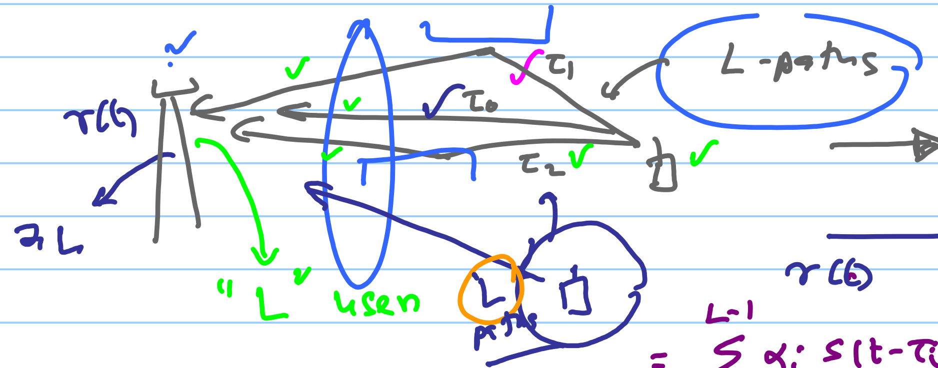
Lesson 8, Class 1 -- Mar. 31, 2022

Note Title

Recall

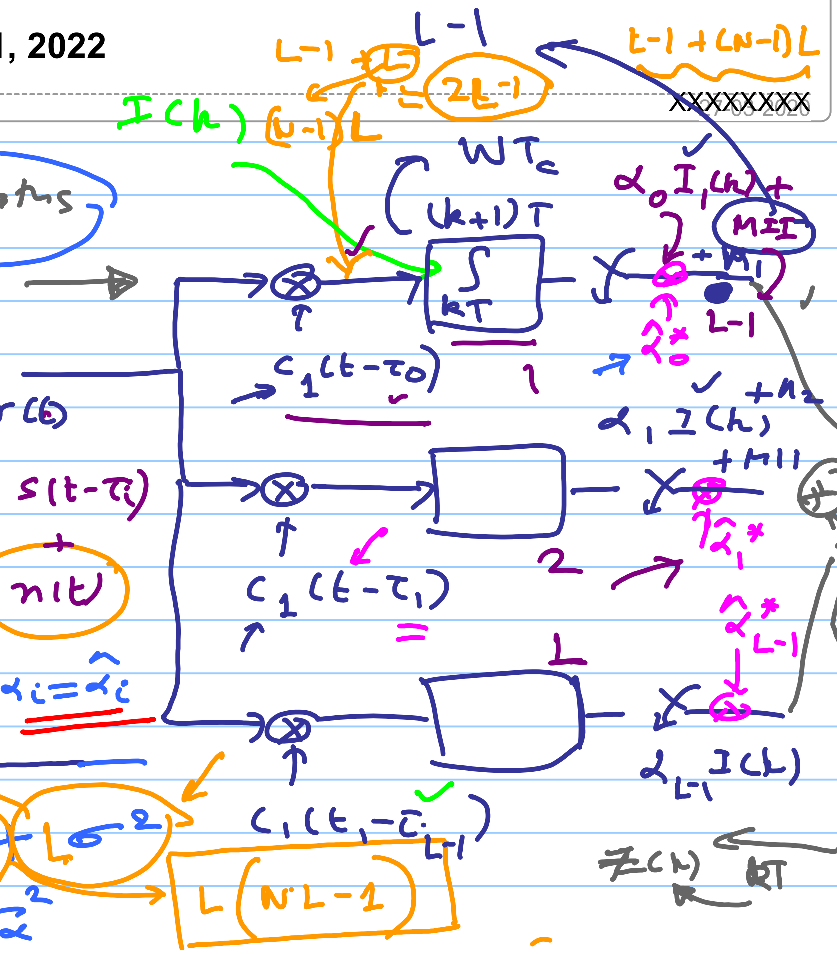
single-aper.



"L" ~ "chip-time" resolvable multipath components

(2)

$r(t) = \sum_{i=0}^{L-1} \alpha_i s(t-\tau_i) + n(t)$



$$\text{SINR} = \frac{\text{W.P} \sum_{i=0}^{L-1} |\alpha_i|^2}{L(L-1)\cdot P(\hat{\alpha}_i^* \alpha_j)^2 + L\sigma^2} \qquad \alpha_i = \hat{\alpha}_i$$

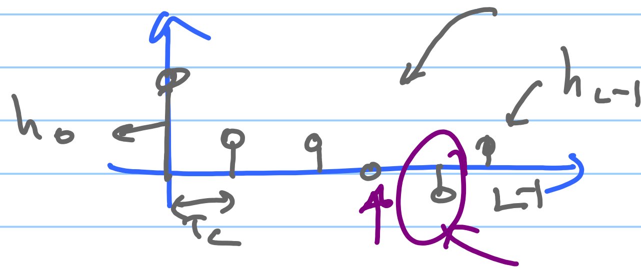
σ_α^2

$L(NL-1)$

MRC combining

$$\frac{wP\, L\cdot(\cdot)}{\underset{(NL-1)L}{(N-1)P} + L\sigma^2} \approx \frac{wP\, C_0'}{(NL-1)P + \sigma^2}$$

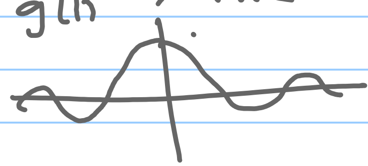
$(L-1) + (N-1)L = NL-1$



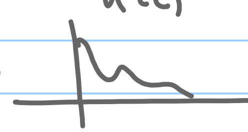
RAKE $\longrightarrow$ Chip-level equalization.

Recall: Nyquist-Pulse Shaping

$g(t) \rightarrow$ SRRC



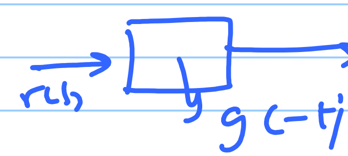
$\longrightarrow$



$\longrightarrow$

$$r(t) = \sum I(k)\, \underbrace{g'(t-kT)}_{g(t) * h(t)} + n(t)$$

$g' \neq$ SRRC

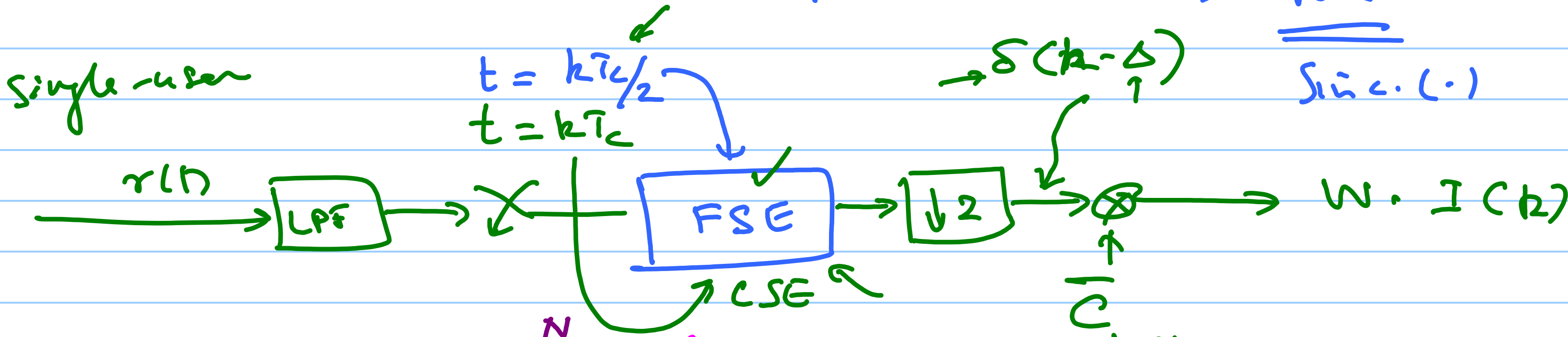


$(g(t) * h(t)) * \underline{\underline{w(t)}}$ → Nyquist pulse-shape

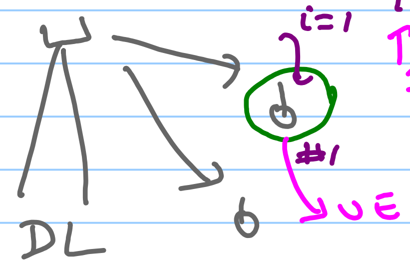
Equaliser

$\underline{\underline{RC}}$

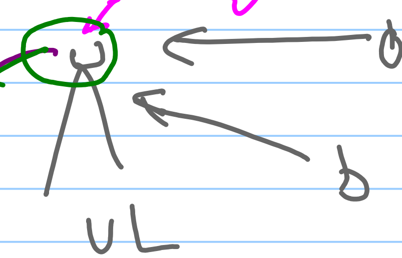
Sinc.(·)



$$r_1(t) = \sum_{i=1}^{N} h_1(t) * s_i(t)$$



$$\ddot{r}(t) = \sum_{i=1}^{N} h_i(t) * s_i(t) + n(t)$$



EE5141 Jan-Apr, 2022

Lesson 8, Class 2 -- Apr. 04, 2022

→ Introduction to Block Modulation

Uplink Multiple Access

(*) Effect of relative delay on the UL between different user waveforms.

DS-CDMA

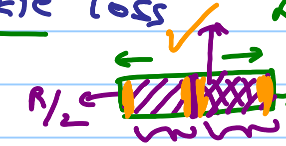
MU intf. → $c_1 \leftarrow s_1(t)$, $s_2(t) - c_2 \longrightarrow s_1(t), \; s_2(t-\tau_1)$ APSI-136

(*) Multipath propagation → optimal Rx → $\sum \alpha_i \, \delta(t-\tau_i)$ ← τ_{max}

MP intf. → $s_1(t) * h_1(t)$ → $P \approx \left(\sum_{i=0}^{L-1} |h_i|^2 \right) \cdot P_1$ $h_i, \; i = 0 \cdots L-1$

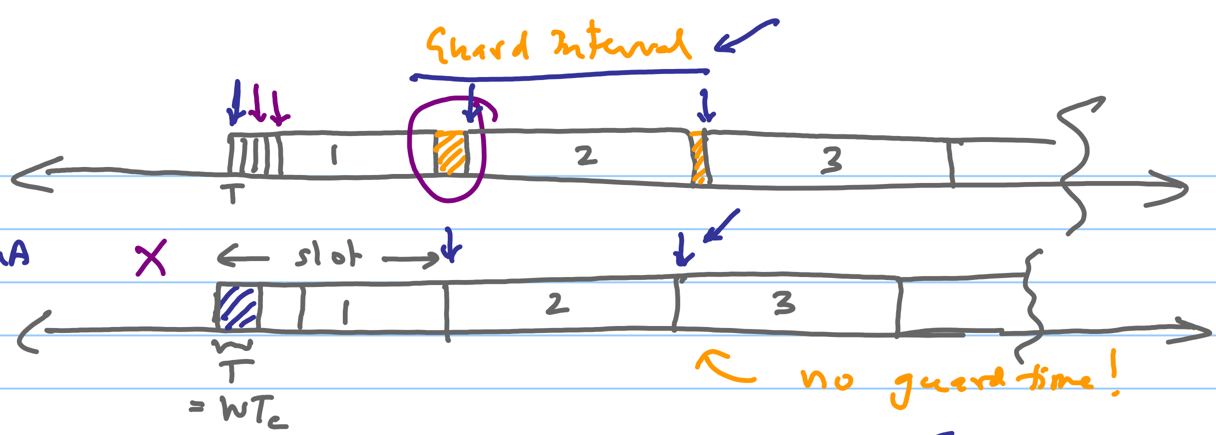
(✓) Narrow-banding possible without any sum-rate loss $< I_1(h) + I_2(h) >$

↳ Flexibility of Resource Allocation.



TDMA

DS-CDMA



1995

TDS-CDMA

LAS Codes

$\bar{c}_1 \rightarrow c_1(t)$

$\bar{c}_2 \rightarrow c_2(t)$

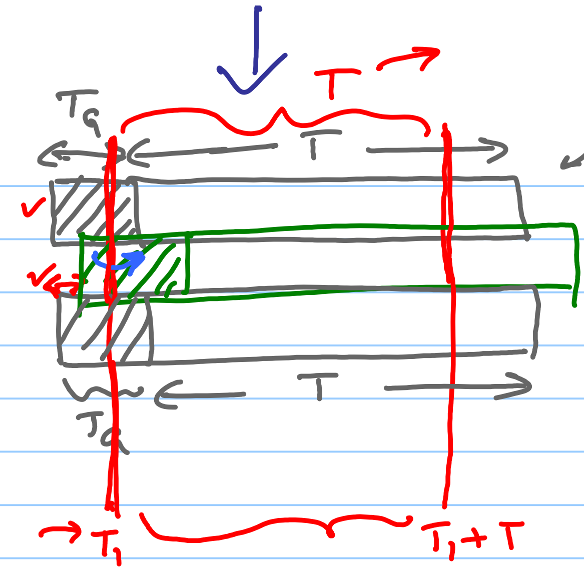
$$\int_0^T c_1(t)\, c_2(t)\, dt = 0$$

$$\not\Rightarrow \int_0^T c_1(t)\, c_2(t-\tau_0)\, dt \neq 0$$

IBI

N

$\boxed{N' > N}$



$c_1(t) \to T + T_G$

$c_2(t) \to T + T_G$

$N \to N'$

$$\int_{T_1}^{T_1 + \boxed{T}} c_1(t)\, c_2(t)\, dt = 0 \; ?$$

(N) or N'

$c_1(t-\tau_1)\; c_2(t-\tau_2)$

"Eigen functions"

DFT basis

Yes!

$$\bar{x}[n]_{N\times 1} \xrightarrow{F_{N\times N}} \bar{X}(k)_{N\times 1}$$

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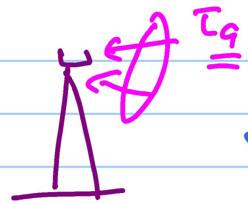
Lesson 8, Class 3 -- Apr. 05, 2022

* UL, LOS links

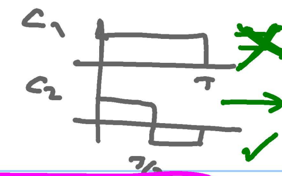


$\underline{\underline{T_g}}$

$c_2(t) \to \bar{c}_2$ $c_1(t) \to \bar{c}_1$ $\bar{c}_1 \longleftrightarrow \bar{c}_{25}$

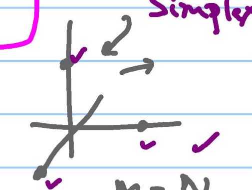
$s_1(t)$ $s_2(t)$

"multi-code"



$T \to N$

$T_g \to N_g \to \underline{\underline{N_{cp}}}$

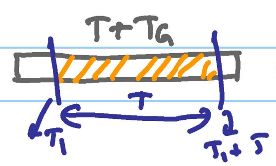


Simplex

$M = N$

i, j

	Continuous time	Discrete time
1 (*)	$c_i(t) \to \int_0^{NT_c \leftarrow T} c_i(t)\,dt = 0$;	
2 (*)	$\int_0^T c_i(t)\, c_j(t)\, dt = \begin{cases} N, & i=j \\ 0, & i \neq j \end{cases}$	

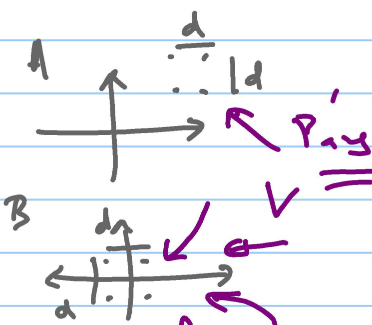


$T + T_G$

T

T_1 $T_1 + T$

$(T_0 \leq T_G)$



A

B

d d_1

P_{avg}'

$\underline{\underline{V}}$

P_{avg}

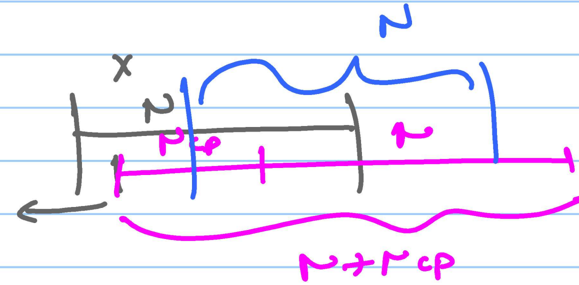
$P_e \leftarrow$ PAPR

3 (*) $\displaystyle\int_{T_1}^{T_1+T} \underbrace{c_i(t)\; c_i(t-\tau_0)}_{r(t)}\,dt = e^{-j2\pi f\tau_0}$

correlation

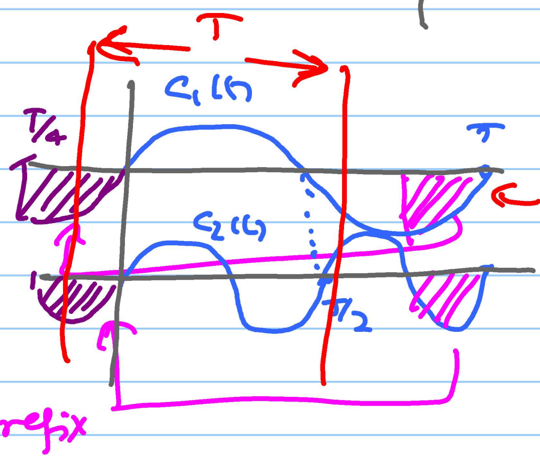
4 (*) $\displaystyle\int_{T_1}^{T_1+T} c_i(t)\; c_j(t-\tau_0)\,dt = 0$ $\quad \boxed{\tau_0 \le T_G}$

$i \neq j$



$\sin(2\pi f T)$

$\sin(2\pi f \, T/2)$



$$\int_0^T c_1(t)\, c_2(t)\, dt = 0$$

$$\int_0^T \sin(2\pi f T)\; \sin(\pi f T + \tau_0)\, dt$$

$$S_i(t) \longrightarrow I_i(k)\cdot C_i(t) \longrightarrow \text{DFT basis}$$

$$F_{N\times N} = \left[\begin{array}{cccc} | & | & & | \\ \bar{q}_0 & \bar{q}_1 & \cdots & \bar{q}_{N-1} \\ | & | & & | \end{array}\right]_{N\times N} \begin{bmatrix} 0 \\ I_1(k) \\ I_2(k) \\ \vdots \\ I_{N-1}(k) \end{bmatrix}_{N\times 1}$$

$\to$ DFT-matrix

$\times$

$N-1$

$$F_{N\times N} \to \begin{bmatrix} F' \\ \cdots \\ F \end{bmatrix}_{(N+N_q)\times N}$$

$\}\, N_q$

Lesson 8, Class 4 -- Apr. 08, 2022

In discrete time:

$c_i(t) \longrightarrow \bar{q}_i$ ← $N \times 1$

- $m \to mT_s \to$ time
- $n \to$ subcarrier (tone) index
- $k \to$ block

$e^{j\frac{2\pi}{N} m\cdot n}$, $m = 0, \dots, N-1$

$$\underbrace{\begin{array}{cccc} n=0 & n=1 & \cdots & n=N-1 \end{array}}_{} \quad \begin{bmatrix} | & | & & | \\ \bar{q}_0 & \bar{q}_1 & \cdots & \bar{q}_{N-1} \\ | & | & & | \end{bmatrix}_{N\times N} \begin{bmatrix} 0 \\ I(1) \\ I(2) \\ \vdots \\ I(N-1) \end{bmatrix}_{N\times 1}$$

$$\bar{q}_0 = \begin{bmatrix} e^{j\frac{2\pi}{N}0\cdot 1} \\ e^{j\frac{2\pi}{N}1\cdot 1} \\ \vdots \\ e^{j\frac{2\pi}{N}(N-1)\cdot 1} \end{bmatrix}$$

IDFT Matrix $N\times N$ = F

W

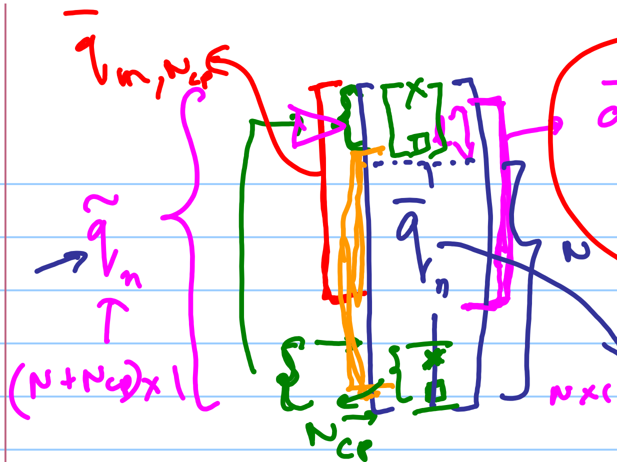
$N = \frac{\text{SINR}}{} = \text{dB}$

$\to N \approx \frac{W}{4}$

$N-1$ users.

$$\bar{q}_0 \cdot 0 + \bar{q}_1 \cdot I(1) + \bar{q}_2 \cdot I(2) + \cdots \bar{q}_{N-1} I(N-1)$$

$N \times 1$

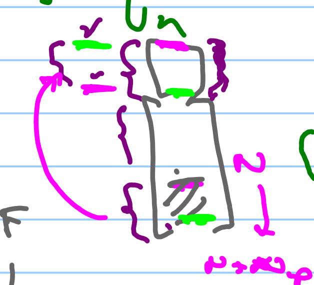


$\bar{q}_{n,\ell} \to \boxed{\ell = 0, 1, \ldots N_{cp}}$ $\quad \tilde{q}(n-\ell)$

N out of $N+N_{cp}$ $\Rightarrow$ contiguous

$$\bar{q}_n = \begin{bmatrix} \bar{q}_n(0) \\ \bar{q}_n(1) \\ \vdots \\ \bar{q}_n(N-1) \end{bmatrix} \to m$$

circular extension of $\bar{q}_n$



$T \xrightarrow{\frac{1}{\sqrt{N}}} F$

Balanced FFT

✓ Prop #1 :

$$\sum_{m=0}^{N-1} \bar{q}_n(m) = \begin{cases} 0, & n = 1, 2, \ldots N-1 \\ \sqrt{N}, & n = 0 \end{cases}$$

✓ Prop #2 :

$$\bar{q}_n^H \bar{q}_{n'} = \begin{cases} 0, & n \neq n', \quad n, n' \in \overbrace{\{0, 1, \ldots N-1\}}^{N} \\ 1, & n = n' \end{cases}$$

Prop #3 :

$$\bar{q}_n^H \, \bar{q}_{n,\boxed{l}} = e^{-j\frac{2\pi}{N} n \boxed{l}}$$

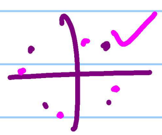
$0 \le l \le N_{cp}$

3

Prop #4 :

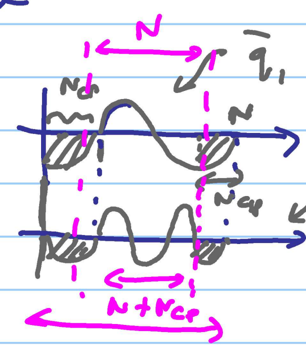
$$\bar{q}_n^H \, \underbrace{\bar{q}_{n',l}} = 0 \,, \quad n \ne n'$$

$0 \le l \le N_{cp}$



Multi-path case

Multi-user case



FFT window.

$I(n)\,\bar{q}_n + I(n')\,\bar{q}_{n'}$

$\rightarrow \otimes \rightarrow I(n)$

$\bar{q}_n$

$I(n)\,\bar{q}_{n,l} + I(n')\,\bar{q}_{n',l'}$

$\rightarrow \otimes \rightarrow \boxed{\Sigma} \rightarrow$

$\bar{q}_n$

$I(n)\, e^{-j\frac{2\pi}{N} n l}$

1

$$\int_0^T \sin(2\pi f_0 t) \cdot \sin(4\pi f_0 (t-\tau_0))\, dt$$

$f_0 = \frac{1}{T}$

$0 \le \tau_0 \le \frac{T_{cp}}{N_{cp}}$

WSSUS

EE5141 Jan-Apr, 2022

Lesson 8, Class 5 -- Apr. 11, 2022

Recall: $e^{j\omega_c t}\left(u(t) - u(t-T)\right)$

LTI

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$ — linear conv

circular conv.

$h(t) \Updownarrow H(f)$

$e^{j\omega_c t}$ ← eigen fn.

$\omega_c = 2\pi f_c$

$\delta(f - f_c)$

$$\boxed{y(t) = \underbrace{H(f_c)}_{re^{j\theta}} \cdot e^{j2\pi f_c t}}$$

$A\bar{v} = \lambda \bar{v}$

$$\boxed{\bar{x}(N) = \sum_{i=1}^{N-1} I(i)\, \tilde{\bar{q}}(i)}$$

$\tilde{\bar{q}}(i) \rightarrow (N + N_{cp}) \times 1$

$\bar{q}(i) \rightarrow N \times 1$

multi-tap

$I(n)\, \bar{q}_n \rightarrow \boxed{\{h_\ell\}} \rightarrow \oplus \rightarrow \tilde{y} \rightarrow \langle \bar{q}_n \rangle$

noise

$\tilde{y}(n)$

$\tilde{z}$

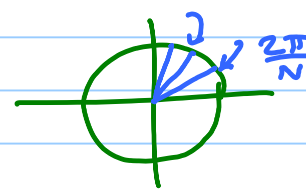
$$H(z) = \sum_{\ell=0}^{L-1} h_\ell z^{-\ell}$$

$L \leq N_{cp}$

$$\underbrace{\left(h_0 + h_1 e^{-j\frac{2\pi}{N}n \cdot 1} + h_2 e^{-j\frac{2\pi}{N}n \cdot 2} + \dots\right)}_{H\left(e^{j\frac{2\pi}{N}n}\right)} I(n)$$

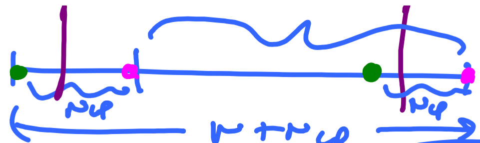
Prop #3

$n' \neq n$



$\leftarrow N \rightarrow$

N



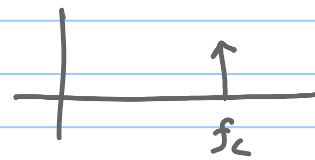
$N \to 2^K$ ✓ $\quad 2^{k_1}\ 3^{k_2}\ 5^{k_3}$

$x(n) \to \boxed{h_\ell} \to \quad y(n) = h_\ell * x(n)$

$\tilde{x}(n) \to \boxed{h_\ell} \to \quad \underbrace{y(n) = h_\ell \circledast x(n)}$

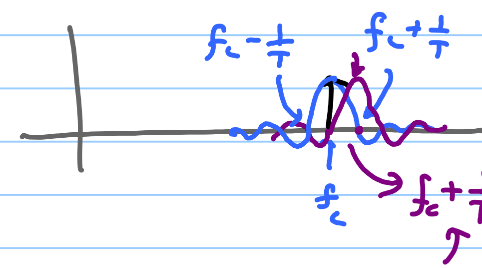
$\bar{x}(n) \longrightarrow F_c\, \bar{x}(n) \longrightarrow \bar{y}(n)$

$e^{j\omega_c t} \rightleftharpoons$



;

$e^{j\omega_c t}\underbrace{(u(t) - u(t-T))}$ ✓



$\overbrace{\underbrace{\bar{q}_{k_0}, \bar{q}_{k_1}, \bar{q}_{k_2} \cdots \bar{q}_{k_{N-1}}}_{\text{OFDM}}}$

$N \to$ <u>samples</u>

WiMax :

Sub-carrier Bandwidth $\Delta f = \underbrace{10}\ \text{KHz}$ ✓ 15 KHz

$T_{cp} \rightarrow \frac{T}{8}, \frac{T}{16}, \frac{T}{32}$

$\Rightarrow$ useful symbol duration $= \frac{1}{\Delta f} = \frac{1}{10\,\text{KHz}} = 100\,\mu\text{sec}$; T_u

$\Rightarrow$ OFDM symbol duration $= T_u + T_{cp} = T$

based on your $\tau_{rms} \approx \sigma$

$\tau_{rms} \approx 1\,\mu\text{sec} \rightarrow T/16$

$\pm 3\sigma$ || $3\,\mu\text{sec}$ ||

$BW \simeq N \times \Delta f$.

$k \rightarrow$ Block #

$m \rightarrow$ Time

1 to N $\leftarrow n \rightarrow$ Subcarrier #

0 to N-1

$N \rightarrow$ # of subs/block $\rightarrow (N-1)$

0

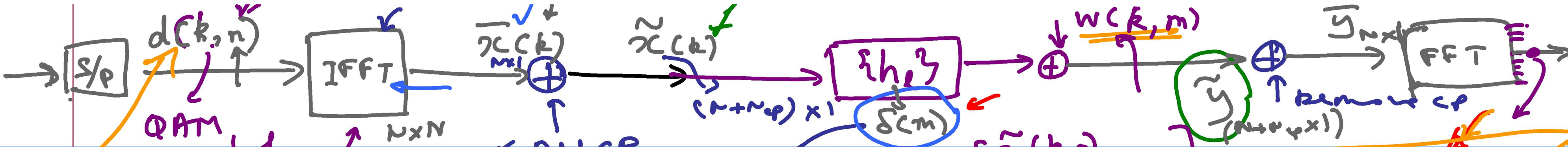
OFDM Transceiver Chain

$\tilde{x}$ $\rightarrow \tilde{x}$

$[\text{HHH}][\text{———}|\text{HHH}]$

N_{cp}

$\left[F^{*} \right] \vec{d} = \tilde{x}$ $\leftarrow \vec{F}$



L=3

N+Ncp

$$\tilde{x}(k) = \begin{bmatrix} \tilde{x}(k,0) \\ \tilde{x}(k,1) \\ \vdots \\ \tilde{x}(k, N+N_{cp}-1) \end{bmatrix}$$

m

$$y(k,n) = H(k,n) \cdot d(k,n) + W'(k,n)$$

$(n = 1 \dots N-1)$

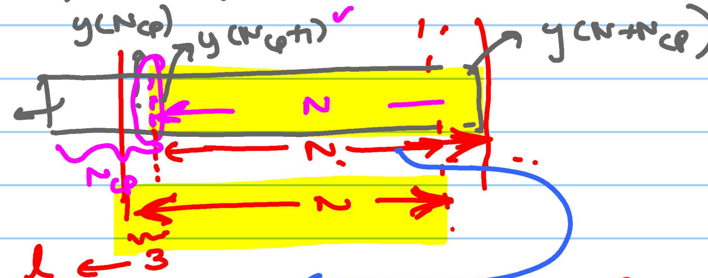
$$\frac{1}{\sqrt{N}} \underbrace{\begin{bmatrix} | & | & & | \\ \bar{q}_0 & \bar{q}_1 & \cdots & \bar{q}_{N-1} \\ | & | & & | \end{bmatrix}}_{F^H} \underbrace{\begin{bmatrix} d(k,0) \\ d(k,1) \\ \vdots \\ d(k,N-1) \end{bmatrix}}_{\bar{d}(k)}$$

N×1

$$\underset{N\times 1}{\bar{x}(k)} = \frac{1}{\sqrt{N}} F^H \bar{d}(k)$$

h^m s.ck

y(1)



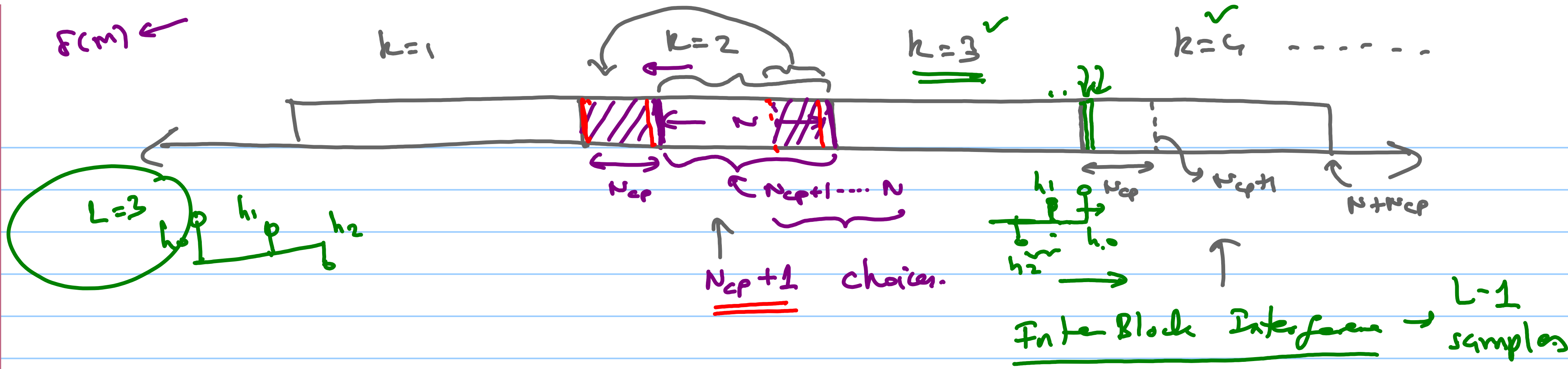
"critical window"

$H(k,n) = 1 \quad \forall n$

$d(1) \quad d(2) \quad d(3) \; \cdots \; d(N-1)$

$\eta(n)$ $\quad m = 0 \cdots N-1$

$$y(n) = e^{-j\frac{2\pi}{N} n \cdot 3} \cdot d(n) + w'(n)$$



EE5141 Jan-Apr, 2022

Lesson 8, Class 7 -- Apr. 18, 2022

OFDM → Synchronisation → Freq. (Carrier) Sync.
→ Timing Sync.

OFDM → Channel Estimation

$$\bar{q}^H_{1,\theta(m)} = \left[e^{-j\frac{2\pi}{N}(1\cdot 0+\theta(0))} \quad e^{-j\frac{2\pi}{N}(1\cdot 1+\theta(1))} \quad \cdots \quad e^{-j\frac{2\pi}{N}(1(N-1)+\theta(N-1))} \right]$$

(*) Impact of Freq. offset on OFDM Rx

$$F^H = \underset{N}{\left[\ \bar{q}_0 \ \ \bar{q}_1 \ \cdots \ \bar{q}_{N-1}\ \right]} \quad m \text{ (time)}$$

"n"

$$\bar{q}_1 = \begin{bmatrix} e^{j\frac{2\pi}{N}1\cdot 0} \\ e^{j\frac{2\pi}{N}\cdot 1\cdot 1} \\ \vdots \\ e^{j\frac{2\pi}{N}\cdot 1\cdot (N-1)} \end{bmatrix} \quad n=1,\ m=0$$

.001 ppm

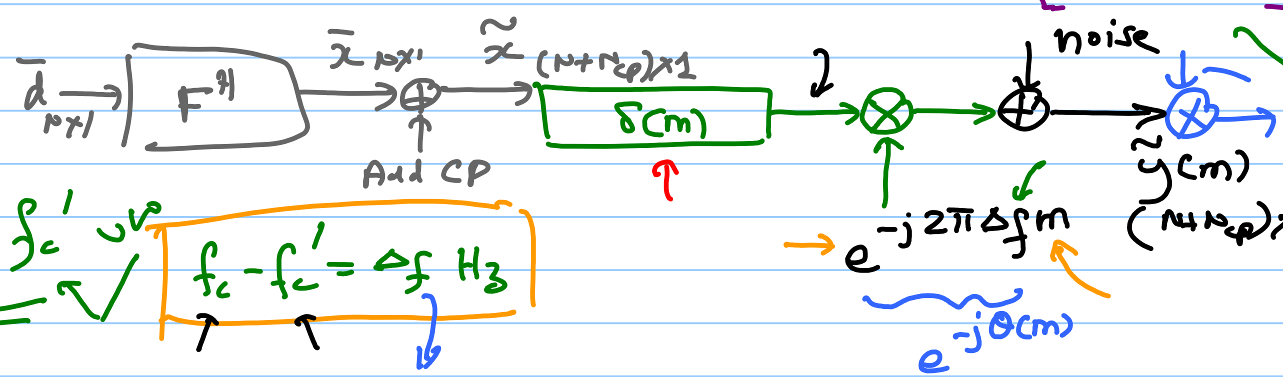
1 ppb

Carrier Freq. Offset → CFO → Δf

$f_c - f_c'$ (VCO)

$\pm f_D$ max

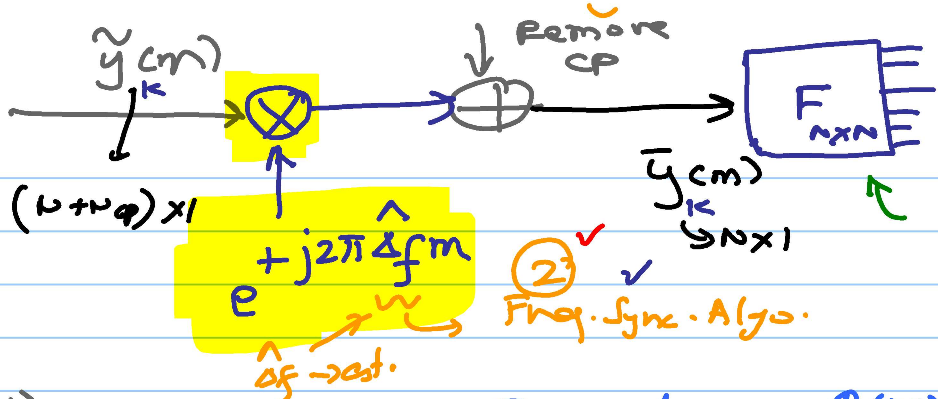
$v/(T)$



f_c $\quad f_c'$

$\boxed{f_c - f_c' = \Delta f \ Hz}$

1) Timing Sync. ← critical window.



$$Y[k,n] = H[k,n]\cdot d(k,n) + W[k,n] \quad \forall n$$

② Freq. Sync. Algo.

$$\Delta f - \hat{\Delta f} = \tilde{\Delta f}$$

1) $\therefore \quad \bar{q}_{1,\theta(m)}^{H} \; \bar{q}_1$ (CFO) $\quad d(1)$.

$$\begin{cases} \neq 1 & \theta(m) \neq 0 \\ = \frac{1}{N}\sum_{m=0}^{N-1} e^{j\left(\frac{2\pi}{N}\cdot 1\cdot m - \theta(m)\right)} e^{-j\frac{2\pi}{N}1\cdot m} \\ = \frac{1}{N}\sum_{m=0}^{N-1} e^{j\theta(m)} = \,? \end{cases}$$

$n=1$

unwrapped phase

$2\pi\tilde{\Delta f}m$

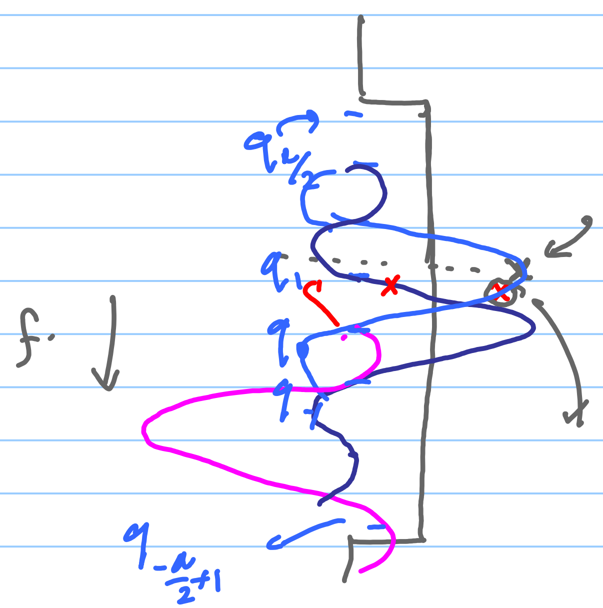
$d(2)$

$$\Rightarrow \quad \bar{q}^{H}_{r_1,\theta(m)} \, \bar{q}_{r_2} \begin{cases} \neq 0, & \theta(m) \neq 0 \\ = \dfrac{1}{N} \displaystyle\sum_{m=0}^{N-1} e^{j\left(\frac{2\pi}{N}(n'-n)m - \theta(m)\right)} \end{cases}$$

$d(1)$ $r_1 \to n$, $r_2 \to n'$

$= 0$

$2 - 1$



$\Delta f = 0$

$\tilde{\Delta f} = 0$

$|C| \longrightarrow$ ISI in single-carrier (symbol-by-symbol) comm.

10 kHz

$\Delta f_{sub} = 15$ kHz

$\tilde{\Delta f} < 1\%$ of Δf_{sub}

< 150 Hz

kHz

kHz

Lesson 8, Class 8- Apr. 21, 2022

(*) Delay, Self. (n, n) $\quad 0 \le l \le N_{cp}$

$$\frac{1}{\sqrt{N}} \bar{q}_{r_n}^{H} \cdot \frac{1}{\sqrt{N}} \bar{q}_{r_{n,l}} = e^{-j\frac{2\pi}{N} n l} \cdot d(n) \longrightarrow \text{scalar}$$

(*) Delay, Mutual

$$\frac{1}{\sqrt{N}} \bar{q}_{r_n}^{H} \cdot \frac{1}{\sqrt{N}} \bar{q}_{r_{n',l}} = 0 \qquad n' \ne n$$

(*) CFO, Self

$$\frac{1}{\sqrt{N}} \bar{q}_{r_n}^{H} \cdot \frac{1}{\sqrt{N}} \bar{q}_{r_{n,\theta(m)}} = \frac{1}{N} \cdot \underbrace{\sum_{m=0}^{N-1} e^{j\theta(m)}}_{\ne N \quad = 0}$$

$\theta(m) = 2\pi \tilde{\Delta f} m$

"n" $\to$ time index

$\underbrace{\bar{q}_{r_{N \times 1}}}_{m = 0 \ldots N-1}$

$f_c - f_c' = \Delta f$

$\hat{\Delta f} - \Delta f = \tilde{\Delta f}$

eg: $f_{sub} = 10\,KHz \Rightarrow T_u = 100\mu$ $\quad T_{cp}$

$\Delta f = 10\,KHz$ $\quad 100\mu$

$\Rightarrow$ 2

(*) CFO, mutual

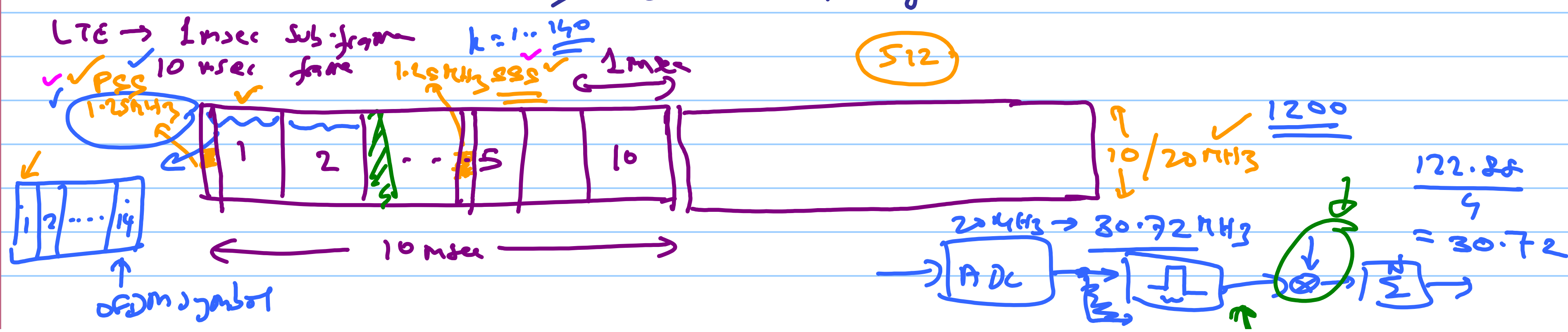
$$\frac{1}{\sqrt{N}} \bar{q}_n^{H} \frac{1}{\sqrt{N}} \bar{q}_{n', \theta(m)} = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(n'-n)m - \theta(m)} \cdot e$$

$n+1 \rightarrow$

$n \rightarrow$

$n-1 \rightarrow$

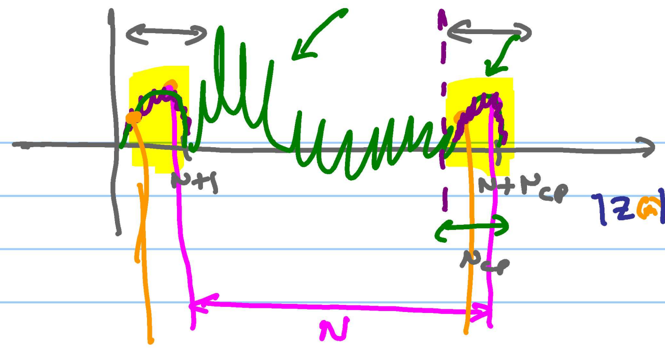
Timing Sync :
- "Cross-correlation" with a "Sync" pattern
- CP-Correlation.
- Schmidl-Cox Algo.

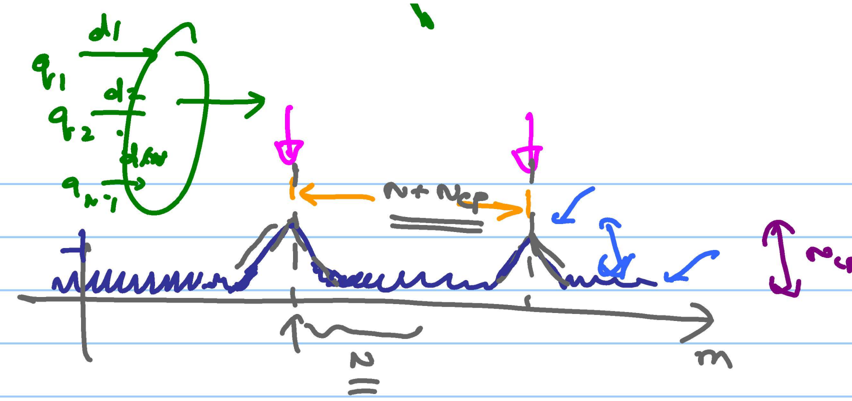


* AWGN

$\tilde{y}(k,m)$

$N+N_{cp}$





1) CP Corr:

$$Z[k,m] = \sum_{i=m-N_{cp}}^{m} \tilde{y}^*[k,i]\, \tilde{y}[k,i-N]$$

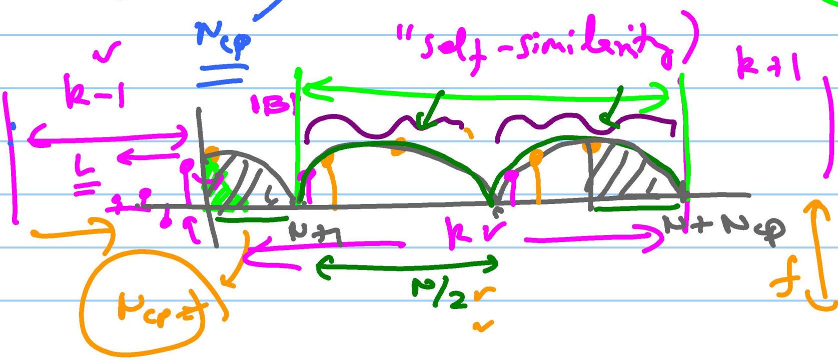
$m = 0, \ldots, N+N_{cp}-1$

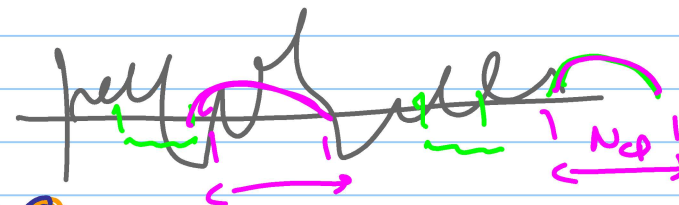
$N+N_{cp}$

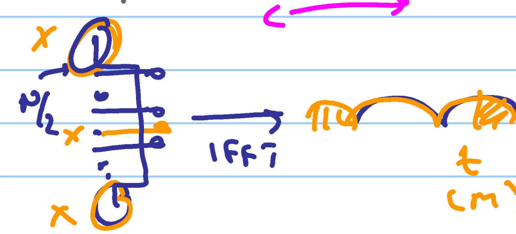
CFO

2) Schmidl-Cox

$\tilde{y}[k,m]$





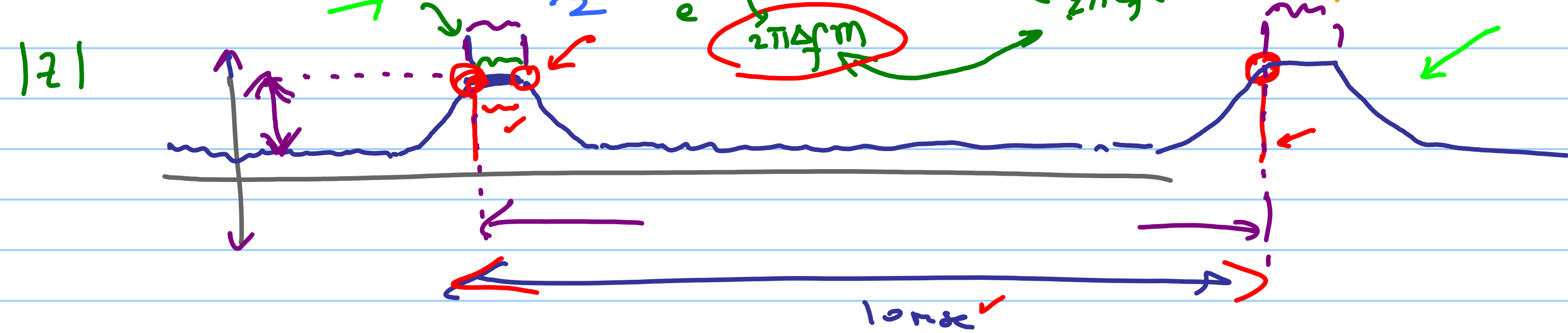


$N_{cp} \to N/16$

Δf

$$\to z[k,m] = \frac{N}{2}\left\{ \sum_{i=m-N/2}^{m} \tilde{y}^*(k,i)\, \tilde{y}^*(k, i-N/2) \right.$$

$e^{j\theta(m)}$ $2\pi\Delta f m$ $2\pi\Delta f(m-N/2)$ N_{cp}



$\Rightarrow$ What happens when $h[n] \neq \delta[n]$?

$$h[n] = \left\{ \sum_{l=0}^{L-1} h_l\, \delta(n-l) \right. \checkmark \qquad h[n,m] = \sum_{l=0}^{L-1} h_l^m\, \delta(n-l) \quad \times$$

$L(m)$

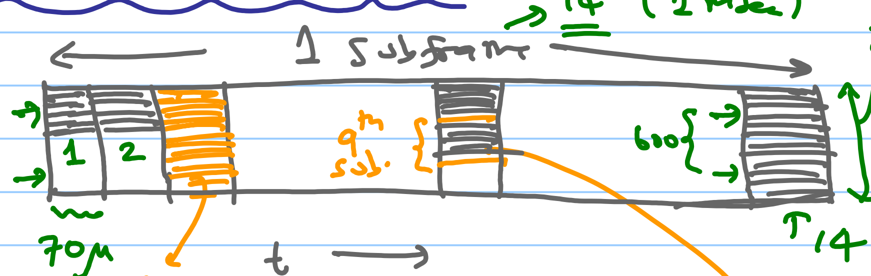
$\underline{\underline{L_{max}}}$

$\{h_l\}$ L-Taps

Lesson 8, Class 9 -- Apr. 22, 2022

(OFDMA)

→ OFDM Channel Estimation



600 → 500, 100; 600 → 70, 530

35μ

"600 pilots" → "Block pilots"

"comb" pilots, "embedded" pilots

$\times 15$ 1024 → 15.36 MHz f_s

10 MHz

600 × 15 kHz = 9 MHz

$\frac{2}{14}$ × 70 = 980μ

15 kHz

20 MHz → $\frac{1200}{2048}$

10 MHz → $\frac{600}{1024}$

5 MHz

60 MHz

F192

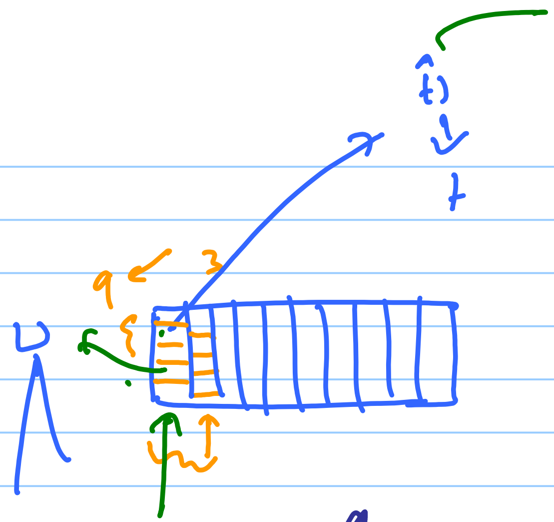
"~~scalable OFDMA~~"

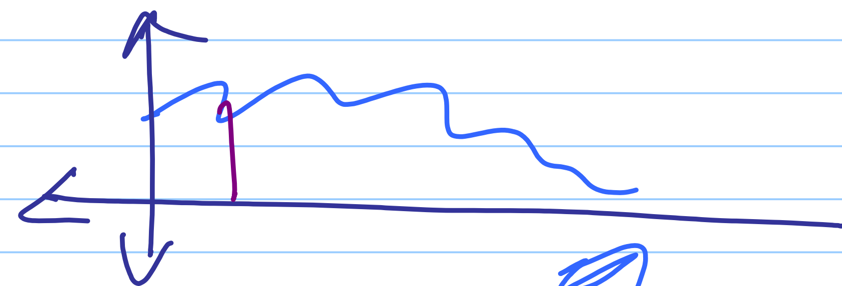
15 kHz

500 MHz → 120 kHz

"Multiple Numerology" → Δf_{sub} 15 kHz → 30 → 60 → 120 kHz

→ 4096

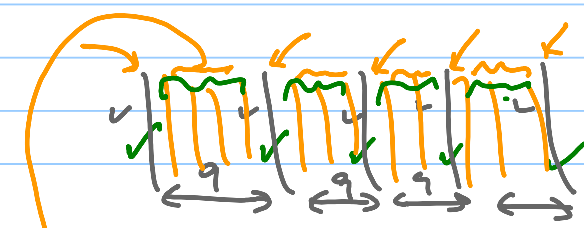




$H(p;n)$

3

2



"sparsity" $\Rightarrow$ "L"

"N"

$\hat{H}[k,n]$

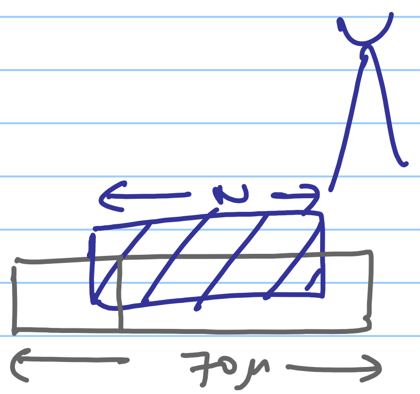
$\hat{H}[k,n+q]$

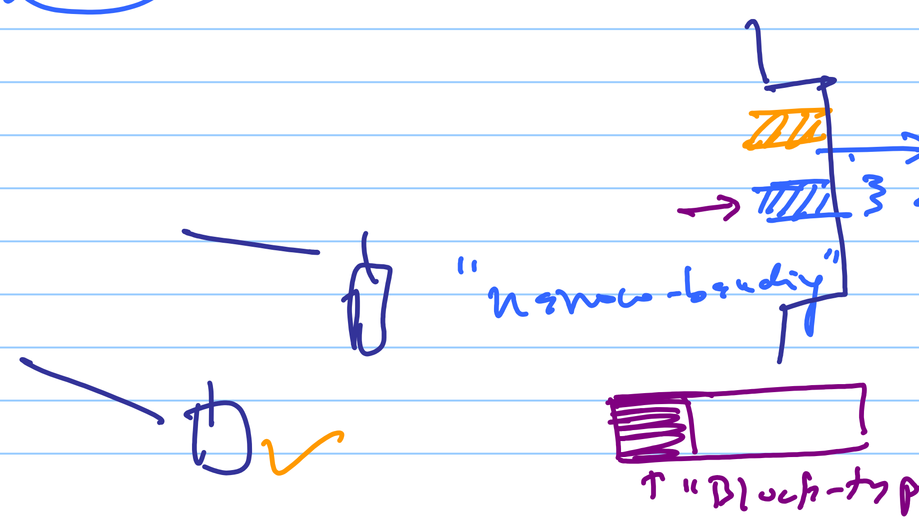
$H[k,n] \underset{DFT}{\rightleftharpoons} h[k,m]$

$\rightarrow$ ZF $e \Leftarrow$ @ $D_i(\omega)$.

$$Y[k,n] = H[k,n] \cdot d_p[k,n] + W[k,n]$$

$$\frac{Y[k,n]}{d_p[k,n]} = \hat{H}[k,n]$$





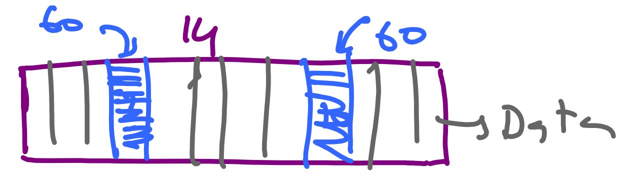
5 PRBs $\Rightarrow$ 60

600

60

12 sub $\rightarrow$ PRB

= ×14



→ "DFT- Spread OFD
- MA

→ "SC-FDMA"

Block Modulation

Multi-carrier BM

Single Carrier BM

Zero padding

UW

CP

OFDM/OFDMA

Conventional

Generalised SC