1(a)

(a) $V_o = b_0 I_R + b_1 I_C$

$$= b_0 \left(\frac{1}{1+SRC} \right) V_i + b_1 \left(\frac{SRC}{SRC+1} \right) V_i$$

$$= b_0 \left(\frac{1}{1+s/\omega_p} \right) V_i + b_1 \left(\frac{b_1 s/\omega_p}{1+s/\omega_p} \right) V_i$$

$$V_o/V_i = \frac{b_0 + b_1 s/\omega_p}{1+s/\omega_p}$$

(b) $V_o = b_0 I_C + b_1 I_R + b_2 I_C$

$$V_o/V_i = b_0 \left[\frac{sC}{sC + 1/R + 1/sL} \right] V_i + b_1 \left[\frac{1/R}{sC + 1/R + 1/sL} \right] V_i + b_2 \left[\frac{1/sL}{sC + 1/R + 1/sL} \right] V_i$$

$$= b_2 \left[\frac{(s/\omega_p)^2}{(s/\omega_p)^2 + s/Q\omega_p + 1} \right] V_i + b_1 \left[\frac{s/Q\omega_p}{(s/\omega_p)^2 + (s/Q\omega_p) + 1} \right] V_i + \frac{1}{(s/\omega_p)^2 + (s/Q\omega_p) + 1} V_i$$

$$= \frac{b_0 + b_1 (s/Q\omega_p) + b_2 (s/\omega_p)^2}{1 + \frac{s}{Q\omega_p} + (s/\omega_p)^2}$$

2(a) Butterworth filter

— 8 oder filter.

→ $\omega_p = \sqrt{\omega_r^2 + \omega_i^2}$ $\quad Q = \frac{\omega_p}{2\omega_r}$ $\quad \omega_r$: real part of pole

ω_i = imaginary part of pole

$\omega_{p1} = \sqrt{1.1031^2 + 0.2194^2} = 1.1247$

$\omega_{p2} = 1.1246$

$\omega_{p3} = 1.1247 \quad \omega_{p4} = 1.1246$

$Q_1 = 0.509 \quad Q_2 = 0.6013 \quad Q_3 = 0.9 \quad Q_4 = 2.563$

Chebyshev

5 oder filter.

$\omega_{p1} = 0.9941 \quad \omega_{p2} = 0.655 \quad \omega_{p3} = 0.2895$

Inverse Chebyshev

$\omega_{p1} = 1.1366 \quad \omega_{p2} = 1.288 \quad \omega_{p3} = 1.42$

$Q_1 = 2.0217 \quad Q_2 = 0.6817$

$\omega_{z1} = 3.0671 \quad \omega_{z2} = 1.895$

Eliptic

4th oder filter

$\omega_{p1} = 0.6014 \quad \omega_{p2} = 0.999$

$Q_1 = 0.8955 \quad Q_2 = 4.145$

$\omega_{z1} = 3.525 \quad \omega_{z2} = 1.6095$

(b) Cascade order.

(i) bilinear before biquads (if exists)

(ii) smaller Q biquads before larger Q biquads

Bessel Filter

$\omega_{p1} = 1.165$ $\omega_{p2} = 1.052$ $\omega_{p3} = 0.9843$ $\omega_{p4} = 0.943$

$\omega_{p5} = 0.9202$.

$Q_1 = 1.506$ $Q_2 = 0.858$ $Q_3 = 0.652$ $Q_4 = 0.557$ $Q_5 = 0.513$

You should set $A_s = -40dB$ at approx $\Omega_s = 2.33 rad/s$

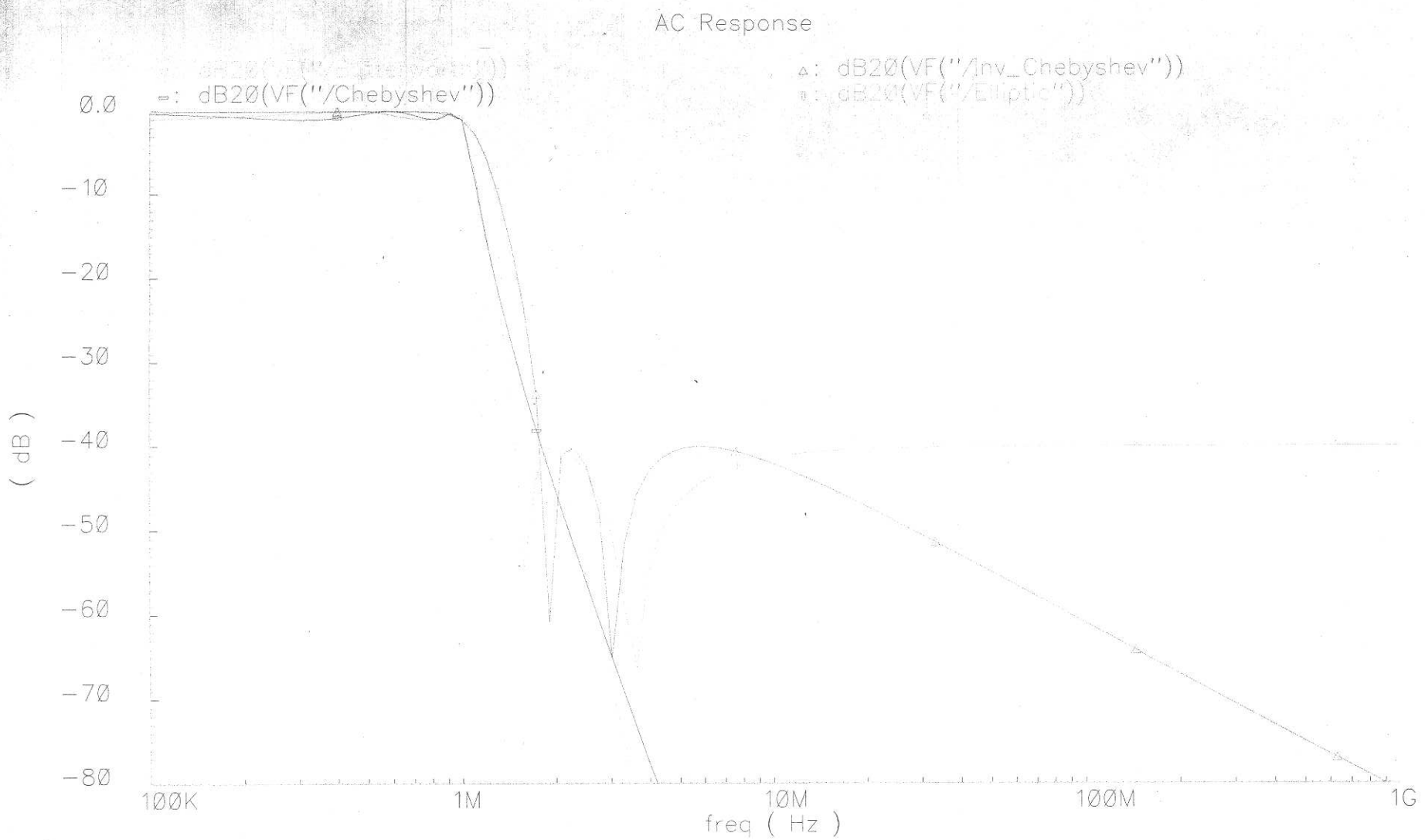
so $\Omega_p \geq \frac{2.33}{3} rad/s$ with $A_p \approx -8dB$

so it will not meet the specs.

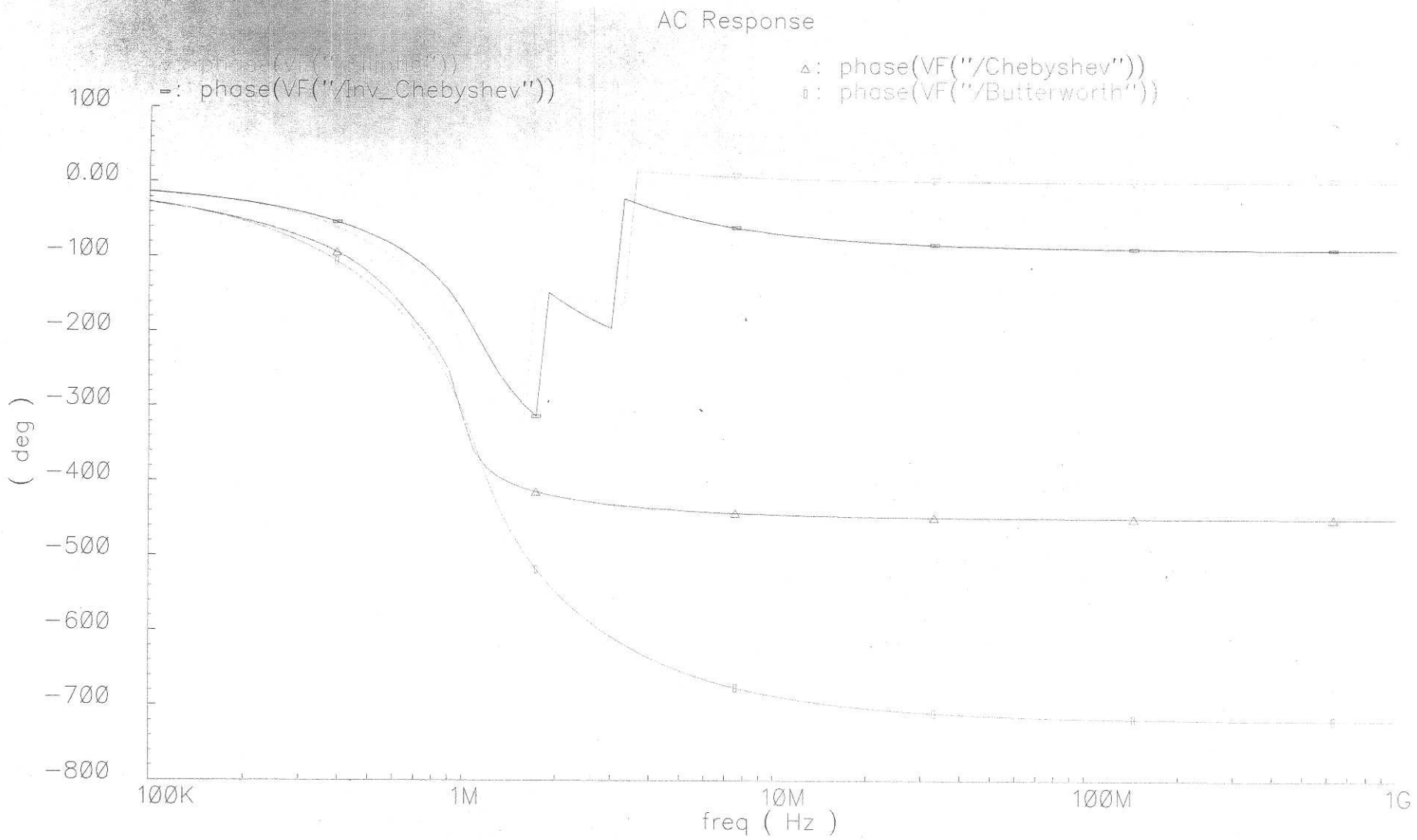
(c) Max gain of each filter stage is 1. So max possible input voltage should be 1 volt.

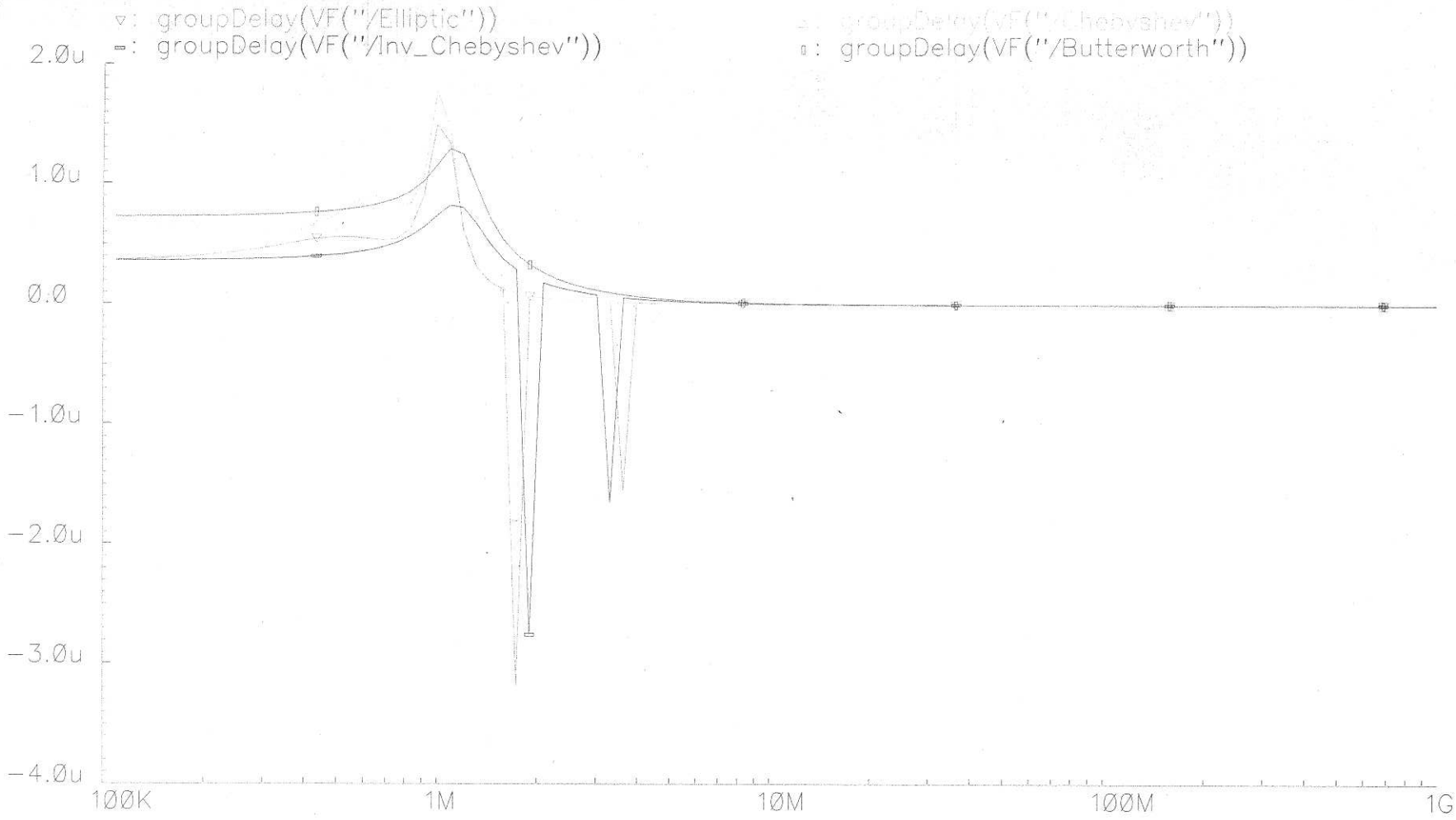
(d)

	Butterworth	Chebyshev	Inv-Chebyshev	Elliptic	Bessel
f_{pmax}	1.124 M	0.994 M	1.42 M	0.999 M	1.16 M
Q_{max}	2.56	5.55	9.02	4.74	1.5
Group delay (max variation)	550 ns	1.15 μs	300 ns	1 μs	50 ps


AC Response
▫: dB20(VF("/Chebyshev"))
▵: dB20(VF("/Inv_Chebyshev"))
▯: dB20(VF("/Elliptic"))
0.0
-10
-20
-30
-40
-50
-60
-70
-80
(dB)
100K
1M
10M
100M
1G
freq (Hz)

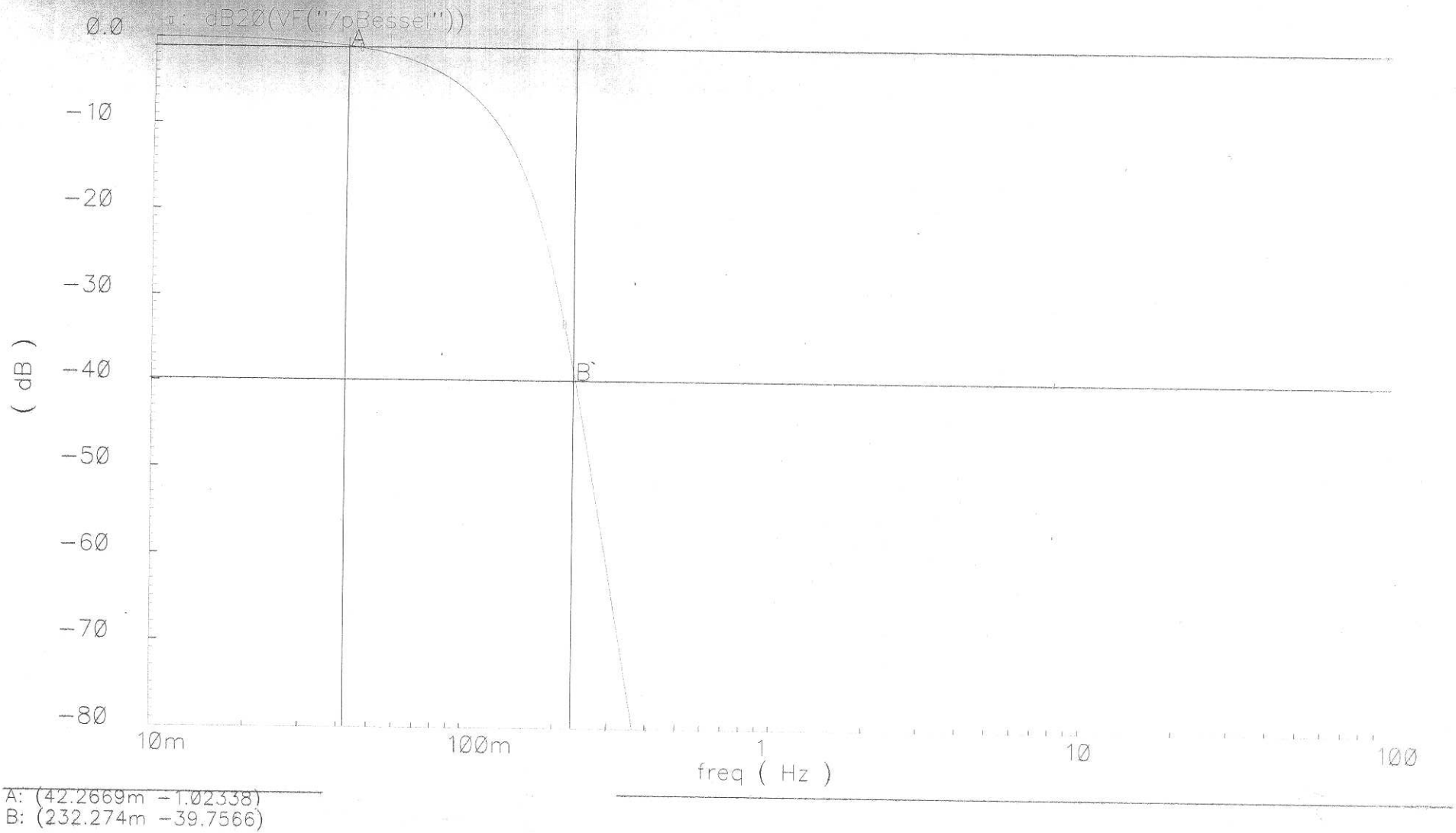
Mag Resp


AC Response
△: phase(VF("/Chebyshev"))
▯: phase(VF("/Butterworth"))
▫: phase(VF("/Inv_Chebyshev"))
100
0.00
-100
-200
-300
-400
-500
-600
-700
-800
(deg)
100K
1M
10M
100M
1G
freq (Hz)

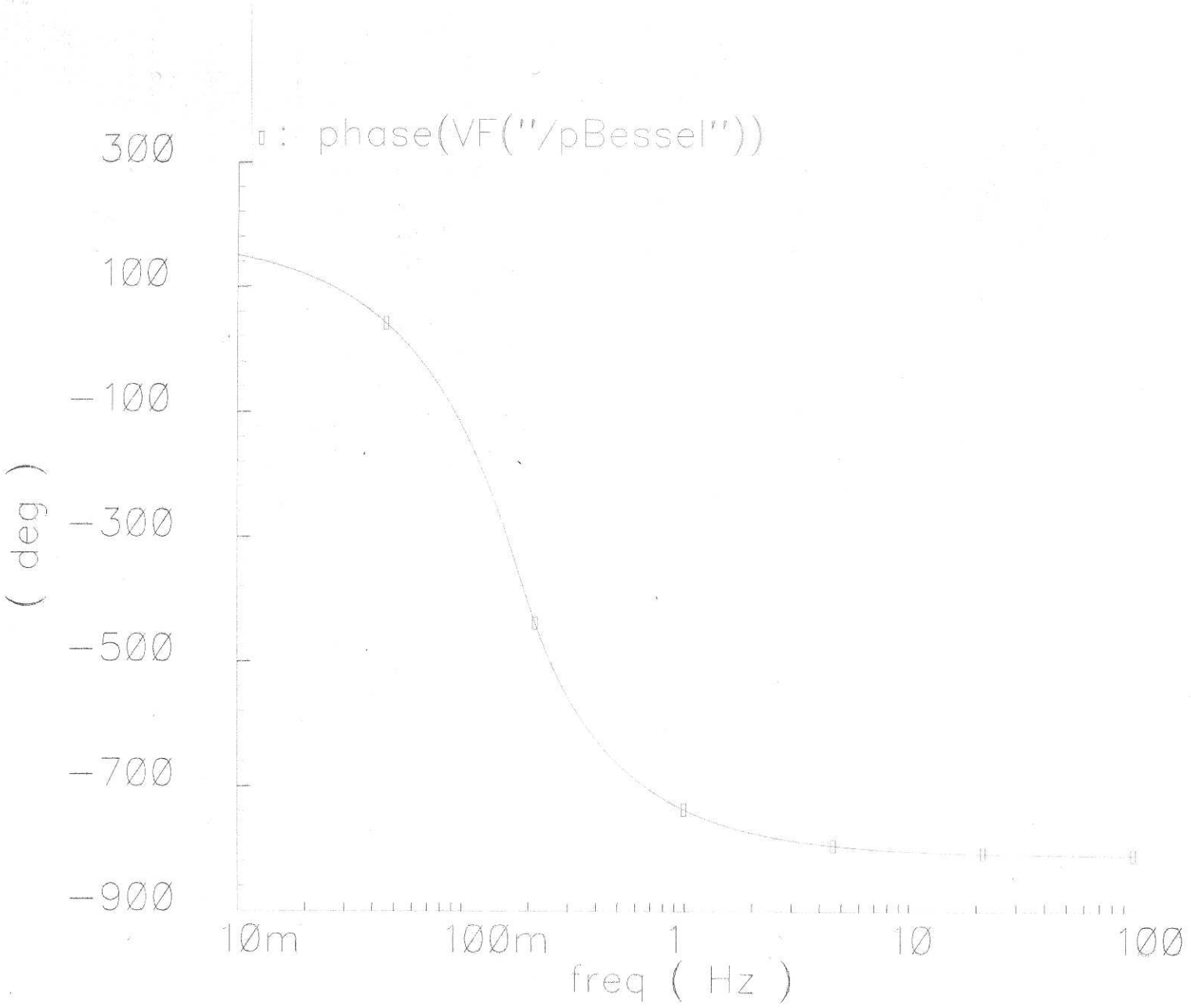

groupDelay(VF("/Elliptic"))
groupDelay(VF("/Inv_Chebyshev"))
groupDelay(VF("/Chebyshev"))
groupDelay(VF("/Butterworth"))
2.0u
1.0u
0.0
−1.0u
−2.0u
−3.0u
−4.0u
100K
1M
10M
100M
1G

Group Delay

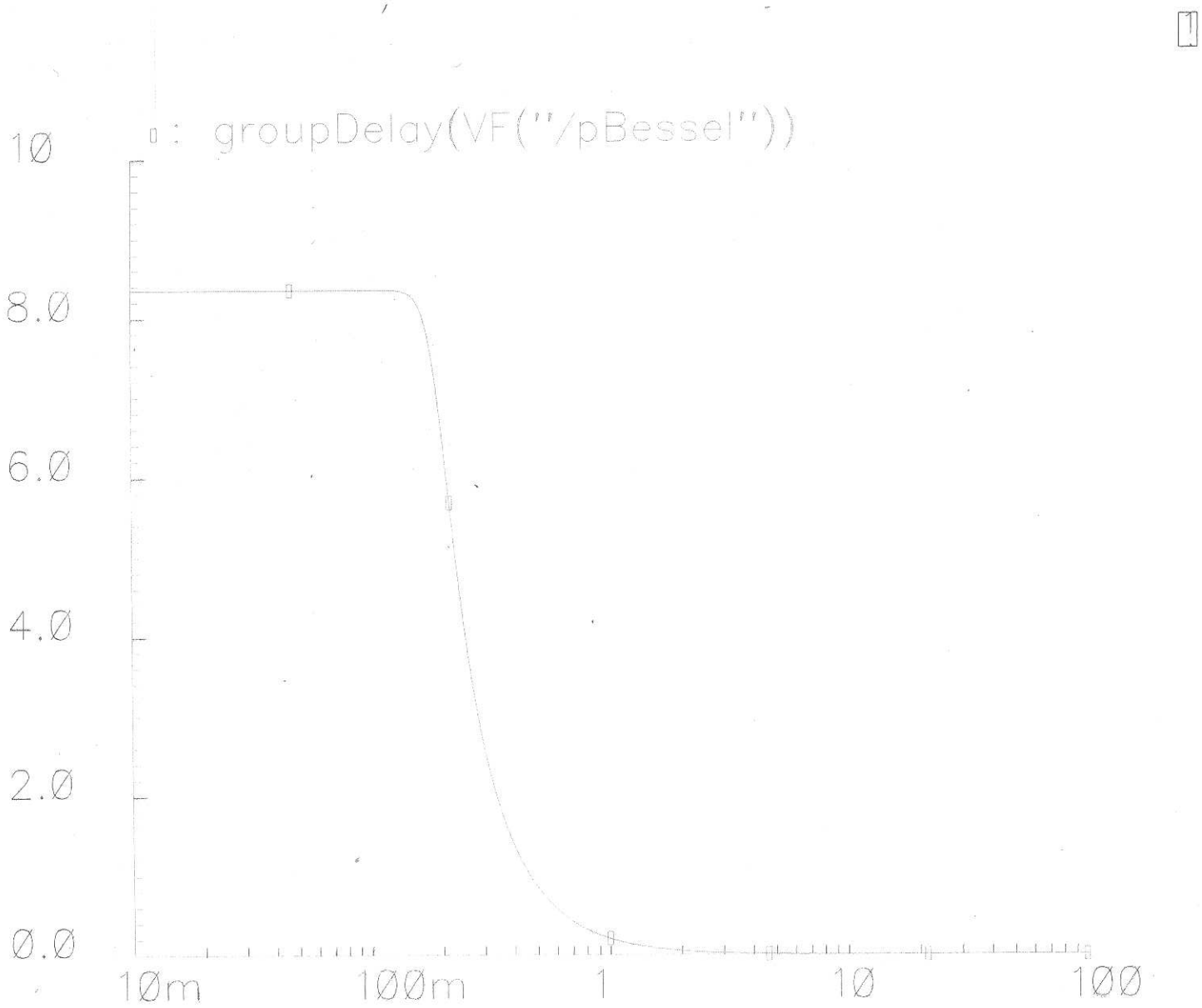
Bessel Prototype



A: (42.2669m -1.02338)
B: (232.274m -39.7566)



Bessel filter


groupDelay(VF("/pBessel"))
10
8.0
6.0
4.0
2.0
0.0
10m
100m
1
10
100