

Problems in Dielectrics

7th February 2007

1. An optical fibre has a refractive index given by

$$n(r) = n_1 + \Delta n \left(\frac{a^2}{a^2 + r^2} \right)$$

where $n_1 = 1.49$, $\Delta n = 0.01$ and $a = 4\mu = 4 \times 10^{-6}$ metres. A small amount of trapped charge $\rho(r)$ is present in the fibre. Given that the refractive index is linked to the dielectric constant by

$$n^2 = \frac{\epsilon}{\epsilon_0}$$

determine the Electric Field that is the result of this charge. **Hint:** Use the fact that both $n(r)$ and $\rho(r)$ depend only on r and invoke symmetry.

2. A plastic window is made of plastic with a dielectric constant $\epsilon(x)$ that is symmetric in x . The window has a point charge Q embedded in its middle. If the window extends from $-L$ to $+L$ along x and is infinite along y and z , determine the total Electric Flux

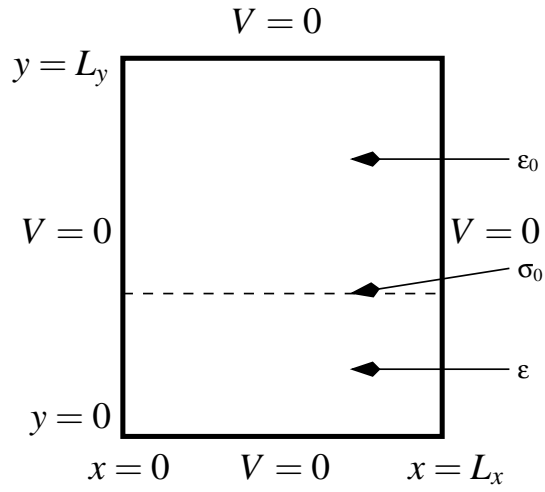
$$\int \int E_x(L^+, y, z) dy dz$$

that leaves the window at $x = L$.

3. A rectangular region $0 < x < L_x$, $0 < y < L_y$ is surrounded by grounded walls. A charge sheet

$$\sigma(x) = \sigma_0$$

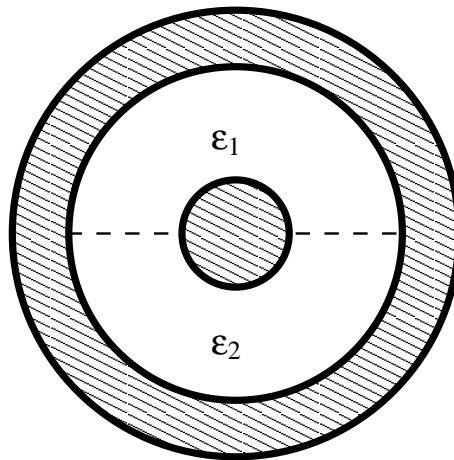
is present at $y = d$. The region $0 < y < d$ is filled with a dielectric ϵ while the region $d < y < L_y$ is filled with air (permittivity ϵ_0).



How much charge is induced on the top wall ($y = L_y$, $0 < x < L_x$)?

4. A cable has an inner wire (radius a) at V volts and a grounded outer wire (inner radius b). The volume between the two wires is filled by two dielectrics:

$$\epsilon(r, \theta) = \begin{cases} \epsilon_1 & 0 < \theta < \pi \\ \epsilon_2 & \pi < \theta < 2\pi \end{cases}$$



- (a) Solve for the potential between the two wires.
 (b) Obtain an expression for the induced charge on the inner wire.
 (c) If the dielectrics had been concentric cylinders, i.e.,

$$\epsilon = \begin{cases} \epsilon_1 & a < r < r_0 \\ \epsilon_2 & r_0 < r < b \end{cases}$$

how would the analysis have changed?