Assignment 1 of EC301 (2009 Jan)
Due: 5 Jan 2008

January 1, 2009

Please do the problems prior to the tutorial on monday morning. The scilab code should be uploaded to the assignment site. There will be a small quiz at the end of the class, based on these problems.

1. An ring of charge of radius a is centred on the origin and lies in the $x-y$ plane. The charge density of the ring is ρ Coulombs per metre.

   (a) Obtain the z-component of the Electric field. Plot it - does it have a maximum?

   (b) Obtain the potential along the z axis. Plot it - does it monotonically decrease in z? Note that potential is given by

   $$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\rho a d\theta}{\sqrt{a^2+z^2}}$$

   We will derive this next week.

   (c) What is the heaviest object with charge q that can be levitated by this annulus?

2. Suppose the curved surface of a cylinder is uniformly covered with charge. The cylinder has radius a and length L is made of insulating material. Determine $\vec{E}(z)$ along the axis. **Hint**: Use the formula for a ring of charge and integrate.

   Advanced question: There is no charge along the axis. Can you determine how E_r will behave as you move away from the axis? **Hint:** Use Gauss' Law and connect the flux through the side walls to the flux through the top and bottom ends.

3. Write a Scilab program to randomly place 5000 charges (each 1 pC) in $0 < x < 2$, with the following pdf:

$$p(x) = A\cos\left(\frac{\pi x}{4}\right)$$

where A is chosen to make the probability density integrate to unity. To do this, generate about 10000 pairs of locations (x, p) with x randomly chosen between 0 and 2 and p between 0 and A. Accept the first 5000 locations where $p < p(x)$.

   (a) Choose a region defined by $1 - a < x < 1 + a$. For different a determine the charge enclosed in the region and hence the estimated charge density ρ. Plot ρ vs a and compare to the actual theoretical value expected from the above probability.

   (b) Repeat for a region defined by $0.5 - a < x < 0.5 + a$. Do the plots differ? Discuss the differences.

   This program should be submitted to the assignment website by Sunday night.