Assignment 2 (EC301)
Due: 12th Jan, 2009

Please do the problem prior to the tutorial on monday morning.

1. Three charges, of value q, $-2q$ and q are placed $d\hat{z}$, origin and $2d\hat{x}$, respectively. Compute the potential far from the charges along the y-axis.

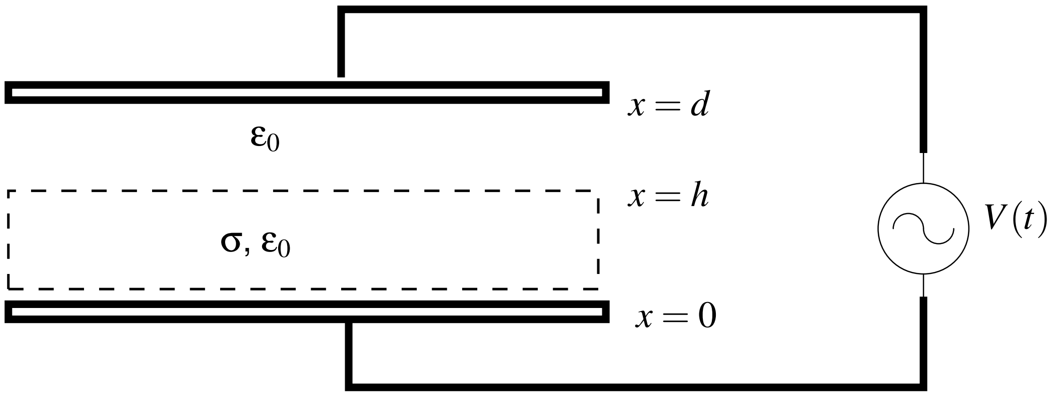
   Hint: You can build the charges out of dipoles.

2. Classify the following vector fields as (a) Electric fields in vacuum, (b) Electric fields in the presence of charge, and (c) not Electric fields

   (a) $\vec{v} = x\hat{x} + y\hat{y}$
   (b) $\vec{v} = x\hat{x} - y\hat{y}$
   (c) $\vec{v} = y\hat{x} + x\hat{y}$
   (d) $\vec{v} = y\hat{x} - x\hat{y}$

   Also, sketch the field lines for the above vector fields.

3. Consider the system shown in the figure.



Its a simple parallel plate capacitor connected to a constant voltage source through a switch. The area of each of the plates is A and the distance between the plates is d. The potential difference across the voltage source is V. There is also a slab of thickness, h, and conductivity, σ, present in between the two capacitor plates as shown. Thus, there are four surfaces, S_1, S_2, S_3, S_4. S_1 is the grounded plate at $x = 0$, S_2 is the lower surface of the slab at $x = 0$, S_3 is at $x = h$ and S_4 is at $x = d$. The region $h < x < d$ is air.

- Let us now close the switch. The potential difference between S_1 and S_4 is now non-zero and is same as that across as the voltage source. Thus, an electric field is created in this region, $0 \leq x \leq d$. Can you obtain an expression for this electric field? Also obtain the charge densities on the four surfaces (two at $x = 0$, one at $x = h$ and the fourth at $x = d$).
  Hint: At steady state, the total electric field inside a conductor is zero (Why?).

In the above problem, the charge densities on the four surfaces were zero before the switch was closed. After the switch is closed, charges start appearing on these surfaces and we can obtain expressions for the charge densities. But for charge to appear on the surfaces, it is obvious that it has to flow to these surfaces from somewhere. Now, since electrons always have finite speed, this charge build up should have taken some time. Thats interesting. We seem to have missed something important while solving the above problem. Time to reboot.

(a) Let us open the switch and allow the accumulated charges on the surface to drain out. Now close the switch at $t = 0^+$. The potential difference between S_1 and S_4 is once again non-zero and is same as that across as the voltage source. Thus, an electric field is created in this region, $0 \leq x \leq d$. Can you estimate this electric field in this region immediately after the switch is closed? Can you also obtain expressions for the electric potential?

Hint: Remember that we need to be careful about time now. Immediately after the switch is closed, we consider charge build up only on the surfaces, S_1 and S_4. This is valid when the wires connecting the voltage source to these plates have a much higher conductivity compared to that of the slab. Thus, immediately after closing the switch, there are no charges on S_2 and S_3.

(b) For $t > 0$, the free electrons in the slab will start moving due to the electric field. This will constitute a current with a current density, J. This causes a charge build up on the surfaces S_2 and S_3. This charge build up also changes the charge density on S_1 and S_3 (Why?). And while all this is happening, the electric field is also changing. I am sure by now things have started looking hopelessly complicated. Lets have some patience and, one by one, write down equations that relate the quantities under consideration. We have the charge densities on the four surfaces (say $\sigma_1, \sigma_2, \sigma_3, \sigma_4$), then there is the current, J, and then the electric field, E. Remember that all these quantities are now functions of time. Write down as many relations connecting them as you can, and then retain only those that matter. This should have simplified things a bit and the problem should start looking tractable now. Solve this set of equations and obtain closed form solutions for these various quantities as functions of time.

Hint: $\sigma_1 = -\sigma_4$ and $\sigma_2 = -\sigma_3$ (Why?). E can be assumed to be uniform in $0 < x < h$ and $h < x < d$ since the conductivity is assumed not vary with space and we are neglecting fringing effects in an otherwise plane geometry. First write the relation between J and E. Then connect the charge densities to the current density, J. Close the system of equations by connecting E to the charge densities through Gauss' Law.

(c) As we know, after a long long time, the electric field in the conductor must decay to zero. And whenever we have a decay w.r.t. time, there is an associated time constant. What is the expression for the time constant for the decay of electric field inside the slab? What is the value of this time constant for $d = 1mm$, $h = 0.3mm$. Let the connecting wires be made of copper and the slab of sulphur? Is the time constant really that long?

Advanced question: Currently, Prof. Jayashankar's Ph.D. student is studying the behaviour of insulators at milli and μHz frequencies, i.e., the response of a dielectric when the applied field is

$$E_x = E_0 e^{j\omega t}, \qquad \omega = 2\pi \times 10^{-3}\text{rad/sec}$$

Can you determine the range of values of σ for which the behaviour of the material at milli and μHz is different from its D.C. response (take ε to be ε_0)? Incidentally, one of the best insulators we know of, quartz, has a conductivity of 10^{-17}S/m.