# Problems in Electrostatics

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1. Which of these Electric fields can be due to static charges. Determine the charge density that created the valid fields.

   (a) $\vec{E}(\vec{r}) = e^{-y^2}\hat{x}$

   (b) $\vec{E}(\vec{r}) = e^{-x^2}\hat{x}$

   (c) $\vec{E}(\vec{r}) = \sin(kr)\hat{r}$ ($r, \theta, z$ coordinate system)

   (d) $\vec{E}(\vec{r}) = \left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)/r^3$ ($r, \theta, \phi$ coordinate system)

2. A conducting box defined by $0 < x < 1$, $0 < y < 1$ contains charge as specified below. In each case determine the potential $\phi(\vec{r})$.

   (a) $\rho(x,y) = 2x$, grounded walls.

   (b) $\rho(x,y) = sin(\pi y)$, $\phi = 1$ on top wall.

   **Hint:** Remember that any solution of poisson's equation can be combined with Laplace's equation to get the solution.

3. A cylinder has the top half kept at 1V, while the bottom half is grounded:

$$V(\theta) = \begin{cases} 1 \text{ volt} & 0 < \theta < \pi \\ 0 \text{ volt} & \pi < \theta < 2\pi \end{cases}$$

   If the cylinder is infinite along $z$ and has a radius of $a$, determine the potential within the cylinder.

   **Hint:** Write out Laplace's Equation in cylindrical polar coordinates and use separation of variables.

$$\begin{aligned} \frac{1}{rR}\partial_r(r\partial_r R) + \frac{1}{r^2F}\partial_\theta^2 F &= 0 \\ \frac{r}{R}\partial_r(r\partial_r R) + \frac{1}{F}\partial_\theta^2 F &= 0 \\ \frac{r}{R}\partial_r(r\partial_r R) &= -\frac{1}{F}\partial_\theta^2 F = n^2 \end{aligned}$$

   Hence

$$\begin{aligned} r^2R'' + rR' - n^2R &= 0 \\ \partial_\theta^2 F + n^2 F &= 0 \end{aligned}$$

$n$ must be an integer since $F(\theta)$ must be a $2\pi$ periodic function. The solutions are therefore

$$\phi(r,\theta) = c_0 + d_0 \ln \frac{r}{a} + \sum_{n=1}^{\infty} \left( c_n \left(\frac{r}{a}\right)^n + d_n \left(\frac{r}{a}\right)^{-n} \right) (e_n \cos n\theta + f_n \sin n\theta)$$

(a) Determine which of the coefficients $c_k$, $d_k$, $e_k$ and $f_k$ are non-zero for this problem. **Hint:** $\phi$ must be bounded and well behaved in $0 \leq r < a$.

(b) Obtain the general expression for $\phi(r,\theta)$ at $r = a$.

(c) Solve the fourier series problem and determine the coefficients.