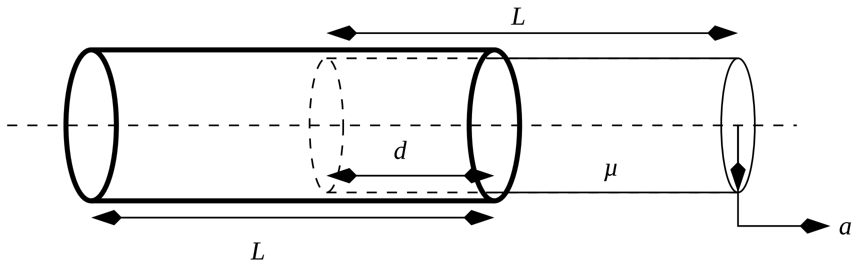
Problems on Faraday's Law, Magnetic Energy and Skin Effect

28th February 2007

Faraday's law by itself rarely is of interest. However, it leads to very interesting problems when combined with mechanics. That is when we get machines, generators and other interesting devices.

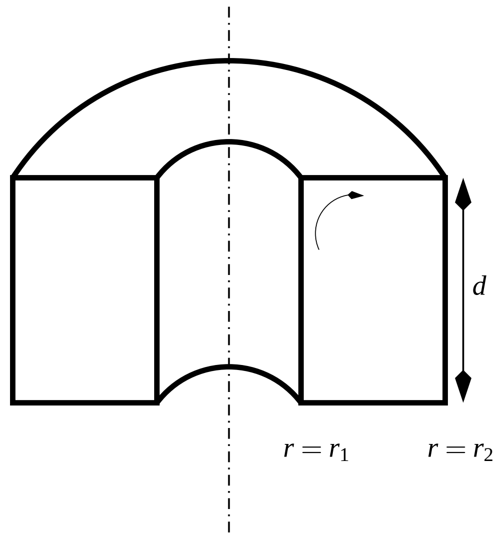
1. A circular loop of radius a is in the y-z plane with its centre at $(0,1,1)$. The loop rotates around the axis $(\hat{y}+\hat{z})\big/2$ at a constant angular frequency ω. If a uniform magnetic field $B_0\hat{z}$ is present, determine the current that will flow in the loop, if it has a resistance R.

2. A cylinder of permeability μ and length L is introduced into a solenoid (also of length L) with an air core. The radius of the cylinder is a, which is also the inner radius of the solenoid. The length of the cylinder that is inside the solenoid is d. Compute the change in energy if the cylinder is introduced an extra δd. From this determine the force on the solenoid. Note that the current through the solenoid is held constant.



   You can assume that $a \ll d, L-d$. How does this make the problem simpler. **Hint**: Fringing fields are localized fields that connect mismatched steady state fields.

3. A solenoid consists of a torus of rectangular cross-section with inner and outer radii of r_1 and r_2 respectively and height d. Determine the external inductance of the torus if there are n windings per radian.



4. A rectangular loop (length along x is l and along y is d) is rotating about the x-axis. Its initial position is in the $x-y$ plane ($\theta = 0$), with an angular velocity of ω, and a current of zero. A uniform magnetic field $B_0\hat{z}$ is present. If the loop has a resistance of R and moment of inertia I, determine its future angular position, $\theta(t)$. Show that energy is conserved.

   Note: This problem differs from problem 1 in that the $\omega = \dot{\theta}$ is not forced to be constant.

5. A rigid, circular loop of radius a and mass m falls under the influence of gravity. A static magnetic field whose z-component is given by

$$B_z = B_0\left(1 + \frac{a^2}{a^2 + z^2}\right)$$

   is present. The loop has a resistance R, and initially carries a current I_0 (in the $\vec{\theta}$ direction). $z(0)$, the position of the loop at $t = 0$, is zero, $v_z(0) = \dot{z}(0) = 0$ as well.

   (a) Determine $B_r(r,z)$.
   (b) Determine $\vec{j} \times \vec{B}$ force on the loop. What are the components of this force? **Hint:** the loop is falling, which means both electrons and atoms have a velocity along $\hat{z}$, which must be taken into account.
   (c) Obtain an equation for the position of the loop $z(t)$, and its current $I(t)$.
   (d) Imagine you are in the frame of the loop. Obtain the induced emf on the loop, and hence obtain $I(t)$ in terms of $z(t)$. Do these agree with your results in part (c)?
   (e) Does the loop rotate? i.e., there is induced current, but is there torque?

6. An AC current $I_0 e^{j\omega t}$ is driven through a long solenoid of radius a. Determine the distribution of current $j_\theta(r,t)$ that is present. Determine the external and internal inductance of this solenoid. The solenoid has an air core, and has n turns per metre.