Problem 2

DC gain = 2

$\omega_Z = \sqrt{10} \times 10^6 \times 2\pi$

$\omega_p = 1 \times 10^6 \times 2\pi$

$\omega_Z = \sqrt{10}\, \omega_p$

$$H(s) = \frac{2\left[1 + \frac{s^2}{(\sqrt{10}\omega_p)^2}\right]}{1 + \frac{s}{\omega_p Q} + \left(\frac{s}{\omega_p}\right)^2} = \frac{2 + \frac{1}{5}\left[\frac{s}{\omega_p}\right]^2}{1 + \frac{s}{\omega_p Q} + \left(\frac{s}{\omega_p}\right)^2}$$

If we use same filter as designed in Prob 1

Denominator remains same (@ $R = 1.59\,k\Omega$, $C = 100\,pF$)

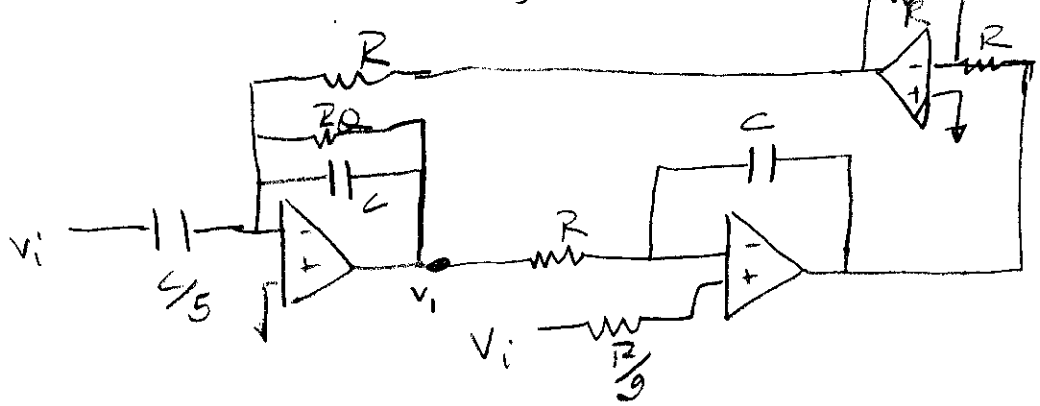
(a) At V_1

To get $\frac{1}{5}\frac{s^2}{\omega_p^2}$ or $\frac{1}{5} s^2R^2C^2$ term in numerator

use V_{i1}, C with capacitor $= \frac{C}{5}$

To get DC gain 2

use V_{i2}, R with a resistance $= \frac{R}{2}$



(b) At V_2

To get s^2 term in the numerator

- Use V_{i2}, C with capacitor value $\frac{C}{\sqrt{5}}$

  to get $-\left[\frac{1}{5}s^2C^2R^2 + \frac{1}{5}\frac{sCR}{Q}\right]$ term in the numerator.

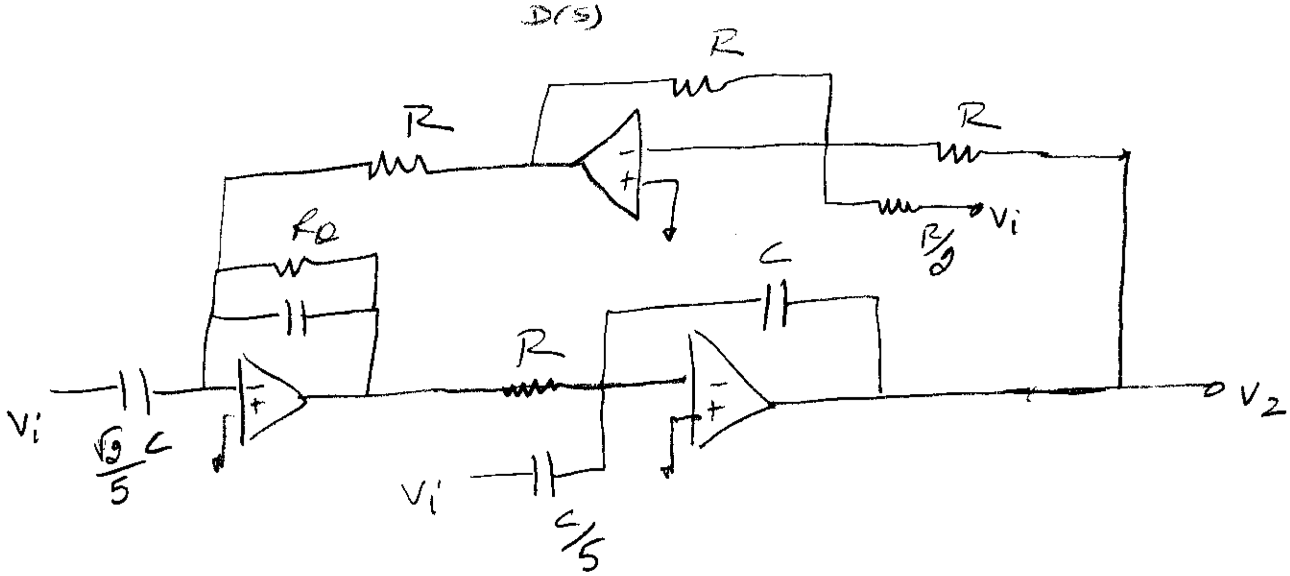
- Use V_{i1}, C with capacitor value $\frac{C}{5} \times \frac{1}{Q}$ $\left[\frac{\sqrt{2}}{5}C\right]$

  to get $\frac{1}{5}\frac{sCR}{Q}$ term in the numerator to cancel out $-\frac{1}{5}\frac{sCR}{Q}$ term due to V_{i2}, C.

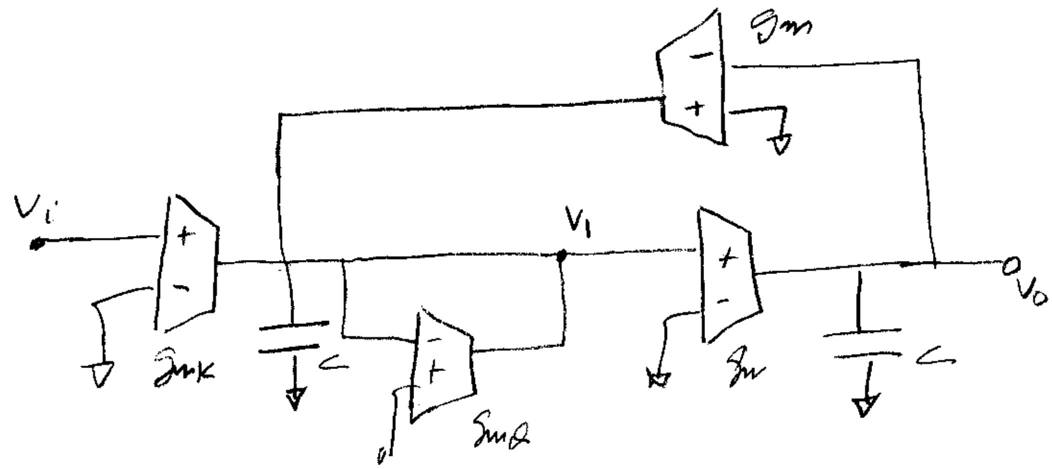
- Use V_{i3}, R with resistance $R/2$ to get DC gain 2

$$H(s) = \underbrace{\frac{-\frac{sCR}{5Q} + \frac{1}{5}(sCR)^2}{D(s)}}_{\text{due to } V_{i1}, C} + \underbrace{\frac{\frac{sCR}{5Q}}{D(s)}}_{\text{due to } V_{i2}, C} - \underbrace{\frac{2}{D(s)}}_{\text{due to } V_{i3}, R}$$

$$= \frac{2 + \frac{1}{5}(sCR)^2}{D(s)}$$



Problem 3



$$\frac{V_o}{V_i} = \frac{g_{mK}}{g_m} \frac{1}{\left(\frac{sC}{g_m}\right)^2 + \frac{sC}{g_m}\frac{g_{mQ}}{g_m} + 1}$$

DC gain $= \frac{g_{mK}}{g_m} = 10$ $\qquad Q = \frac{g_m}{g_{mQ}} = \frac{1}{\sqrt{2}}$ (Butterworth filter)

$\therefore$ g_m is the smallest conductance.

$g_m = 10\,\mu S \qquad g_{mQ} = 10\sqrt{2}\,\mu S \qquad g_{mK} = 100\,\mu S$

Also $\quad \omega_p = 3\times10^6 \times 2\pi$ rad/s $\quad$ [For Butterworth filter 3dB bandwidth = ω_p]

$= g_m/C$

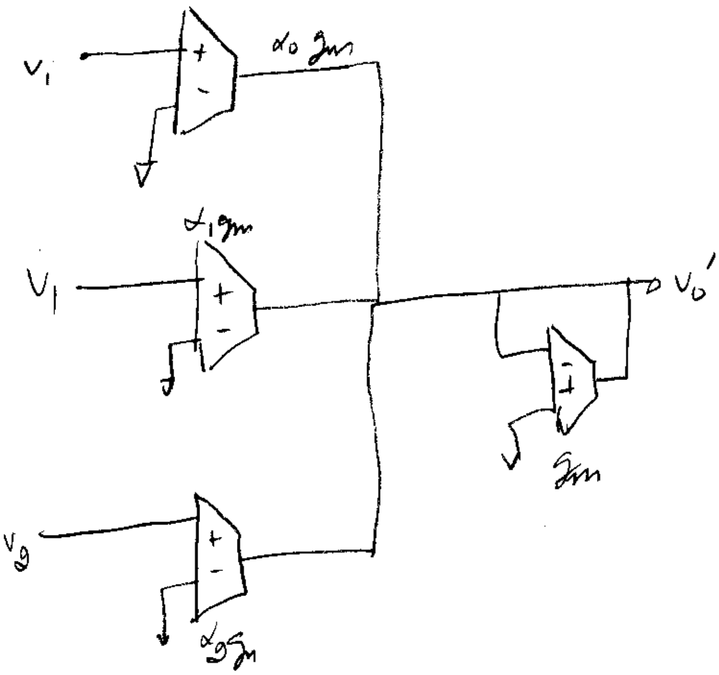
$\therefore C = 0.53\,pF$

(b) $\quad \omega_z = 10\,MHz \times 2\pi \qquad \omega_p = 3\,MHz \times 2\pi$

$\omega_z = \frac{10}{3}\omega_p$

$$H(s) = \frac{10\left[1 + \frac{s^2}{\omega_z^2}\right]}{1 + \frac{s}{Q\omega_p} + \left(\frac{s}{\omega_p}\right)^2} = \frac{10 + \frac{9}{10}\frac{s^2}{\omega_p^2}}{1 + \frac{s}{\omega_p Q} + \left(\frac{s}{\omega_p}\right)^2}$$

Voltage Summing Technique



$$\frac{V_0'(s)}{V_i'(s)} = \frac{(\alpha_0 + K\alpha_2) + (\alpha_0 + \alpha_1 K Q)\frac{s}{Q\omega_p} + \alpha_0\left(\frac{s}{\omega_p}\right)^2}{1 + \frac{s}{Q\omega_p} + \left(\frac{s}{\omega_p}\right)^2}$$

$\alpha_0 + K_1\alpha_2 = 10$

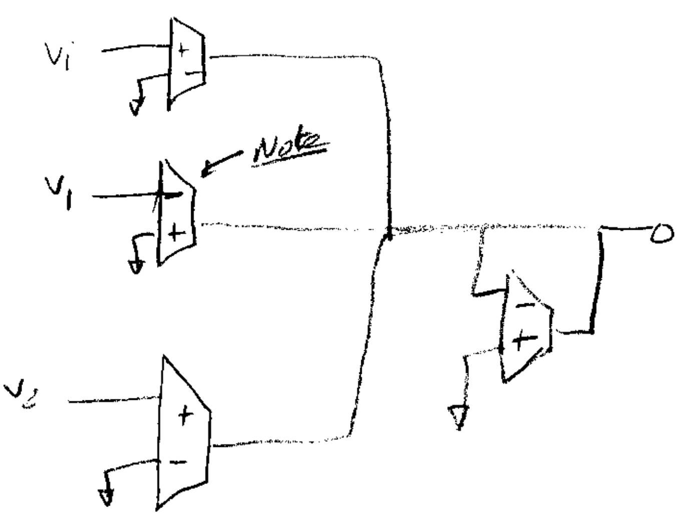
$\alpha_0 + KQ\alpha_1 = 0$

$\alpha_0 = 9/10$

$\alpha_0 = 9/10$

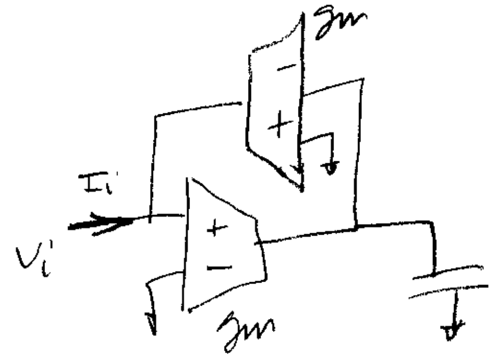
$\alpha_1 = -\frac{9}{100}\sqrt{2}$ [so apply V_1 to inverting terminal of $\alpha_1 g_m$]

$\alpha_2 = 91/100$



Problem 4

$L \equiv$

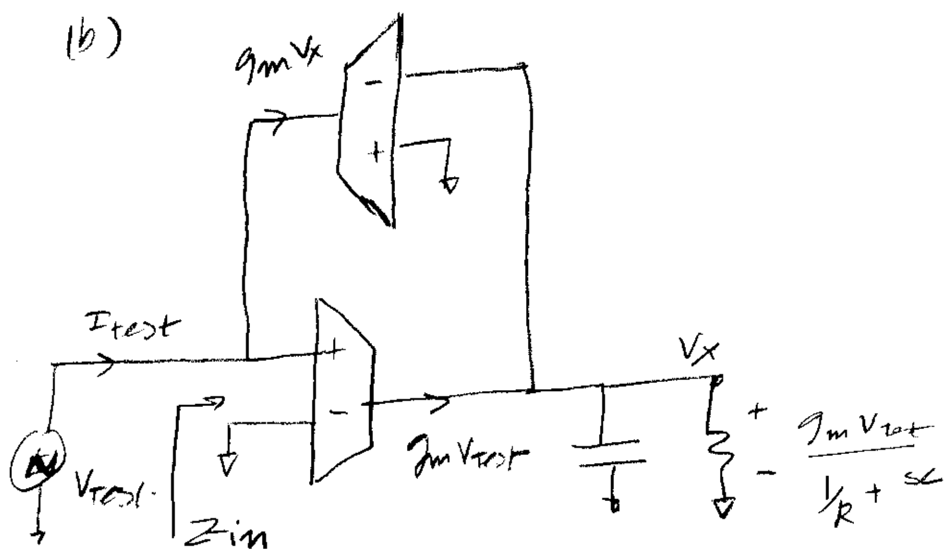


$$L = \frac{C}{g_m^2} = 100\,H$$

$$C = 100\,pF$$

$$\Rightarrow g_m = 1\,\mu S$$

(b)



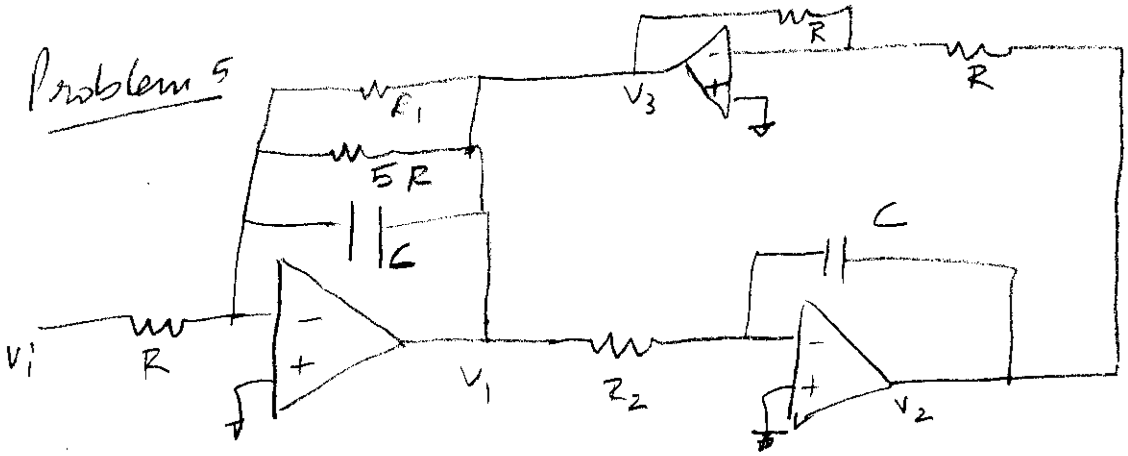
$$V_x = (g_m V_{Test})\left[\frac{1}{1/R + sC}\right]$$

$$\therefore I_{test} = g_m^2 V_{Test} \times \frac{1}{1/R + sC}$$

$$Z_{in} = \frac{V_{Test}}{I_{test}} = \frac{1/R + sC}{g_m^2} = \underbrace{\frac{1}{g_m^2 R}}_{\text{Resistance}} + \underbrace{\frac{sC}{g_m^2}}_{\text{capacitance}}$$

$Z_{in} \equiv$

Problem 5



(a) Use KCL for all 3 opamps

$$\frac{V_i}{R} + V_i sC + \frac{V_1}{5R} + \frac{V_3}{R_1} = 0 \qquad -(1)$$

$$V_3 = -V_2 \qquad -(2)$$

$$V_2 = -\frac{1}{sCR_2} V_1 \qquad -(3)$$

Eliminate V_2, V_3

$$\frac{V_1}{V_i} = \frac{-\frac{R_1 R_2 C}{R} s}{R_1 R_2 C^2 s^2 + \frac{s R_1 R_2 C}{5R} + 1}$$

(i) $R_1 = R_2 = R$

$$\frac{V_1}{V_i} = \frac{-sRC}{s^2(R^2C^2) + \frac{1}{5}RCs + 1}$$

(ii) $R_1 = 3R \quad R_2 = \frac{1}{3}R$

$$V_1/V_i = \frac{-sRC}{s^2R^2C^2 + \frac{1}{5}sR + 1}$$

There's no difference

(b) Eliminate V_1, V_3 from (1), (2) and (3) to get

$$V_2/V_i = \frac{5R_1}{5R_1R_2R\,C^2s^2 + R_1R_2Cs + 5R}$$

(i) $$V_2/V_i = \frac{1}{R^2C^2s^2 + \frac{1}{5}RCs + 1}$$

(ii) $$V_2/V_i = \frac{3}{R^2C^2s^2 + \frac{1}{5}RCs + 1}$$

Different DC Gain But same BW, ω_c, poles etc

$$\frac{V_3}{V_i} = -\left[\frac{V_2}{V_i}\right]$$

(i) $V_3/V_i = \frac{-1}{R^2C^2s^2 + \frac{1}{5}RCs + 1}$ $\qquad$ $V_3/V_i = -3\times \frac{1}{R^2C^2s^2 + \frac{1}{5}RCs + 1}$

$\max\left|\frac{V_2}{V_i}\right| = \max\left|V_3/V_i\right|$

$Q = 5 > 1/\sqrt{2}$ ∴ Peaking in frequency response

$\max\left|\frac{V_2}{V_i}\right|$

$\frac{d}{d\omega}\left(\left|V_2/V_i\right|\right) = 0 \Rightarrow \omega_{peak} = \omega_p\sqrt{1-1/2Q^2} = \omega_p\sqrt{1-\frac{1}{50}}$

$\approx \omega_p \qquad \approx \omega_p$

(i) $R_1 = R$, $R_2 = R$

∴ $\max\left(\left|V_2/V_i\right|\right) \approx Q$ $\qquad$ [Remember $\left|\text{Transfer Func}\right|_{\omega=\omega_p} = Q$ for low pass filters]

(ii) $R_1 = 3R$, $R_2 = R/3$

$\max\left(\left|V_2/V_i\right|\right) \approx 3\times Q = 15$

(c) To maintain opamp in linear region max output voltage for the opamps should be $< 1V$.

for opamps 2,3 $|V_3| = |V_2| = \left|\frac{V_2}{V_i}\right| \times V_i < 1$ Volt for all ω

(i)

$\max\left|\frac{V_2}{V_1}\right| = 5$ at $\omega \approx \omega_p$

$5\, V_{ip} < 1$

∴ $V_{ip} < 1/5$ Volts

(ii) Similarly $\quad 15\, V_{ip} < 1$

$V_{ip} < 1/15$ Volts