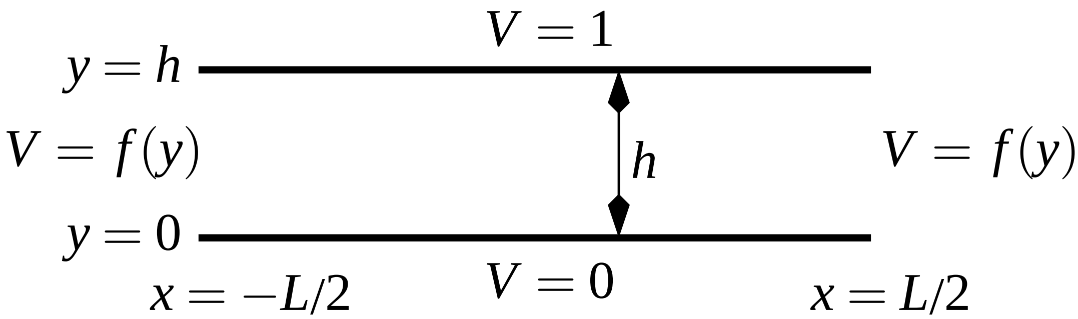
# Fringing fields of a capacitor

22nd January 2007




We start with the known $1-D$ solution to this problem

$$\phi(x,y) = \frac{y}{h} \tag{1}$$

This solution satisfies Laplace's equation and also satisfies the boundary conditions on the top and bottom walls. However, it assumes that the potential on the side walls is given by

$$V(y) = \frac{y}{h}$$

which is, in general, not true. Suppose the actual potential that is present in the side is $f(y)$. We therefore need to add a correcting piece to Eq. 1 that satisfies

- $V = 0$ on top and bottom walls, and
- $V = f(y) - y/h$ on both side walls.

This problem is very similar to the problem we have already done. The solution is given by

$$\phi(x,y) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{h}y\right) \frac{\cosh\left(n\pi x/h\right)}{\cosh\left(n\pi L/2h\right)} + \frac{y}{h}$$

where cosh functions are used along $x$ since I want symmetric, exponential solutions. The $c_n$ are given by

$$c_n = \frac{2}{h}\int_0^h \left(f(y) - \frac{y}{h}\right) \sin\left(n\pi y/h\right) dy$$

The potential $f(y)$ along the sides of the parallel plate capacitor is unknown, but lies between 0 and 1 (why?). For $L \gg 2h$, the solution can be simplified to

$$\phi(x,y) = c_1 e^{-\frac{\pi}{h}\left(\frac{L}{2}-x\right)} \sin\left(\frac{\pi}{h}y\right) + \frac{y}{h}$$

This is graphed in the following program. The function `fpot` defines a trivial fringing field due to only the first component.

2a $\langle * \text{ 2a}\rangle\equiv$

```
function phi=fpot(x,y)
  phi=c1*exp(-%pi*a*(1-abs(x)))*sin(%pi*y);
endfunction
```

We obtain the potential for $L_x/d = 10$. $x$ and $y$ are normalized to their respective sizes.

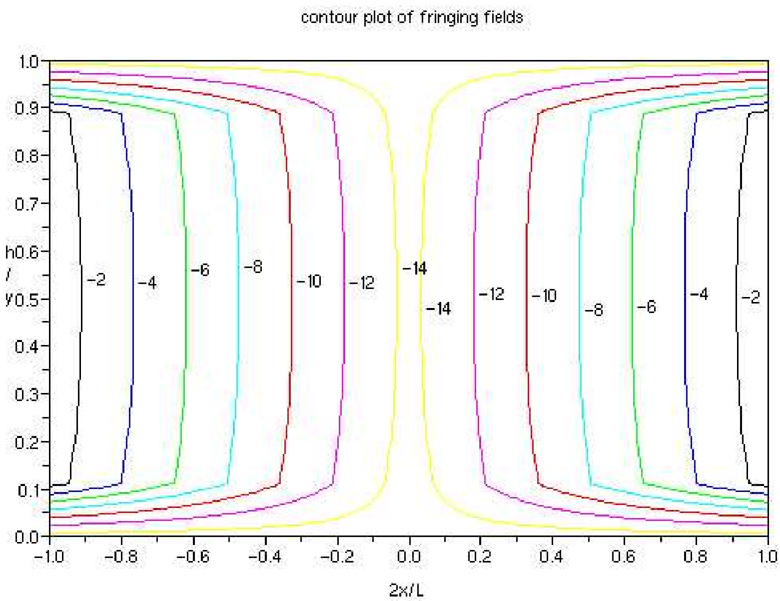
2b $\langle * \text{ 2a}\rangle{+}{\equiv}$

```
phi=zeros(10,100);
a=10;
c1=-0.5/%pi;
x=linspace(-1,1,100);
y=linspace(0,1,10);
fphi=abs(feval(x,y,fpot));
```

The computed potential has zeros. Since I want a log plot, I remove very small values and then take the logarithm, and plot the same.

2c $\langle * \text{ 2a}\rangle{+}{\equiv}$

```
fphi(find(fphi<1e-15))=1e-15;
fphi=log10(fphi);
contour(x,y,fphi,-2:-2:-14);
xtitle("contour plot of fringing fields",...
  "2x/L","y/h");
```




As can be seen, the fringing potentials fall off rapidly and are only significant at the very edges. This is why a parallel plate capacitor can be well modelled as a $1-D$ device.