Code for Week 1

January 10, 2009

$$P = \int_0^x p dx' = \frac{2A}{\pi} \sin(\pi x/2)$$

Clearly $A = \pi/2$. However, to generate the pdf we simply look for points where the random number is less than $\cos(\pi x/2)$.

In the program, n_0 is the number of charges to generate. It is kept as a variable to allow us to see how that affects the curve.

1a ⟨*Q3* 1a⟩≡

```
n0=500000;
N=n0;
n=0;
```

We keep increasing the number of points generated till we get n0 valid ones.

1b ⟨*Q3* 1a⟩+≡

```
while n<n0;
  N=N*2;
  x=rand(N,1,"uniform")*2;
  p=rand(N,1,"uniform");
  ii=find(p<cos(%pi*x/4));
  n=length(ii);
end
ii=ii(1:n0);
x=x(ii);
p=p(ii);
```

We select a set of window sizes from 0.5 to $0.1/\sqrt{n_0}$. for each a, we determine ρ

2a ⟨*Q3* 1a⟩+≡

```
a=logspace(log10(0.1/sqrt(n0)),-0.3,21);
num1=zeros(a);
rho1=zeros(a);
num2=zeros(a);
rho2=zeros(a);
for i=1:length(a)
  aa=a(i);
  num1(i)=length(find(x>1-aa & x<1+aa));
  rho1(i)=num1(i)/(n0*2*aa);
  num2(i)=length(find(x>0.5-aa & x<0.5+aa));
  rho2(i)=num2(i)/(n0*2*aa);
end
```

We now plot the results. Note that the top subplot shows the number of charges in the bin. The vertical line shows where the number of charges in the bin drops below 100.

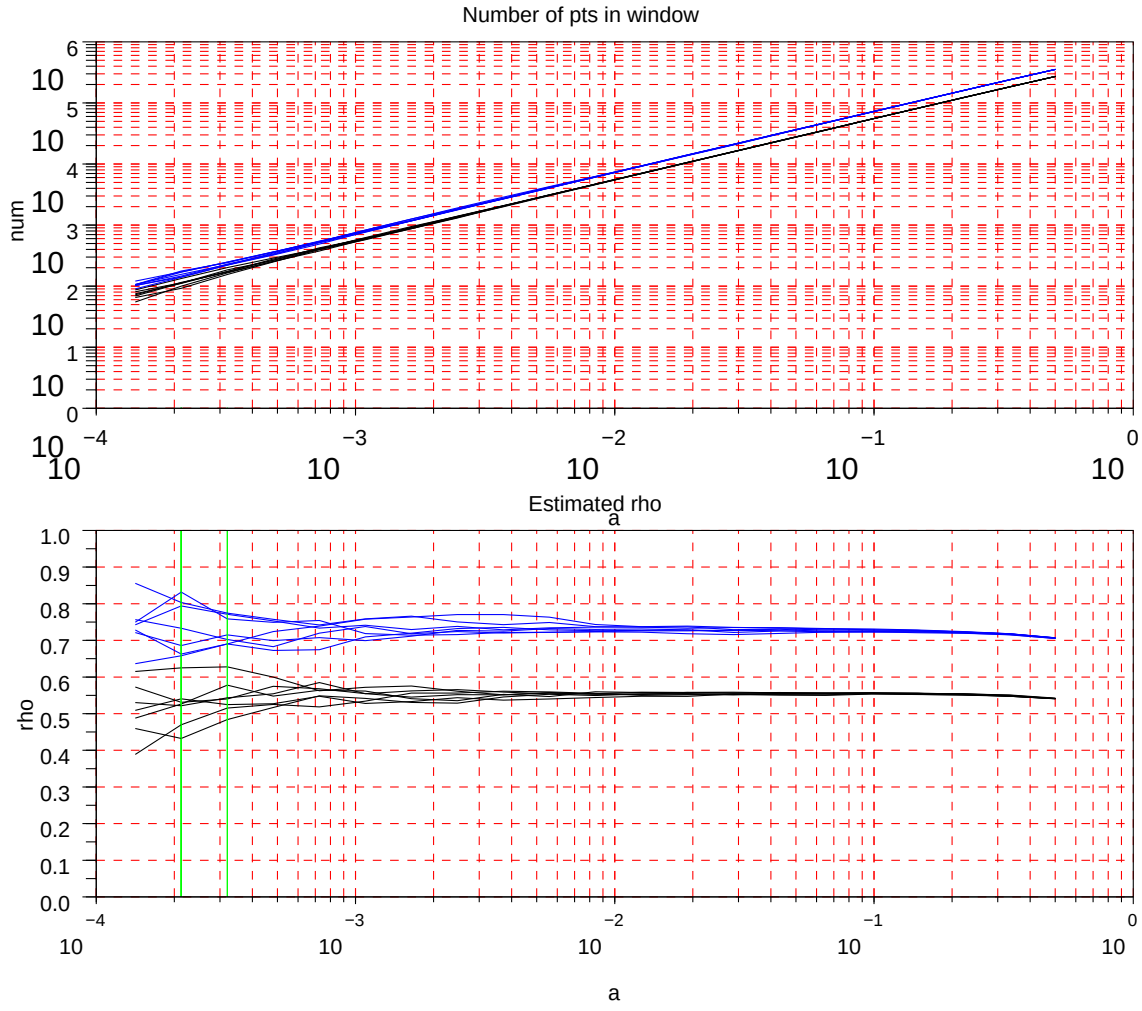
2b ⟨*Q3* 1a⟩+≡

```
i0=min(find(num1>100));
subplot(2,1,1)
xset("font size",3);
plot2d(a,num1,1,logflag="ll");
plot2d(a,num2,2,logflag="ll");
plot2d([1;1]*a(i0),[0;1],3);
xgrid(5);
xtitle("Number of pts in window","a","num");
subplot(2,1,2);
plot2d(a,rho1,1,logflag="ln");
plot2d(a,rho2,2,logflag="ln");
plot2d([1;1]*a(i0),[0;1],3);
xgrid(5);
xtitle("Estimated rho","a","rho");
```

Conclusions

- By running this program many times, it is clear that the black lines (corresponding to binning around $x = 1$) is not sensitive to the bin size. However, the blue lines (corresponding to binning around $x = 0.5$) change as the bin size is reduced. This is because the value of $d^2\rho/dx^2$ is larger at 0.5, which is what tends to make the measurement inaccurate.

- The noise is around ± 0.1 when the number of points in the bin is 100 (the gree vertical line). This is reasonable, since fluctuations should be of order $\sqrt{100}$ or 10%.



- By increasing the number of charges to 5×10^5 we see that there is a bin size range from 0.1 and 0.01 for which the ρ value is costant. Below 0.01 (about 5000 points in the bin) noise starts to corrupt the measurement significantly. It is interesting that different runs show far more variation in the blue curves than the black, *even though* the blue curves represent better statistics (see the upper plots).