Relaxation of a system of charges

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Suppose we have N charges that move under the influence of an Electric field, that is due to their own charge. These charges are constrained to certain volumes where the medium is conducting. What final arrangement do these charges arrive at?

The force equation for these charges is given by

$$m_i \frac{d\vec{v}_i}{dt} = q_i \vec{E} - \nu_i \vec{v}_i$$

where m_i and q_i are the mass and charge of the i^{th} charge, ν_i is the coefficient of friction experienced by that charge and $\vec{v}_i$ is the velocity of that charge. Multiply both sides by $\vec{v}_i$ to get

$$\frac{d}{dt}\left(\frac{m_i}{2}|\vec{v}_i|^2\right) = q_i \vec{E} \cdot \frac{d\vec{r}_i}{dt} - \nu_i |\vec{v}_i|^2$$

Integrating in time from t_1 to t_2 we get

$$\frac{m_i}{2}|\vec{v}_i|^2_{\text{final}} - \frac{m_i}{2}|\vec{v}_i|^2_{\text{initial}} = q_i \phi_{\text{initial}} - q_i \phi_{\text{final}} - \int_{t_1}^{t_2} \nu_i |\vec{v}_i|^2 \, dt$$

i.e., summing over all particles,

$$\text{Final Energy} - \text{Initial Energy} = -\sum_{i=1}^{N} \int_{t_1}^{t_2} \nu_i |\vec{v}_i|^2 \, dt \le 0$$

Thus the total energy of all the particles keeps reducing in time. It only stops reducing when the velocity of all the charges goes to zero. But for that to happen, the field at all those charges should either be zero or should be in a direction that is not allowed by the constraints, i.e., the quantum work function of the material prevents motion of the particles.

This is why we conclude that the field inside a conductor is zero and the field at the surface is normal to the conductor. When externally imposed fields are present, this result fails. Those fields can do work on the system and increase the total energy of the system. When this happens, a new balance is achieved, where the friction force is balanced by the energy injected by the external fields. This is what happens when a conduction current is present. Charges are injected at high potential energy, migrate through the conductor and leave at low potential energy. Thus, every charge supplies $q_i\left(V_{\text{initial}} - V_{\text{final}}\right)$ joules of energy to the system and dissipate $\nu_i u_i L$ joules of energy through friction, where u_i is the average drift velocity of the charge. This is nothing more than the statement

$$-q_i E = -\nu_i u_i$$

which underlies the concept of mobility.