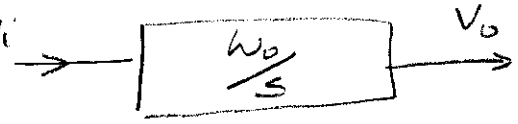
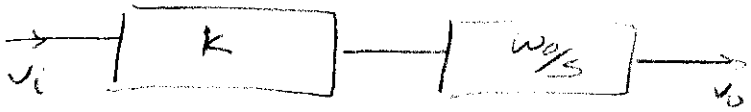
E 4915 HW 4 Analog Filters

Problem 1

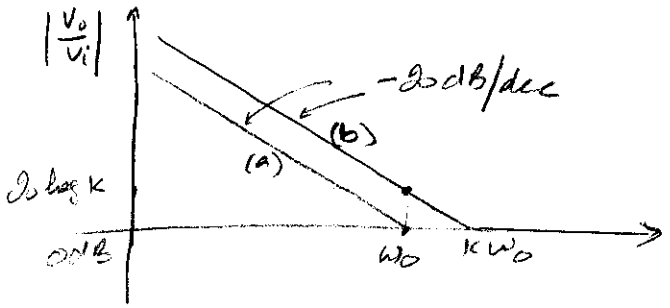
(a) 

$$\frac{V_0}{V_i} = \frac{\omega_0}{s}$$



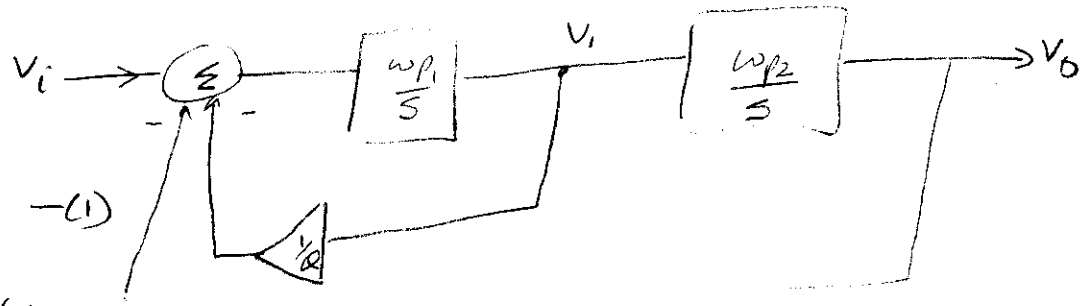
$$\frac{V_0}{V_i} = \frac{K\omega_0}{s}$$

Bode Plot



Unity gain freq of second integrator is K times that of first.

Problem 2



$$V_1 = V_i - \frac{V_1}{Q} - V_0 \quad -(1)$$

$$V_0 = V_1\left(\frac{\omega_{p2}}{s}\right) \quad -(2)$$

Eliminate V_1 from (1) & (2), to get

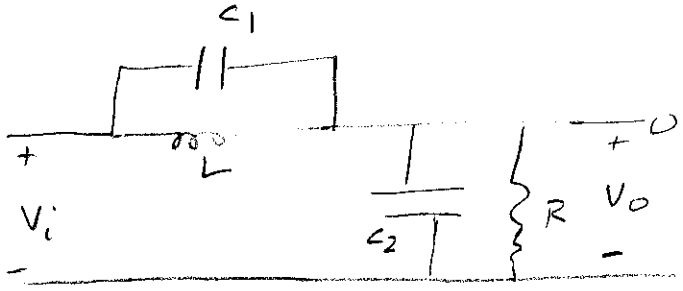
$$\frac{V_0}{V_i} = \frac{1}{\frac{s^2}{\omega_{p1}\omega_{p2}} + \frac{s}{Q_1\omega_{p2}} + 1}$$

Resonant frequency

$$\omega_p' = \sqrt{\omega_{p1}\omega_{p2}}$$

quality factor $Q' = \frac{Q\omega_{p2}}{\omega_p'} = Q\sqrt{\frac{\omega_{p2}}{\omega_{p1}}}$

Problem 3



$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1} = \frac{R \,||\, \frac{1}{sC_2}}{\left(R \,||\, \frac{1}{sC_2}\right) + \left(sL \,||\, \frac{1}{sC_1}\right)}$$

$$= \frac{\frac{R}{1+sRC_2}}{\frac{R}{1+sRC_2} + \frac{sL}{1+s^2RC_1L}} = \frac{1 + s^2LC_1}{s^2(C_1+C_2)L + sL/R + 1}$$

$$= \frac{1 + \frac{s^2}{\left(\frac{1}{\sqrt{LC_1}}\right)^2}}{\frac{s^2}{(C_1+C_2)L} + \frac{s}{R/L} + 1}$$

$\alpha = \frac{1}{\sqrt{2}}$, $C_1 + C_2 = 1nF$

$\omega_p^2 = \frac{1}{(C_1+C_2)L} = (2\pi \times 10^6)^2 \Rightarrow L = 25.3\,\mu H$

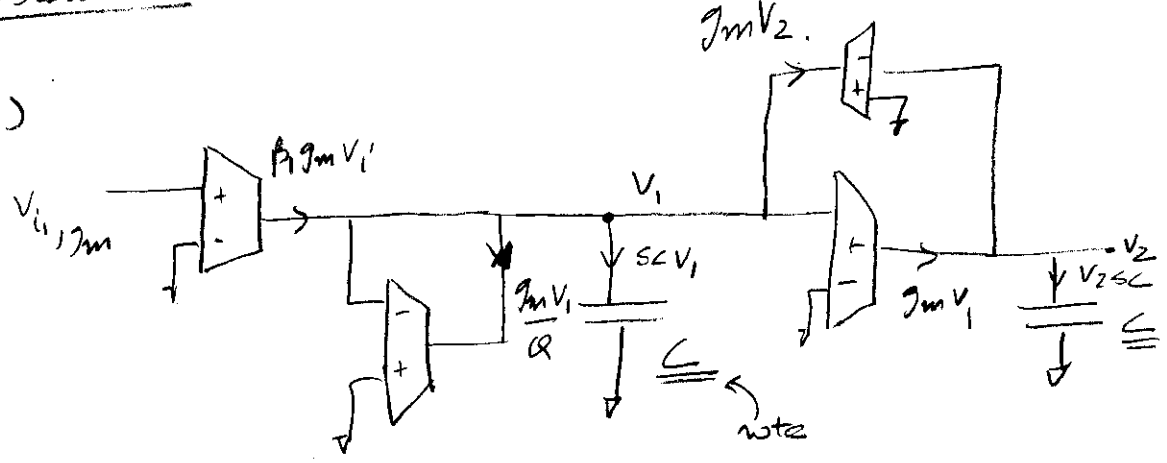
$\omega_z^2 = \frac{1}{C_1 L} = (2\pi \times 10^7)^2 \Rightarrow C_1 = 10\,pF$

$\Rightarrow C_2 = 990\,pF$

$Q = R\sqrt{\frac{C_1+C_2}{L}} \Rightarrow R = 112.4\,\Omega$

Problem 5

(1)



KCL

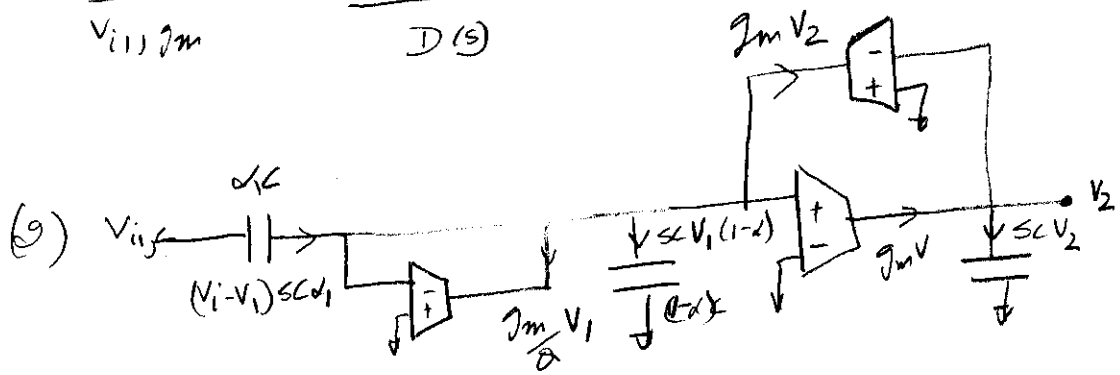
$$\beta_1 g_m V_i = \frac{g_m V_1}{Q} + sCV_1 + g_m V_2$$

$$g_m V_1 = V_2 sC$$

$$\frac{V_1}{V_i, g_m} = \frac{\frac{sC\beta_2}{g_m}}{\left(\frac{sC}{g_m}\right)^2 + \left(\frac{sC}{g_m Q}\right) + 1} = \frac{\left[\frac{sC\beta_2}{g_m}\right]}{D(s)}$$

$$\frac{V_2}{V_i, g_m} = \frac{\beta_1}{D(s)}$$

(2)



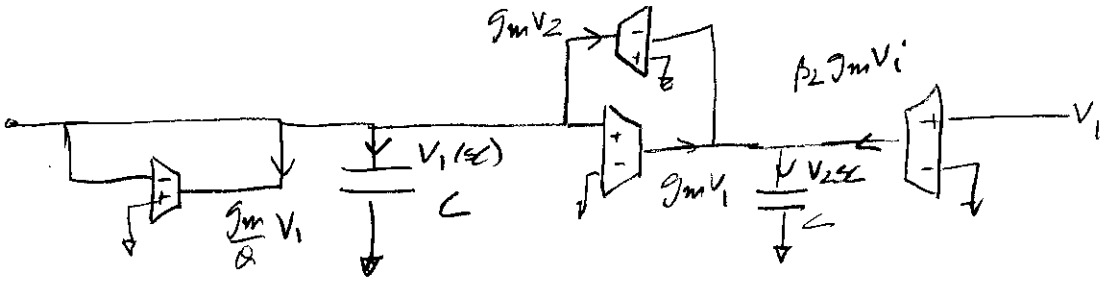
Apply KCL at V_1, V_2

$$(V_i - V_1) sC\alpha_1 = \frac{g_m}{Q} V_1 + sCV_1(1-\alpha) + g_m V_2$$

$$g_m V_1 = V_2 sC$$

$$\frac{V_1}{V_i} = \frac{\alpha_1 \left(\frac{sC}{g_m}\right)^2}{D(s)} \qquad \frac{V_2}{V_i} = \frac{\alpha \left(\frac{sC}{g_m}\right)}{D(s)}$$

(3)

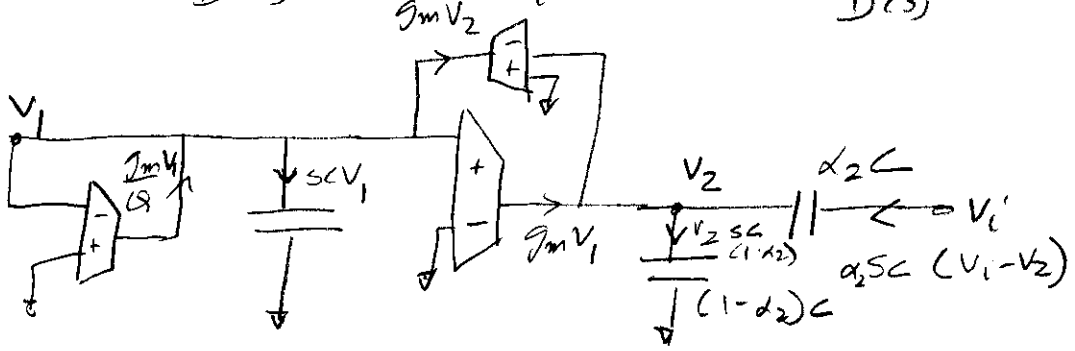


$$V_1 sC + \frac{g_m}{Q} V_1 + g_m V_2 = 0$$

$$V_2 sC = g_m V_1 + \beta_2 g_m V_i$$

$$\frac{V_1}{V_i} = \frac{-\beta_2}{D(s)} \qquad \frac{V_2}{V_i} = \frac{\beta_2 \left(\frac{1}{Q} + \frac{sC}{g_m}\right)}{D(s)}$$

(4)



$$sCV_1 + \frac{g_m}{Q} V_1 + g_m V_2 = 0$$

$$\alpha_2 sC(V_i - V_2) + g_m V_1 = sCV_2(1-\alpha_2)$$

$$\frac{V_1}{V_i} = \frac{-\alpha_2 \frac{sC}{g_m}}{D(s)} \qquad \frac{V_2}{V_i} = \frac{\alpha_2 \left[\frac{sC}{g_m Q} + \left(\frac{sC}{g_m}\right)^2\right]}{D(s)}$$