Problems in Faraday's Law and Wave Equation

April 9, 2008

I was asked to give some more challenging problems as preparation for the exam. These are all problems that are as hard as what I will give in the tests/final exam, and probably harder. (Actually I gave the last problem as a final exam problem but that was when we had two courses in Electromagnetics.) Try these to get practice in solving problems in skin effect and parallel plate waveguides.

1. An air-filled parallel plate waveguide has a thin layer of dielectric ($\varepsilon \gg \varepsilon_0$, μ_0, $\sigma = 0$) of thickness d attached to the bottom plate. The plates are separated by a distance $a \gg d$ (with $\varepsilon d \ll \varepsilon_0 a$)
   - (a) Does a TEM wave exist for this system? Why or why not?
   - (b) Does there exist a mode without a cutoff? Find it.
2. A parallel plate waveguide has thick walls with conductivity σ. Its plates are separated by a and space between is air (ε_0, μ_0, $\sigma = 0$).
   - (a) Can the TEM wave exist? Why or why not?
   - (b) Write down the equations for E_x, E_z and H_y.
   - (c) The presence of H_y implies a surface wall current, J_z. Use the skin depth solution for a plate to determine the profile of E_z and J_z in the plate, and relate H_y to E_z at the wall.
   - (d) What is the direction of E_z at the two walls. Use the boundary condition to write down the form of E_z. Hence obtain a correction to the dispersion equation (you can assume that $k_x \ll \pi/a$).
   - (e) Over what length will the power in the wave drop to half its original value?
   - (f) Suppose the top wall was thin compared to skin depth. How would the solution change? What happens to the wave? (I want a graphical and qualitative answer).
3. A parallel plate waveguide has walls of thickness d and separation a. AC current $I_0 e^{j\omega t}$ is injected into the top plate and returns back to the source through the bottom plate. Ignore the displacement term and treat this as a skin depth problem, and determine the currents and Electric fields in the plates. How does the solution to this problem differ from problem 2 above? Are the current distributions the same or different?
4. A plane wave $E_0 \exp(j\omega t - j\beta z)\hat{x}$ is normally incident on a conducting plate. The plate has a thickness of 2δ. If the medium has $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, and conductivity σ, determine
   - (a) The strength of the reflected wave.
   - (b) The fraction of power that passes right through the plate to the other side.
   - (c) The current distribution in the plate.
5. A round wire of radius a and conductivity σ_1 is soldered to a wire of the same radius but conductivity σ_2. A current I flows through the wires. Determine the fields and currents at the soldered joints. You may ignore the displacement current, and you may assume that cylindrical symmetry is maintained everywhere. Sketch the lines along which the current flows as it crosses the junction. **Note:** This is a hard problem.