E4215: Analog Filter Synthesis and Design
First order filter using an opamp: nonidealities

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12 Feb. 2003

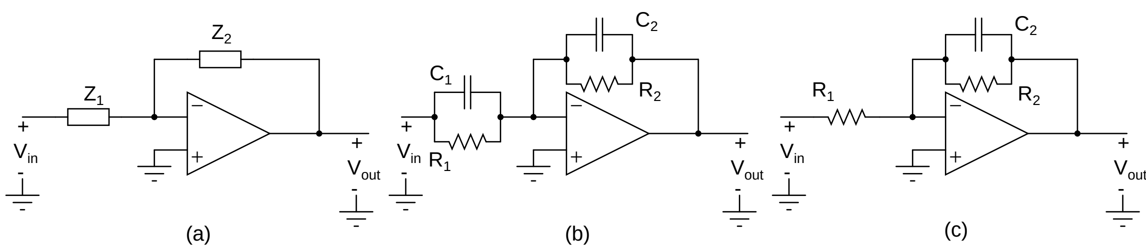

Figure 1: (a) Inverting amplifier with gain $-Z_2/Z_1$, (b) First order filter with a pole and a zero, (c) First order lowpass filter.

Fig. 1(a), with frequency dependent impedances Z_1 and Z_2 can realize a filter. Fig. 1(b) shows a first order filter with a transfer function (assuming ideal opamps)

$$\begin{aligned} H(s) &= \frac{V_o(s)}{V_i(s)} \\ &= -H_0 \frac{1+s/z_1}{1+s/p_1} \\ &= -\frac{R_2}{R_1}\frac{1+sC_1R_1}{1+sC_2R_2} \end{aligned}$$

It has a zero at $-z_1 = -1/C_1R_1$ and a pole at $-p_1 = -1/C_2R_2$[1].

For the filter in Fig. 1(a) using an opamp with a gain $A(s)$,

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2}{Z_1}\frac{1}{1+\frac{1+Z_2/Z_1}{A(s)}}$$

As usual, the reciprocal of the loop gain appears in the denominator as the deviation caused by the opamp.

Substituting the form of Z_1/Z_2

$$\begin{aligned} \frac{Z_2}{Z_1} &= H_0\frac{1+s/z_1}{1+s/p_1} \quad \text{The desired transfer function without the minus sign} \\ 1+\frac{Z_2}{Z_1} &= \frac{1+s/p_1+H_0+H_0s/z_1}{1+s/p_1} \\ \frac{V_o(s)}{V_i(s)} &= -H_0\frac{1+s/z_1}{1+(1+H_0)/A(s)+s/p_1+s/A(s)p_1+H_0s/A(s)z_1} \end{aligned}$$

[1] Note that it is common to refer to "the pole p_1" although the pole is at $-p_1$.

The transfer function reduces to the ideal as $A(s) \rightarrow \infty$.

The effect of different nonidealities are analyzed separately. For small nonidealities, the various effects can be added up to arrive at the final filter parameters.

1 Effect of finite dc gain A_0

Assume $A(s) = A_0$.

$$\begin{aligned}
\frac{V_o(s)}{V_i(s)} &= -H_0 \frac{1 + s/z_1}{1 + (1+H_0)/A_0 + s/p_1 + s/A_0p_1 + H_0s/A_0z_1} \\
H_0' &= \frac{H_0}{1 + \frac{1+H_0}{A_0}} \\
&\approx H_0\left(1 - \frac{1+H_0}{A_0}\right) \\
z_1' &= z_1 \\
p_1' &= p_1 \frac{1 + \frac{1+H_0}{A_0}}{1 + \frac{1+H_0p_1/z_1}{A_0}} \\
&\approx p_1\left(1 + \frac{H_0}{A_0}\left(1 - \frac{p_1}{z_1}\right)\right)
\end{aligned}$$

The dc gain is reduced[2]. The zero remains unchanged. The pole moves to a higher frequency if $p_1/z_1 < 1$ and to lower frequency if $p_1/z_1 > 1$. The following observations can be made.

- The reduction in dc gain and the change in the pole frequency decrease if A_0 increases[3].
- The reduction in dc gain H_0 is worse[2] for larger dc gains H_0.
- The change in pole frequency is worse if the dc gain H_0 of the intended filter is larger.

1.1 Low pass filter ($z_1 = \infty$, Fig. 1(c))

$$p_1' \approx p_1\left(1 + \frac{H_0}{A_0}\right)$$

The filter's pole frequency increases due to opamp's finite dc gain A_0. The relative shift is H_0/A_0. i.e. if a first order filter of gain 10 is designed with an opamp of gain 1000, the pole frequency will be 1 % larger than predicted. *If* A_0 is known accurately, the design can be adjusted to obtain the correct frequency response. i.e. To obtain a pole of 100 krad/s and $H_0 = 10$ using an opamp with accurately known $A_0 = 1000$, $1/R_2C_2$ can be made 1 % smaller (i.e. $= 99$ krad/s) so that $p_1' = 100$ rad/s. Fig. 2(a) shows the magnitude response change with finite dc gain.

[2] Expected from previous analysis of frequency independent inverting amplifiers.

[3] The opamp is closer to ideal.

2 Effect of finite unity gain frequency ω_u

The integrator model $A(s) = \omega_u/s$ is used to determine the effect of opamp's frequency dependence.

$$\frac{V_o(s)}{V_i(s)} = -H_0 \frac{1+\frac{s}{z_1}}{1+\frac{s}{p_1}+(1+H_0)\frac{s}{\omega_u}+\frac{s^2}{\omega_u}\left(\frac{1}{p_1}+\frac{H_0}{z_1}\right)}$$

The result is a second order transfer function. The dc gain is unchanged as the dc gain of the integrator model is ∞. As usual, we check that the expression reduces to the ideal as $\omega_u \to \infty$. We assume that the transfer function is as follows. i.e with a pole at $-p_1'$ which is slightly different from $-p_1$ and a high frequency pole at $-p_h$.

$$\frac{V_o(s)}{V_i(s)} \approx -H_0 \frac{1+\frac{s}{z_1}}{\left(1+\frac{s}{p_1'}\right)\left(1+\frac{s}{p_h}\right)}$$

p_1 and p_h are determined approximately to be

$$\begin{aligned} p_1' &\approx \frac{p_1}{1+(1+H_0)\frac{p_1}{\omega_u}} \\ &\approx p_1\left(1-(1+H_0)\frac{p_1}{\omega_u}\right) \\ p_h &\approx \omega_u \frac{1+(1+H_0)\frac{p_1}{\omega_u}}{1+H_0\frac{p_1}{z_1}} \end{aligned}$$

The pole frequency is reduced. The amount of reduction is related to the dc gain of the filter and the ratio of the pole to the unity gain frequency of the opamp.

- The reduction in pole frequency is worse if the dc gain H_0 of the intended filter is larger.
- The reduction in pole frequency is worse if p_1 increases. i.e. if it is a high frequency filter.

2.1 Low pass filter ($z_1 = \infty$, Fig. 1(c))

$$\begin{aligned} p_1' &\approx p_1\left(1-(1+H_0)\frac{p_1}{\omega_u}\right) \\ p_h &\approx \omega_u\left(1+(1+H_0)\frac{p_1}{\omega_u}\right) \\ \frac{p_h}{p_1'} &\approx \frac{\omega_u}{p_1}\left(1+2(1+H_0)\frac{p_1}{\omega_u}\right) \end{aligned}$$

The ratio p_h/p_1' should be as large as possible to minimize "interference" from the high frequency pole in the region of interest of the filter's operation. This ratio decreases, i.e. the filter gets worse, for larger gain H_0, smaller ω_u, and larger filter pole p_1. Fig. 2(b) shows the magnitude response change with finite unity gain frequency.

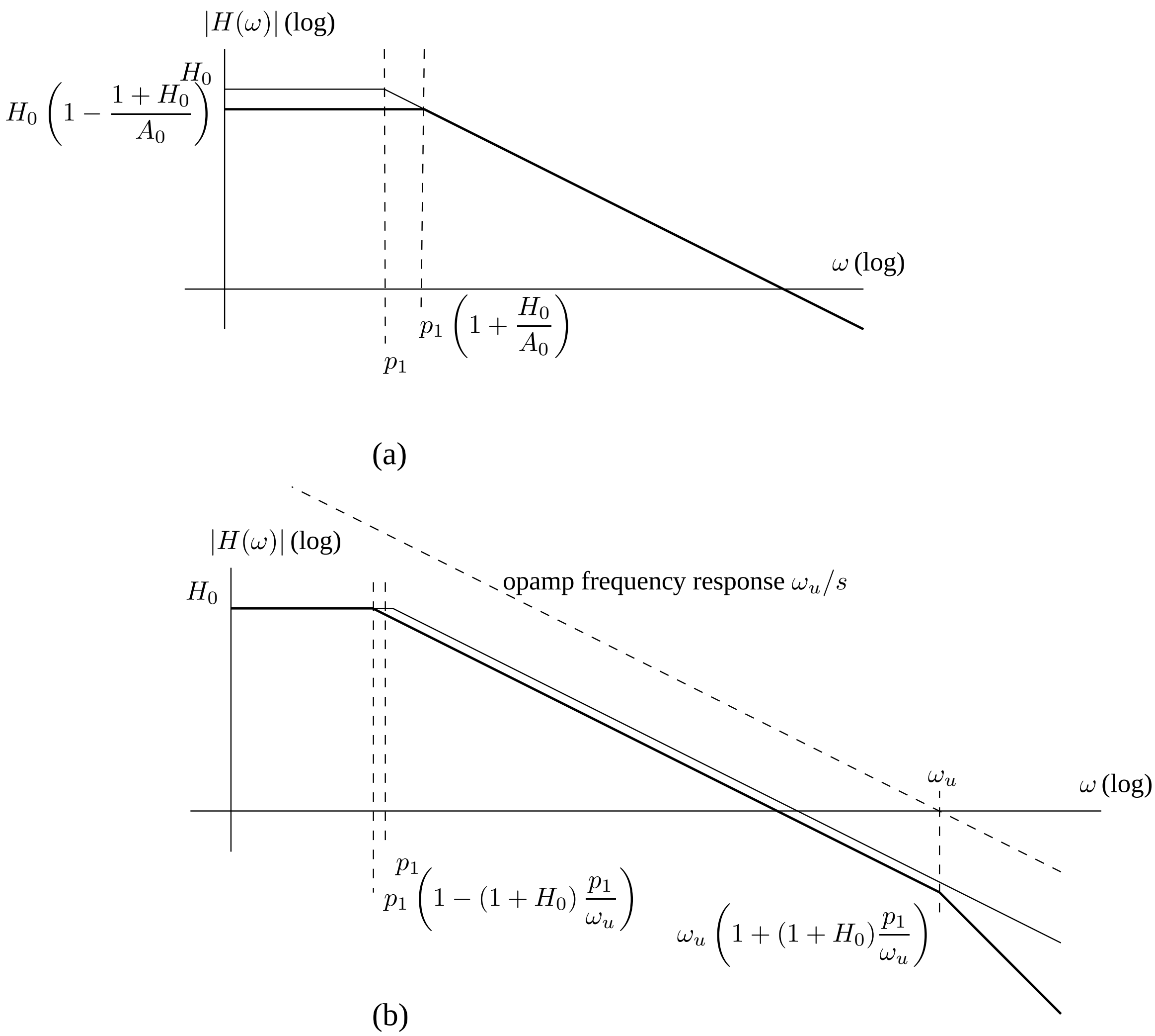

Figure 2: First order lowpass filter: (a) Effect of finite dc gain, (b) Effect of finite unity gain frequency; thick line: with nonidealities