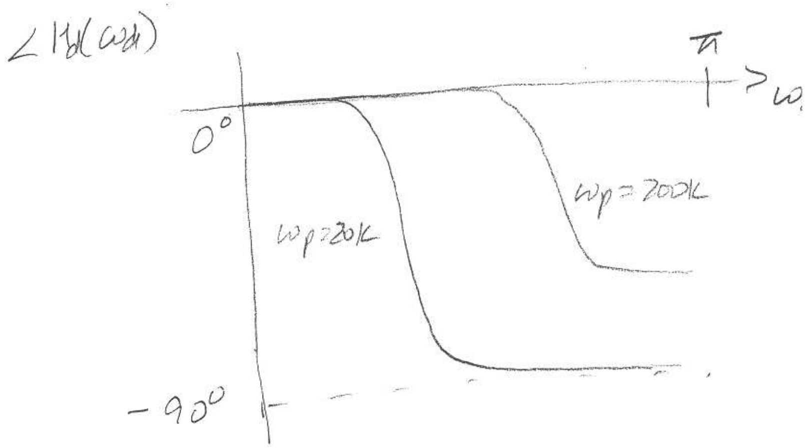
$$|H_d(e^{j\omega_d})| = \frac{4}{1+1.6 \sin^2\frac{\omega_d}{2}}$$

$$|H_d|^2 = \frac{4}{1+15.95^2 \sin^2\frac{\omega_d}{2}}$$

$$\angle H_d(e^{j\omega}) = -\tan^{-1}\left(15.9 \sin\frac{\omega_d}{2}\right)$$

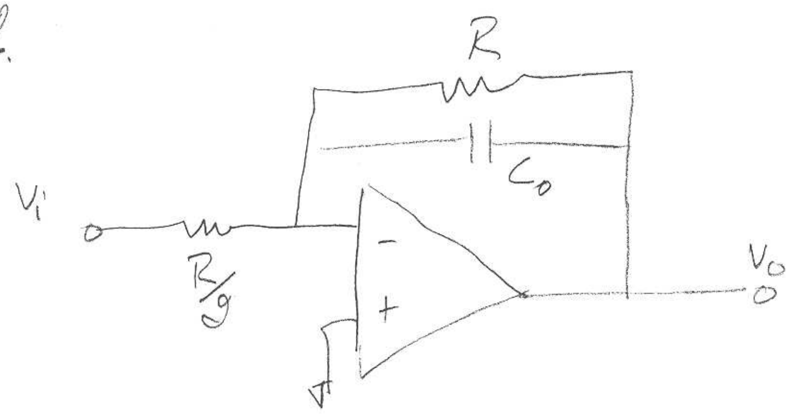
$$\angle |H_d(e^{j\omega})| = -\tan^{-1}\left(1.6 \sin\frac{\omega_d}{2}\right.$$





Problem 2

(a)

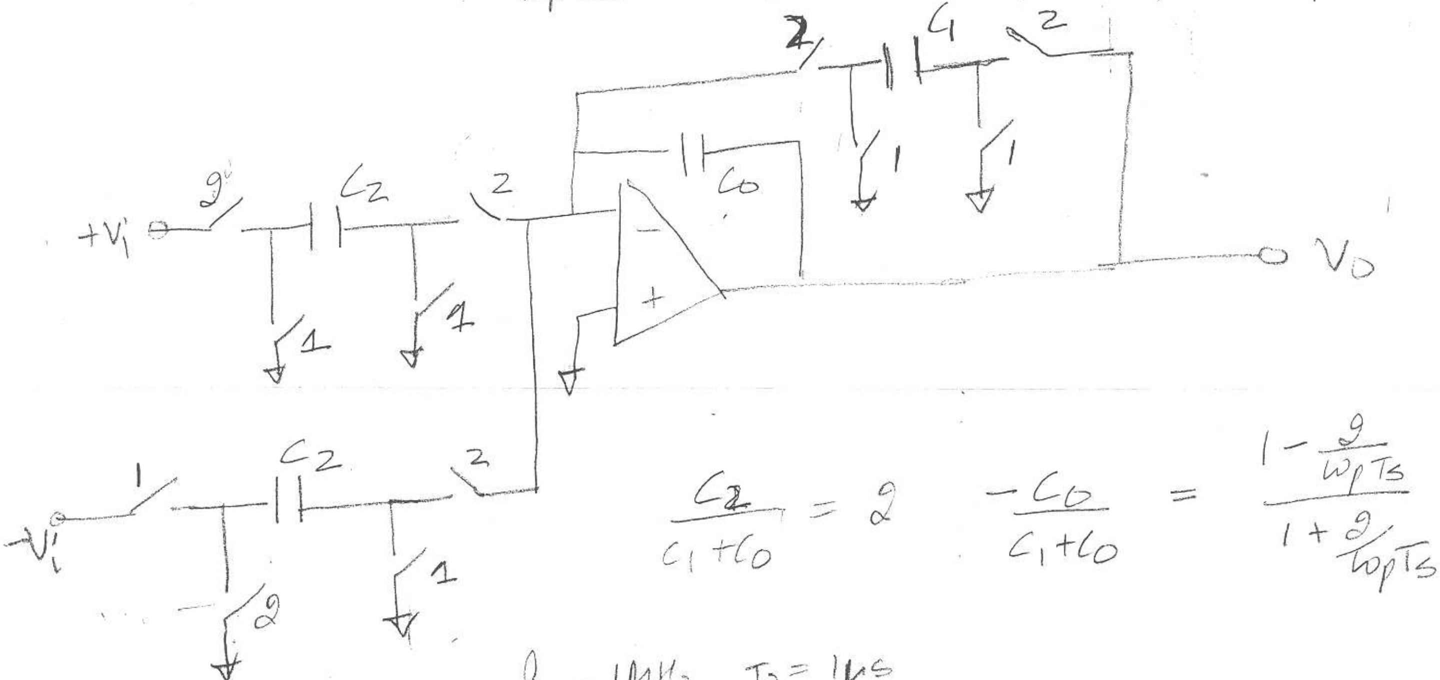


$\omega_p = \frac{1}{RC}$

(i) $\omega_p = 2\pi \times 20K$ $R = 1K\Omega$ $C_0 = 8\,nF$

(ii) $\omega_p = 2\pi \times 20K$ $R = 1K\Omega$ $C_0 = 0.8\,nF$

(b) $$H_d(z) = \frac{2(1+z^{-1})}{\left(1+\frac{2}{\omega_p T_s}\right) + \left(1-\frac{2}{\omega_p T_s}\right) z^{-1}} = \frac{2(1+z^{-1})}{1 + \frac{\left(1-\frac{2}{\omega_p T_s}\right)}{\left(1+\frac{2}{\omega_p T_s}\right)} z^{-1}}$$



$$\frac{C_2}{C_1+C_0} = 2 \qquad -\frac{C_0}{C_1+C_0} = \frac{1-\frac{2}{\omega_p T_s}}{1+\frac{2}{\omega_p T_s}}$$

$f_s = 1MHz$ $T_s = 1\mu s$

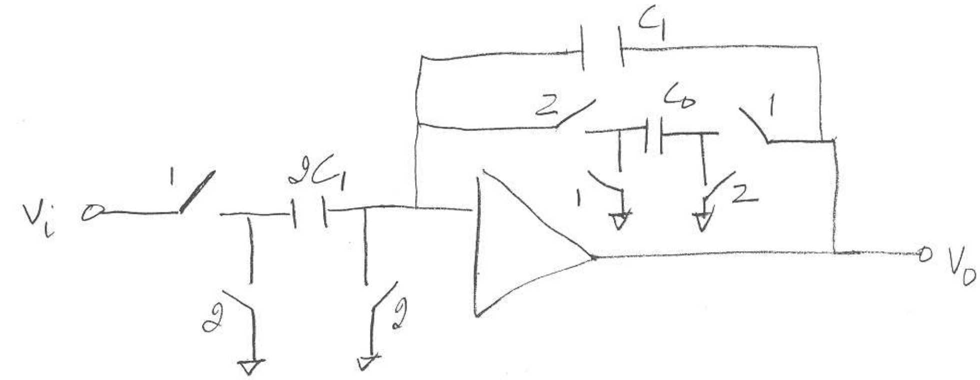
(i) $\omega_p = 2\pi \times 20k$ rad/s $\frac{C_0}{C_1+C_0} = 0.8817$

$C_0 = 1nF$ $C_1 = 0.134\,nF$ $C_2 = 2.268\,nF$

(ii) $\omega_p = 2\pi \times 200K$ rad/s

$C_0 = 1nF$ $C_1 = 3.385\,nF$ $C_2 = 8.77\,nF$

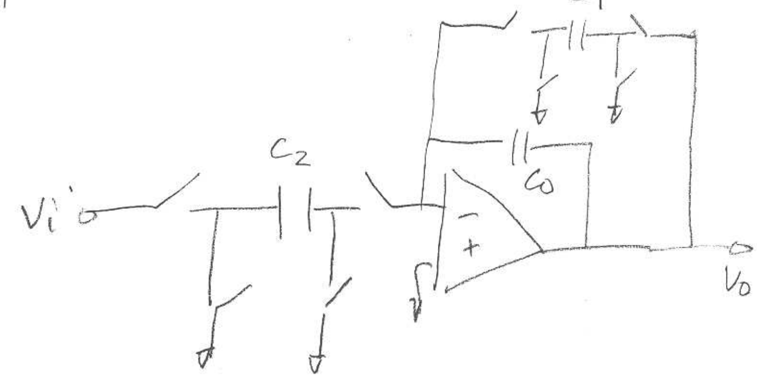
(c) $C_1 = \frac{1}{f_s R}$



(i) $\omega_p = 2\pi \times 20k$ rad/s $\quad C_1 = 1nF$

(ii) $\omega_p = 2\pi \times 200k$ rad/s $\quad C_1 = 1nF$

(d) $H(z) = \dfrac{2\Omega_p T_s}{\Omega_p T_s + z^{1/2} - z^{-1/2}}$



$$\frac{V_o}{V_i} = \frac{-C_2}{C_1+C_0} \frac{1}{1-\frac{C_0}{C_1+C_0}z^{-1}} = \frac{-C_2}{C_1+C_0}\frac{1}{(1-pz^{-1})}$$

$$\frac{C_2}{C_1+C_0} = 2 \quad -(1)$$

$\omega_p = 2\sin^{-1}\left(\frac{\Omega_p T_s}{2}\right)$ $T_s = 1\mu$

$\omega_p = 0.1257$ $\Omega_p = 2\pi \cdot 20k$

$\omega_p = 0.1358$ $\Omega_p = 2\pi \cdot 200$

$$\left|\frac{1}{1-pz^{-1}}\right|_{\omega=\omega_p} = \left|H_d(e^{j\omega_p})\right|_{\omega=\omega_p}$$

$$\left|\frac{1}{1-pz^{-1}}\right|_{\omega=\omega_p} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{|(1-p\cos\omega_p)+j\sin\omega_p|} = \frac{1}{\sqrt{2}} \qquad \frac{1}{(1-p\cos\omega_p)^2+p^2\sin^2\omega_p} = \frac{1}{2}$$

$$1+p^2-2p\cos\omega_p = 2.$$

$\Rightarrow$ $p = .4165$ for $\omega_p = 0.1257$

$p = .4169$ for $\omega_p = 0.1388$

$$\frac{C_0}{C_1+C_0} = .4165 \quad \omega_p = 0.1257$$

$$= .4169 \quad \omega_p = 0.1358$$

Say $C_0 = 1nF$

For $\Omega_p = 2\pi \times 20k$ rad/s

$C_1 = 1.40\ nF$

$C_2 = 4.802\ nF$

For $\Omega_p = 2\pi \times 200k$ rad/s

$C_1 = 1.398\ nF$

$C_2 = 4.7973\ nF$

4(a) (i) LP $N(s)=1$ (ii) BP $N(s) = \frac{s}{2\omega_p}$ (iii) HP $N(s) = \left(\frac{s}{\omega_p}\right)$

(iv) BE $N(s) = 1 + \left(\frac{s}{\omega_p}\right)^2$

(b) $s = \frac{2}{T_s} \frac{1-z^{-1}}{1+z^{-1}}$ $\qquad \omega_p = \frac{1}{10 \cdot T_s}$

LP $\quad H(z) = \dfrac{1}{1+5\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 20\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2}$

BP $\quad H(z) = \dfrac{5\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{1+5\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 20\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2}$

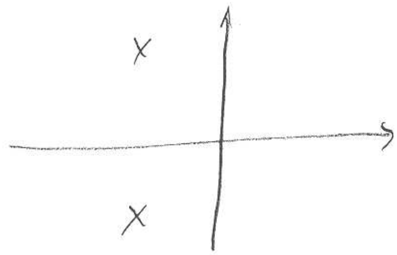
HP $\quad H(z) = \dfrac{20\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2}{1+5\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 20\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2}$

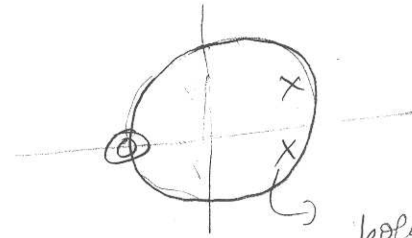
BE $\quad H(z) = \dfrac{1+20\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2}{1+5\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 20\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2}$

(C)

Continuous Time | Discrete Time

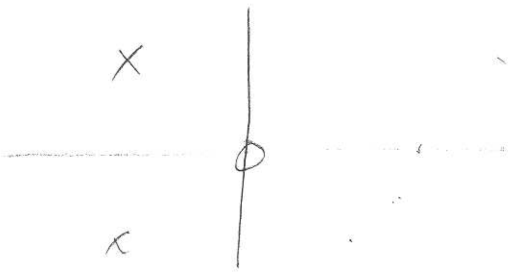
LP

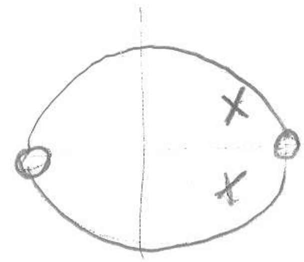




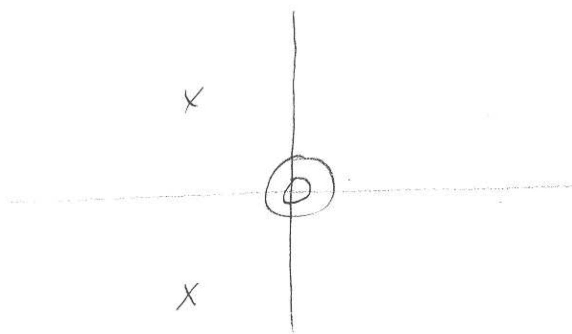
poles will have +ve real part if $f_s > \omega_p$

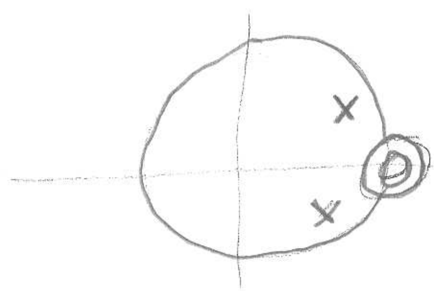
BP



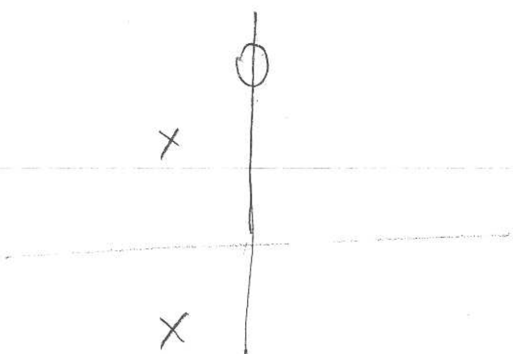


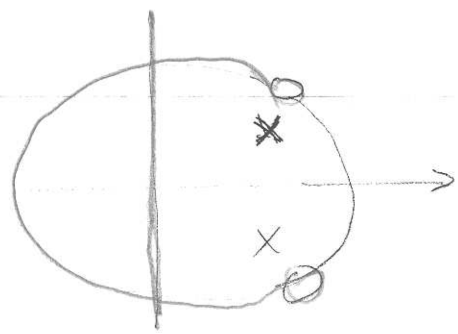
MP



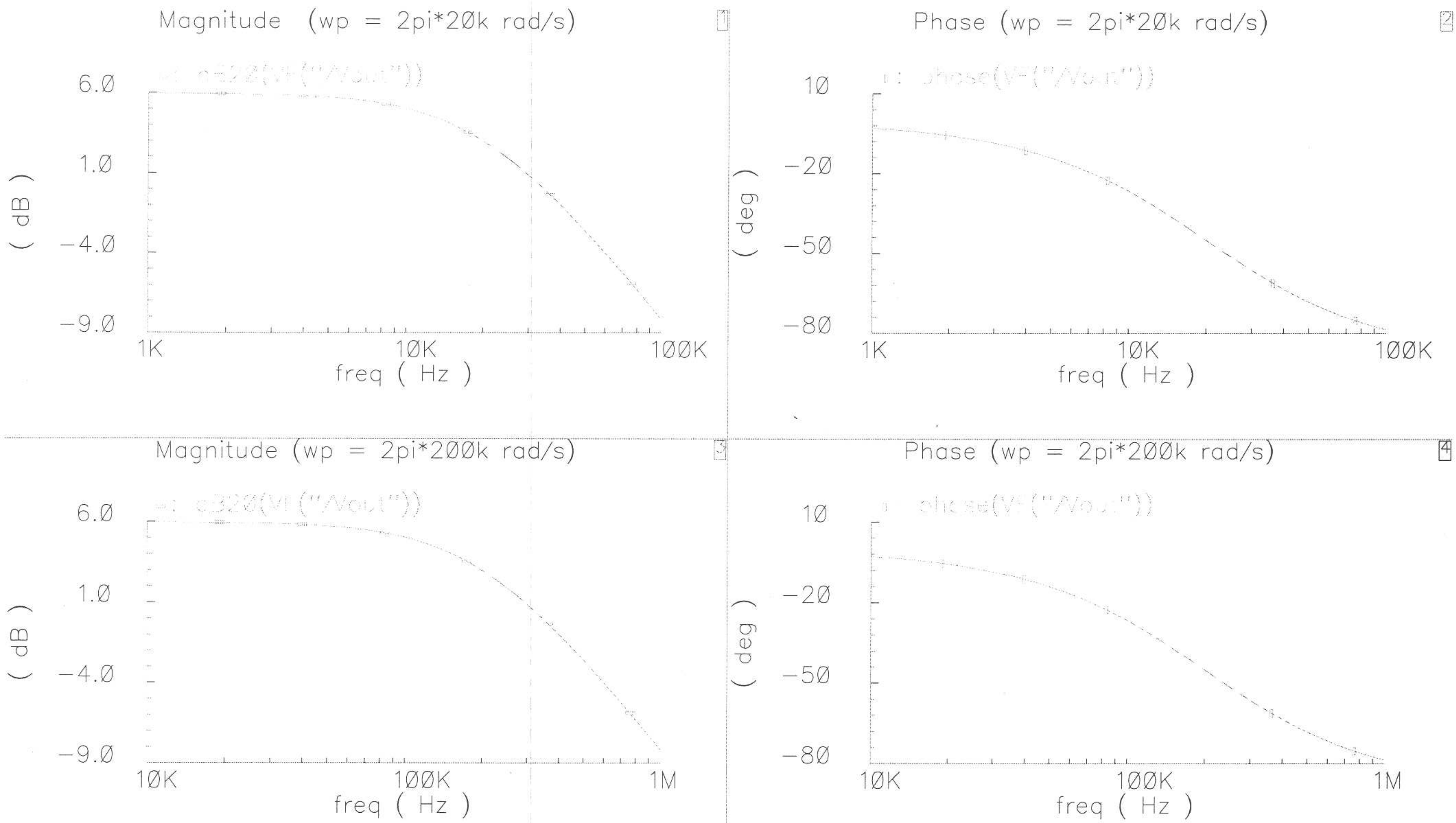


BE



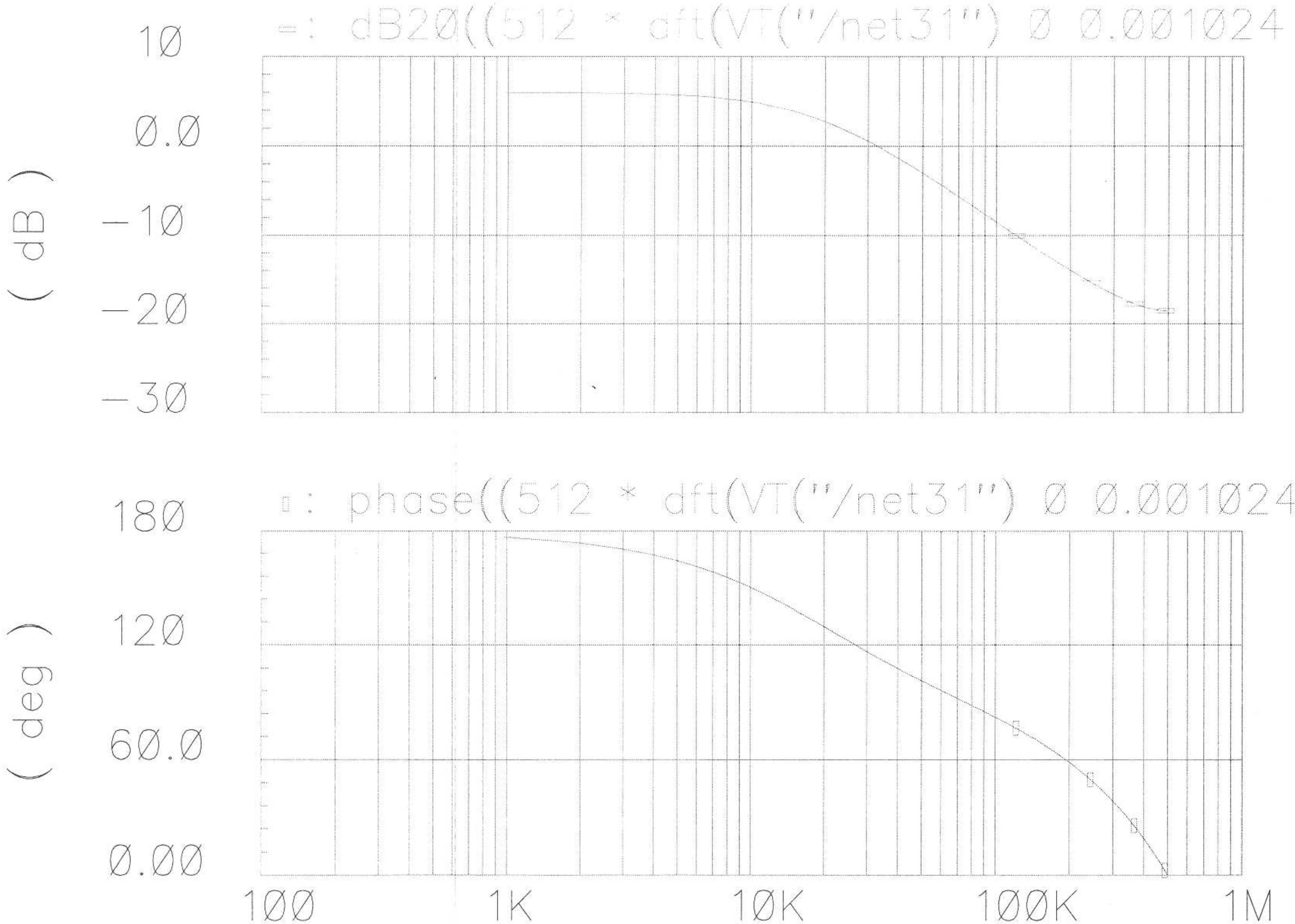


$$z = \frac{1 + \frac{j\omega_z}{f_s}}{1 - \frac{j\omega_z}{f_s}} \qquad |z| = 1$$

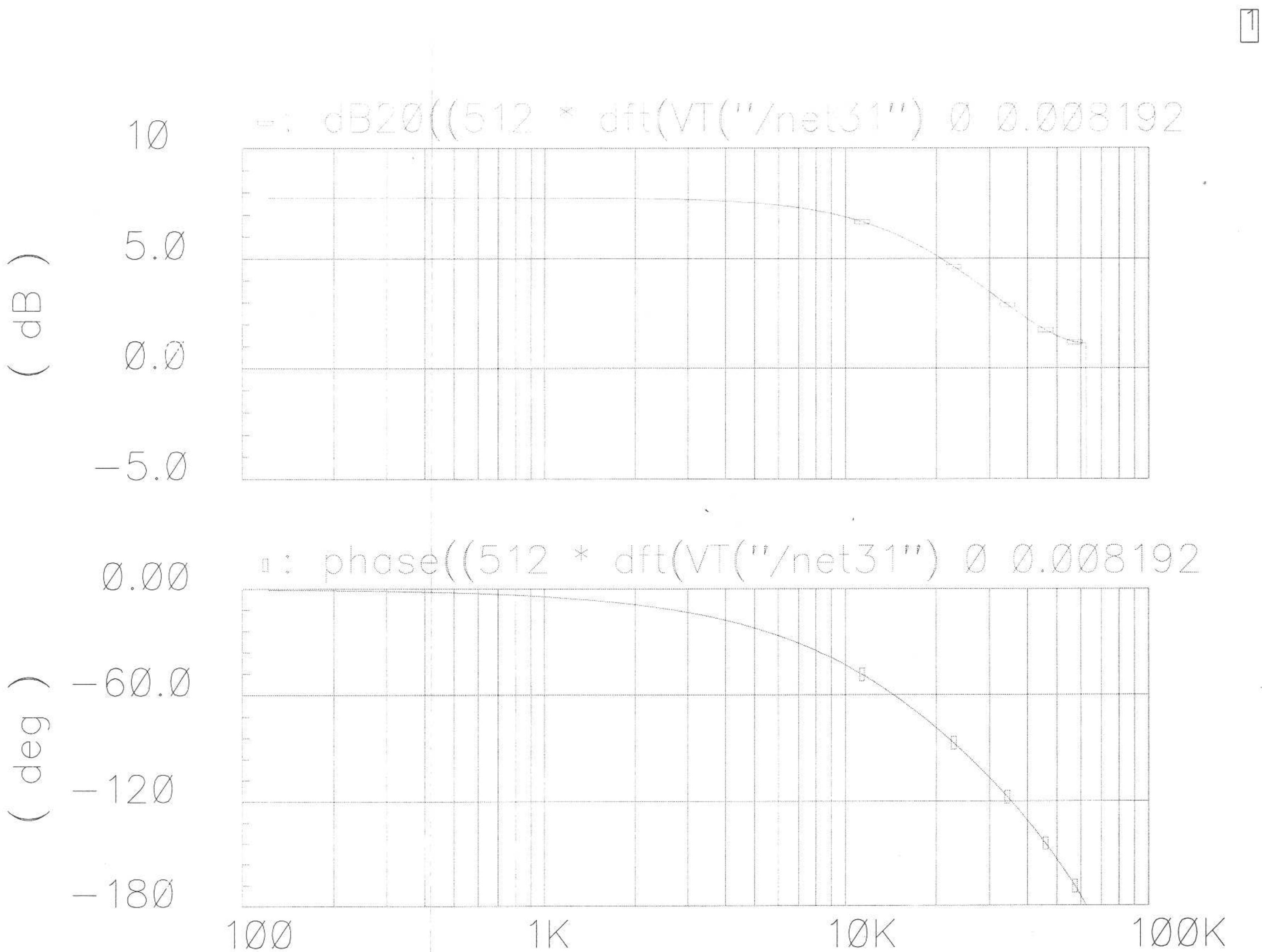


Magnitude (wp = 2pi*20k rad/s)
1
6.0
1.0
-4.0
-9.0
(dB)
1K
10K
100K
freq (Hz)
Phase (wp = 2pi*20k rad/s)
2
10
-20
-50
-80
(deg)
1K
10K
100K
freq (Hz)
Magnitude (wp = 2pi*200k rad/s)
3
6.0
1.0
-4.0
-9.0
(dB)
10K
100K
1M
freq (Hz)
Phase (wp = 2pi*200k rad/s)
4
10
-20
-50
-80
(deg)
10K
100K
1M
freq (Hz)

Prob 2a



2(b) $\omega_p = 2\pi \times 20\,k\,rad/s$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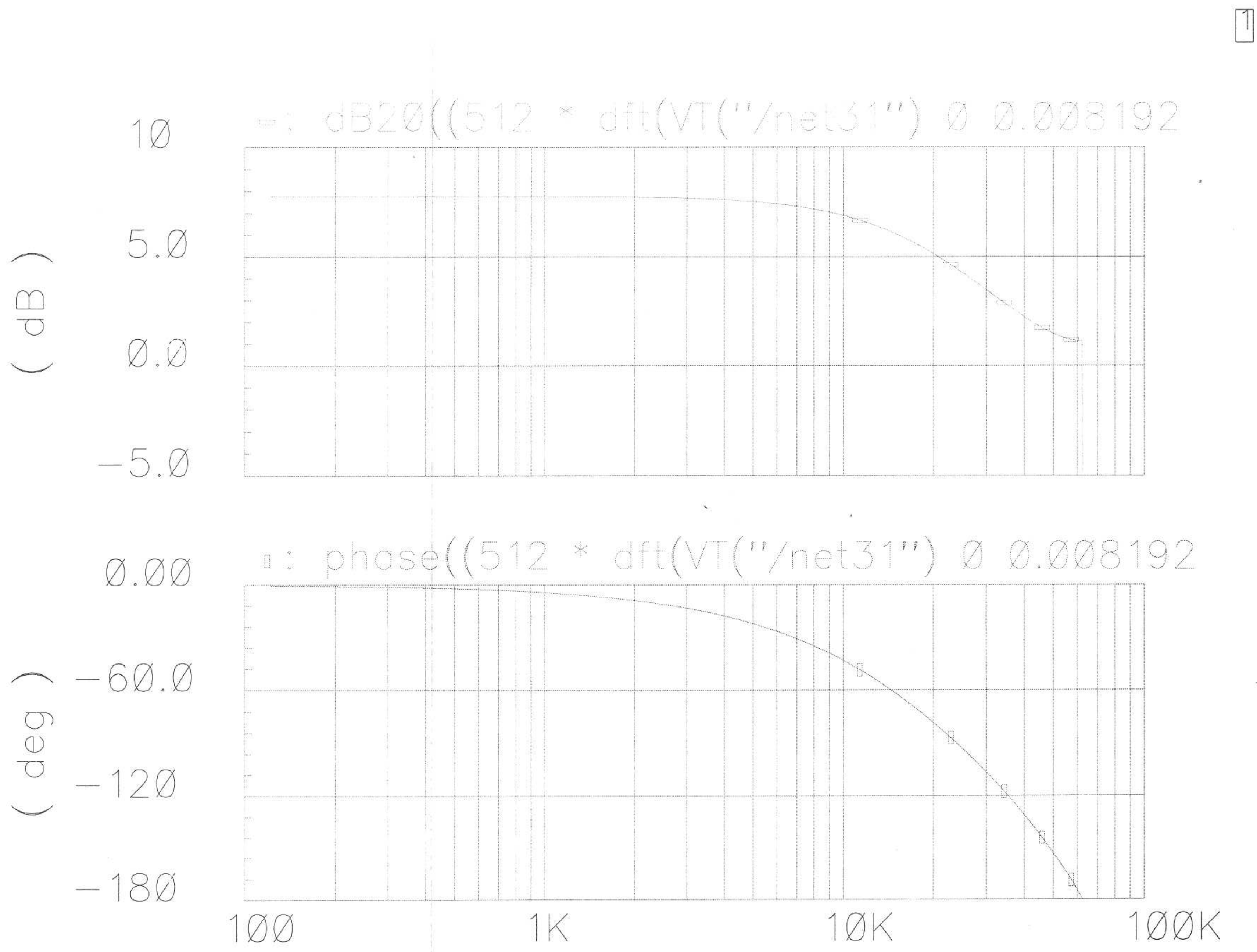
(dB)
10
5.Ø
Ø.Ø
−5.Ø
phase((512 * dft(VT("/net31") Ø Ø.ØØ8192
(deg)
Ø.ØØ
−6Ø.Ø
−12Ø
−18Ø
1ØØ
1K
1ØK
1ØØK

9(b) 20K rad/s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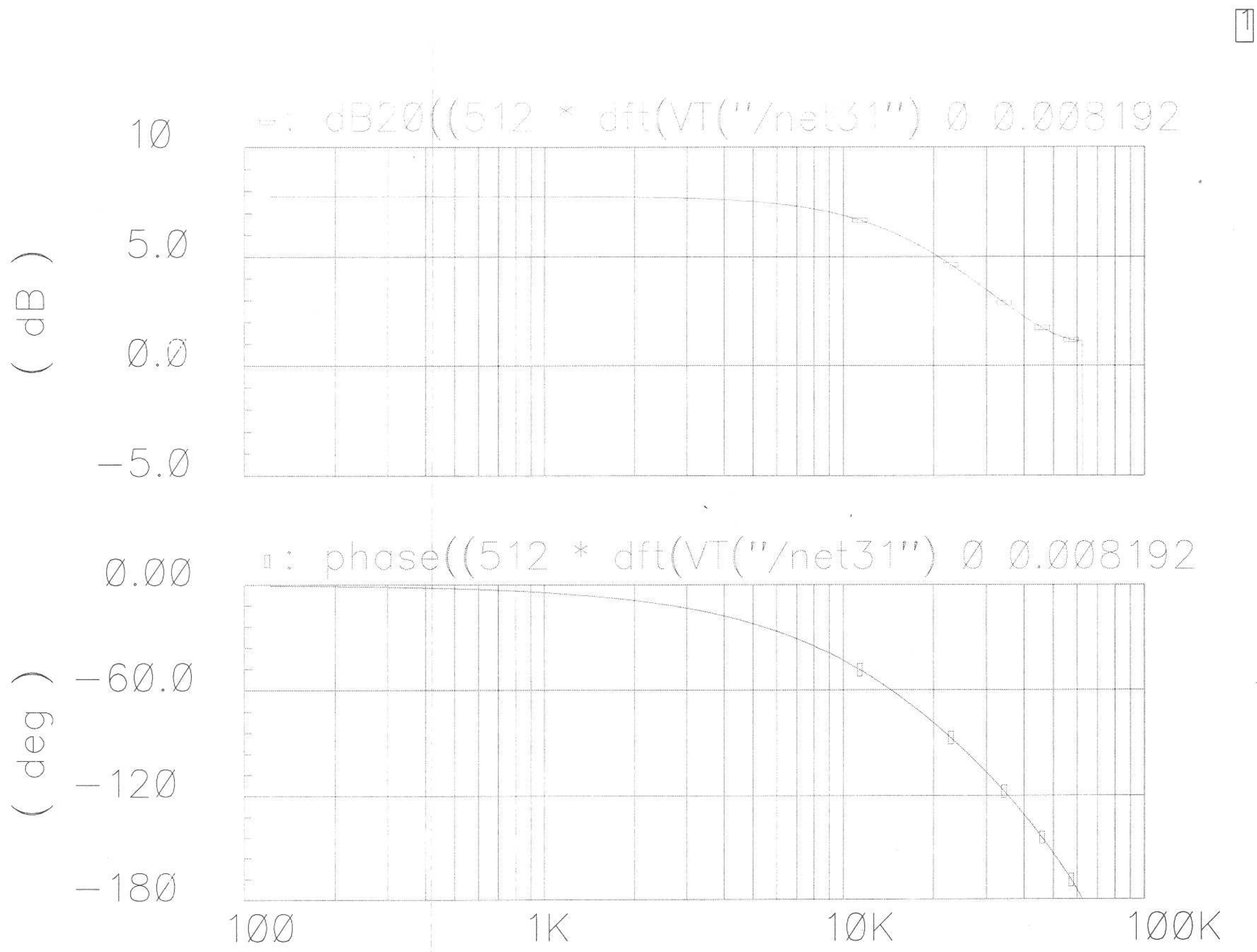
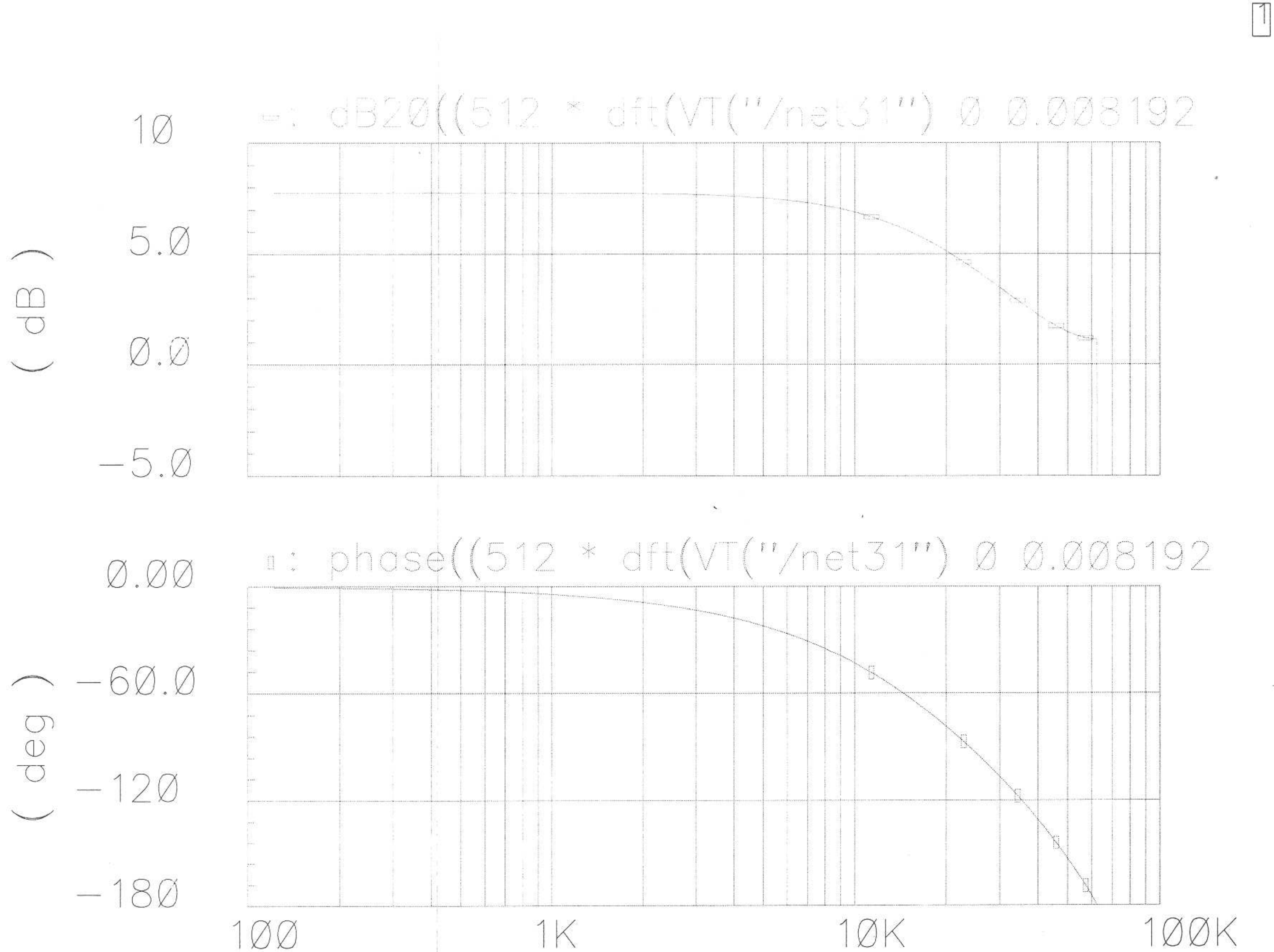
(dB)
10
5.0
0.0
−5.0
□ : phase((512 * dft(VT("/net31") 0 0.008192
(deg)
0.00
−60.0
−120
−180
100
1K
10K
100K

2(b) 20K rad/s


1
□: dB20((512 * dft(VT("/net31") 0 0.008192
(dB)
10
5.0
0.0
-5.0
□: phase((512 * dft(VT("/net31") 0 0.008192
(deg)
0.00
-60.0
-120
-180
100
1K
10K
100K

2(b) 20K rad/s



1
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10
5.0
0.0
-5.0
(dB)
□: phase((512 * dft(VT("/net31") 0 0.008192
0.00
-60.0
-120
-180
(deg)
100
1K
10K
100K

2(b) 20K rad/s