EE5141 Jan-Apr, 2022

Lesson #7, Class 1 -- Mar. 15, 2022

Note Title 15-03-2022

Multi-Access Schemes

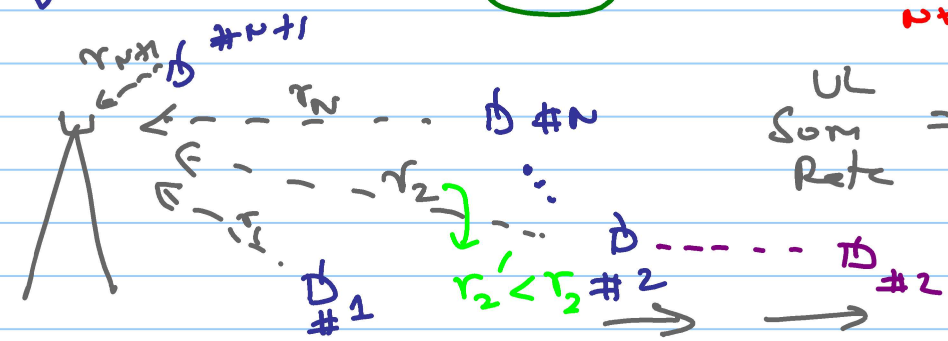
- 2G → TDMA → Yes!
- 3G → CDMA → DS-Spread Spectrum → Need not
- 4G → OFDM / OFDMA → Definitely not!

"narrow banding"

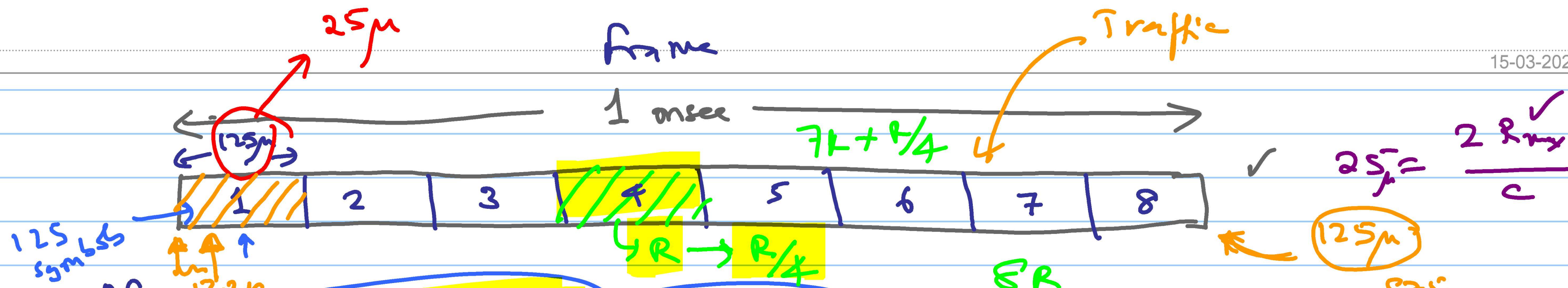
bitrate vs link distance

→ ↓r ↑d

MAC



UL Sum Rate $= \sum_{i=1}^{N} r_i = R_{sum}$ → R'_{sum}

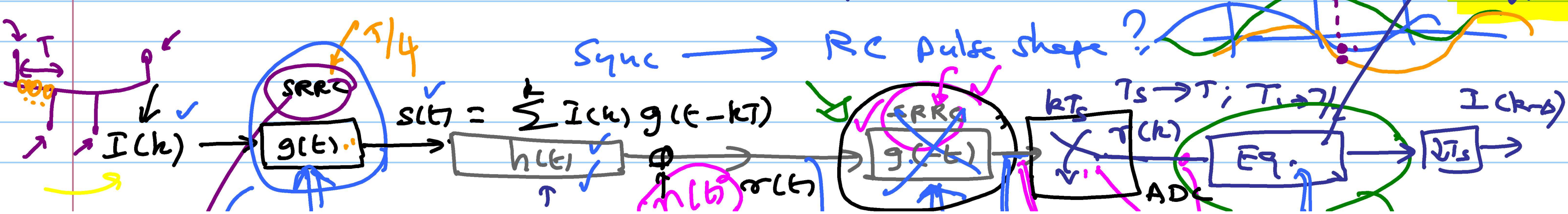


RF Bandwidth = $\Delta f = 1\,MHz$; 1 Msymbols/sec.

(*) Single carrier modulation → symbol period. $T = \frac{1}{\Delta f} = 1\,\mu sec$;

MAPSE → MLSE → VA

(*) Nyquist pulse-shaping × BW = $\frac{|G(f)|^2}{T} \rightarrow g(t)$



Note Title 15-03-2022

$I(k) \rightarrow g(k) \rightarrow$ DAC ×4 $\rightarrow s(t)$

SRRC $g(t)$

interpolator

$|G(f)| \rightarrow$ RC

$g(\cdot)r$

SRRC $\rightarrow |G(f)|^{1/2} \rightarrow g'(\cdot)$

LPF

$B_n > W H_3$

$T_s \leftrightarrow T$

BL

$n'(t)$

$n'(kT_s)$

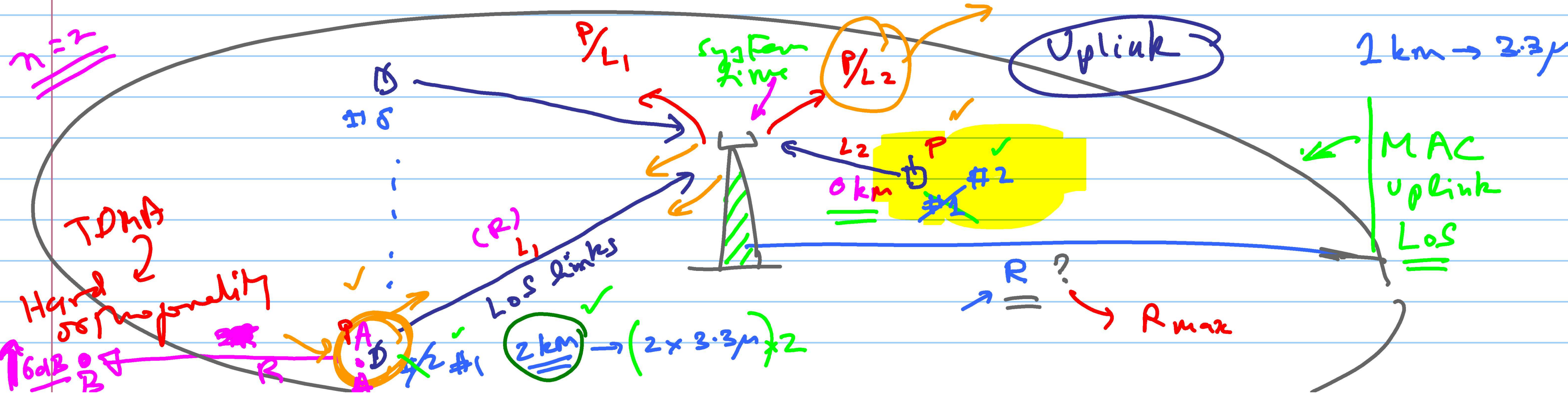
FSE

$h_T \tilde{D}_f$

R_1

R_2

$R_2 < R_1$



Note Title 15-03-2022

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Lesson #7, Class 2 -- Mar. 17, 2022

PRACH

13.2μ

13.2μ

125μ

#1

13.2μ

#2

Guard Time = $\frac{2R_{max}}{c}$

// RTT

100 MHz → 10 nsec

10 MHz

0.1 μsec

T

h(τ) → Freq2(·)

N

Multipath model → DTFT

N = 2^k

IDFT

h[n]

ep

PDP

p ↓D

Multi-path model ⟷ Multi-tap Model

τ_0 τ_1 τ_2 τ_3

70ns

nT_s

100ns

1 μsec ↳ 100ns

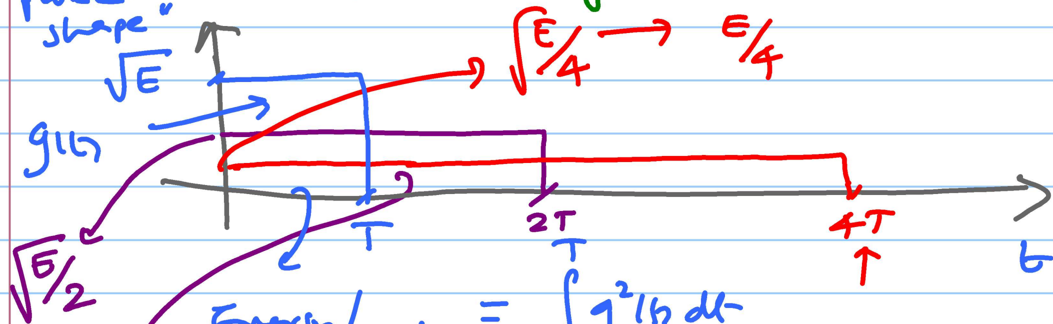
"Sample-spaced" CIR

T_s

To prevent "Collisions"

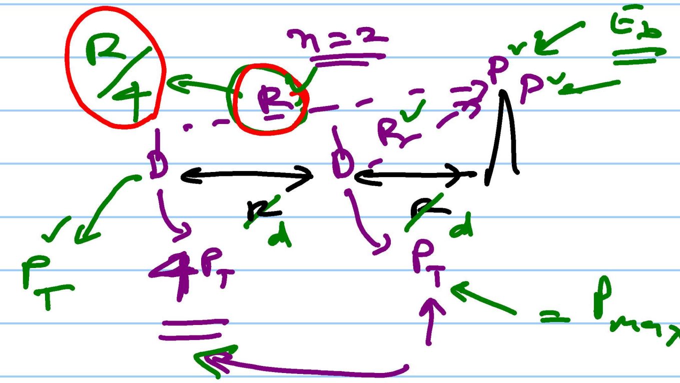
→ Guard Time GT → RTT $= \frac{2R_{max}}{c}$ ✓ → ~ 10 samples ||

"rectangular pulse-shape" → **Timing Advancement** → Ranging. ✓ → How accurate can this be?



Energy/symbol $= \int_0^T g^2(t)\,dt = E.T$ Joules

Energy/symbol $ET = \int_0^{2T} \sqrt{\frac{E}{2}}\,dt =$



TDMA → narrowbanding is possible

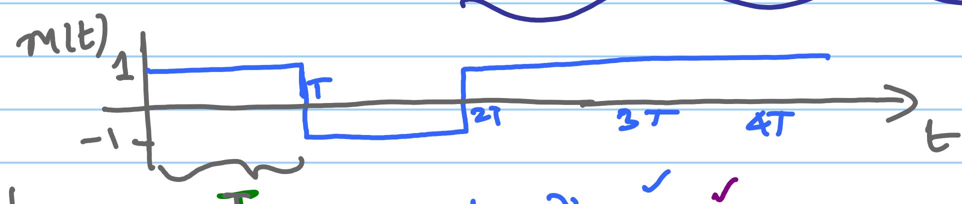
but

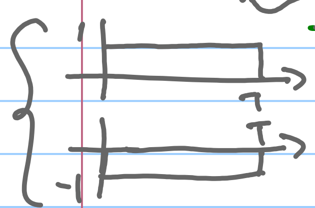
(*) Rate of user will reduce

(*) Sum Rate of system will also reduce?

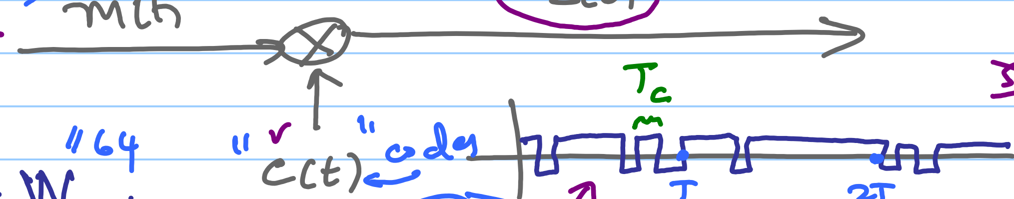
Direct Sequence CDMA (DS-CDMA)

"Bit sequence"





"Power Division"



$\approx \frac{1}{T}$ — m(t)

s(t) $\approx \frac{1}{T_c}$

c(t) "code" $\approx \frac{1}{T_c}$

"chip sequence"

$$\frac{T}{T_c} = W = 2^k$$

"64"

"spreading gain"

$$x_1(t) \cdot x_2(t) \Longleftrightarrow X_1(f) * X_2(f)$$

L_1, L_2

$k \to kT$

$$\bar{S}(k)_{64\times 1} = I(k) \cdot \bar{C}(k)_{64\times 1}, \quad \bar{C}(k) = \begin{bmatrix} c_1(k) \\ c_2(k) \\ \vdots \\ c_{64}(k) \end{bmatrix}$$

64 64 64

I(1) I(2) I(3)

k

$I \otimes \bar{c}$

$\bar{S}(k) \to g(t) \to s(t)$

$g[n]$ + DAC

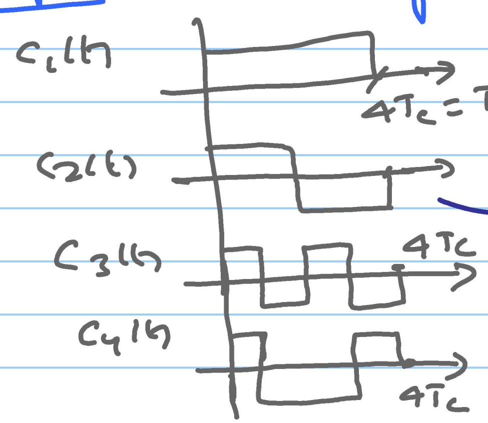
$g[nT_c]$

"chip" shaping filter

Example : "Orthogonal Codes"

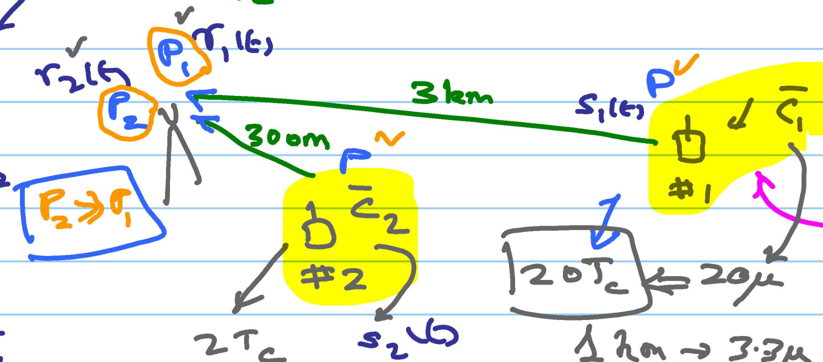
W = 4

$\frac{1}{T_c} = 1\ MHz \Rightarrow T_c = 1\mu sec$

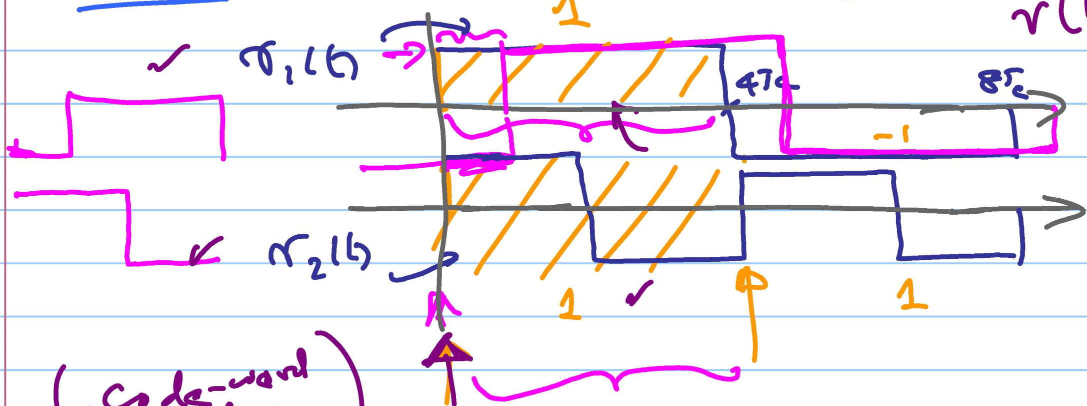


$\left(\frac{1}{T_c}\right)$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \bar{c}_1 \qquad \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = \bar{c}_2 \qquad \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = \bar{c}_4$$



Case 1 : "Accurate Ranging"



$$r(t) = r_1(t) + r_2(t) + n(t)$$

$$r_1(t) = h_1(t) * s_1(t)$$

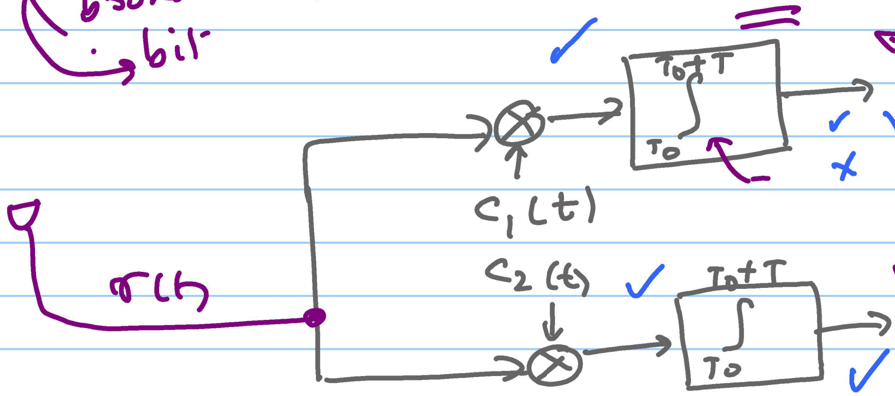
$$r_2(t) = h_2(t) * s_2(t)$$

multipath-path channel

$$h_1(t) = \delta(t - t_1)$$

$$h_2(t) = \delta(t - t_2)$$

$T = 4T_c$



kT: $P_1 I_1(kT) + n_1 + \varepsilon_{21} P_2 I_2(\cdot)$

kT: $P_2 I_2(kT) + n_2 + \varepsilon_{12} P_1 \cdot I_1(\cdot)$

W

W

Case 2: "Inaccurate" Ranging.

"near-far"

$T_c \simeq 1 \mu sec.$

$T_c = 0.1 \mu sec$

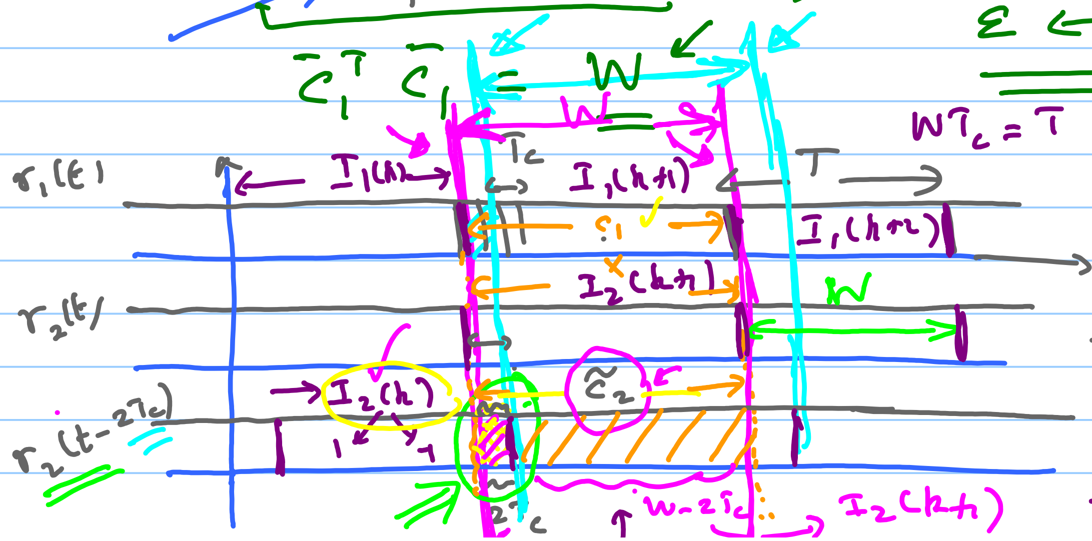
"sub-chip"

$\bar{c}_1^T \bar{c}_2 = 0 \rightarrow$ time aligned

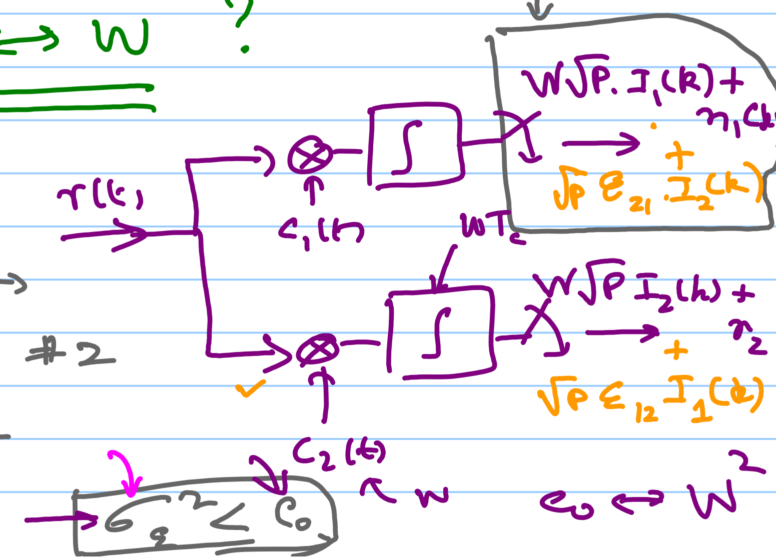
$\bar{c}_1^T \tilde{c}_2 = \varepsilon \rightarrow$ alignment is absent

$\varepsilon \leftrightarrow W$?

$W T_c = T$



#2



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Lesson #7, Class 4 -- Mar. 22, 2022

$2^{42}-1$ "0" 40

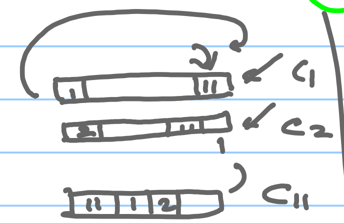
2^{41} "1" 40

$2^{42}-1$

$2^{14}-1$

PN codes
- M-ary sequences → LFSR
- Gold codes →
- Kasami Codes
- Barker

$c_1^T c_2$

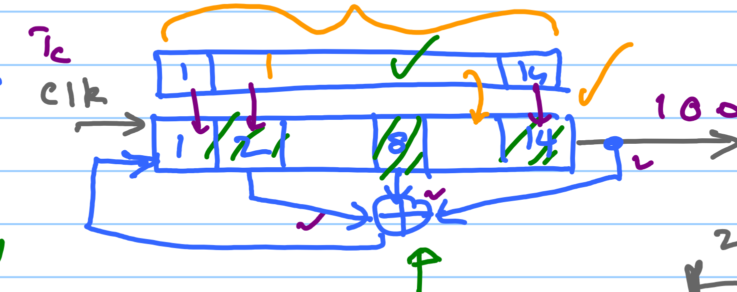


$\bar{c}_i^T \bar{c}_j = \begin{cases} W, & i=j \\ 0, & i\neq j \end{cases}$

ε_{ij} r.v.

$E[\varepsilon_{ij}]=0$

$x_{I_2}(k)$

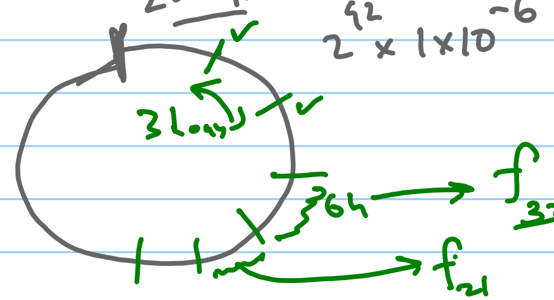


10010...... | 10010. .. |

← $2^{14}-1$ →

2 days

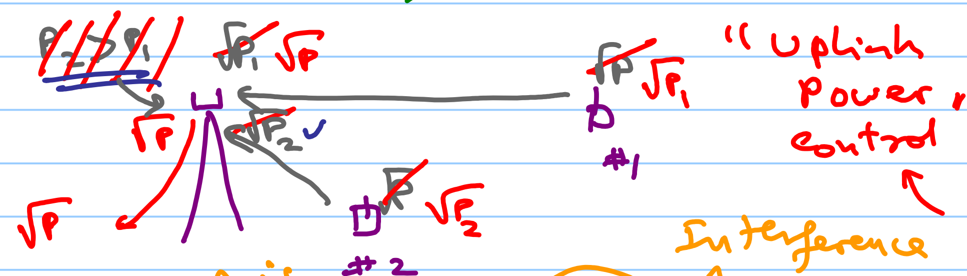
$2^{42} \times 1\times10^{-6}$ sec



FH

Uplink "soft" Capacity of DS-CDMA

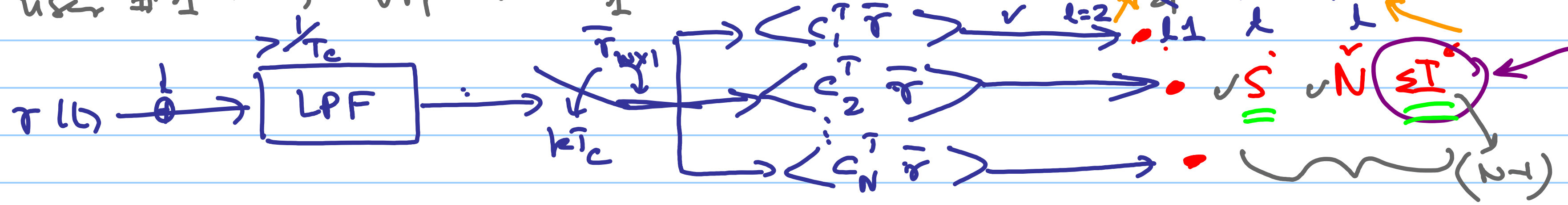
"single cell" ⟶ cellular context



(*) $\bar{c}_i^T \bar{c}_j = \begin{cases} W, & i = j \\ \varepsilon, & i \neq j \end{cases}$

user #1 ⟶ $\sqrt{P_1} \cdot W \cdot I_1(kT) + n(kT) + \sum_{\ell=2}^{N} \varepsilon_{\ell 1} \sqrt{P_\ell} \cdot I_\ell(kT)$ → (1)

noise, Interference



→ (*) Assume "perfect" UL Power Control

$N \rightarrow$ users $\rightarrow \sqrt{P} \Rightarrow P$; All users use same | W | $2W$, $W/2$, $2^{-k}W$ |

$\Rightarrow$ all users have the same bit rate

User #1

$W = T/T_c$;

$\bar{c}_1^T$ $[c_{11}\ c_{12} \dots c_{1W}]$ $\begin{bmatrix} c_{21} \\ c_{22} \\ \vdots \\ c_{2W} \end{bmatrix}$ $\bar{c}_2$ $= \varepsilon_{12} \rightarrow$ $E(\varepsilon_{12}) = 0$, $E[\varepsilon_{12}^2]$

$\varepsilon_{13}\ \varepsilon_{14} \dots \varepsilon_{1W}$

$c_{11} \in \{1, -1\}$

$T/T_c = W$

$r(t) = \sum_{\ell=1}^{N} s_\ell(t-\tau_\ell) + n(t)$

$r(t) \rightarrow \bar{r} \rightarrow \langle \bar{c}_1^H \bar{r} \rangle \rightarrow$

$\bar{r}_k = \sum_{\ell=1}^{N} I_\ell(k) \cdot \bar{c}_\ell(k) \cdot \sqrt{P} + \bar{n}(k)$

I ↔ $\bar{c}$

Lesson #7, Class 5-- Mar. 24, 2022

$\bar{c}_i^T \bar{c}_j$, $i \neq j$

$c_{ij} \in \{+1, -1\}$

$$\bar{c}_1^T \bar{c}_2 = \sum_{n=1}^{W} c_{1n} c_{2n} = \varepsilon_{12}$$

$c_n \in \{1, -1\}$

$E[\varepsilon_{12}] = 0$

$E[\tilde{c}(n)] = 0$

$\sigma^2 = E[\tilde{c}^2] = 1$

$$\Rightarrow E[\varepsilon_{12}^2] = Var.(\cdot) = W \cdot 1 = W$$

Focus on R_x #1

$$\bar{c}_1^T \bar{r} = \bar{c}_1^T \left(\sum_{l=1}^{N} I_l(k) \cdot \sqrt{P}\, \tilde{\bar{c}}_l(k) + \bar{n}(k) \right)$$

$W \times 1$

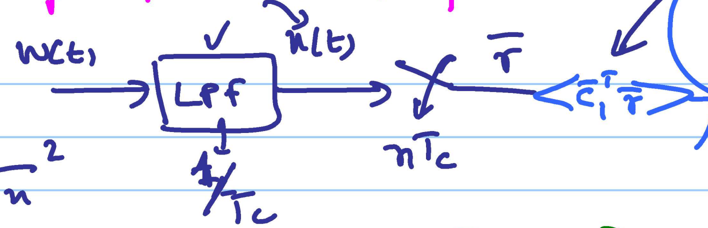
$N \leftrightarrow W$

$$\bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_W \end{bmatrix}_{W \times 1} \rightarrow \sigma_n^2$$

$$= W \cdot \sqrt{P} \cdot I_1(k) + \sqrt{P} \cdot \sum_{l=2}^{N} I_l(k) \cdot \varepsilon_{1l} + n_k$$

$\bar{c}_1^T \bar{n} = \sum_{l=1}^{W} c_{1l} \cdot n_l$; $c_{1l} = +1, -1$; $\tilde{n}_l \to \sigma_n^2$

signal component



intf.

noise

$W \cdot \sigma_n^2$ (A)

$I_k \in \{+1, -1\}$

$k = 1 \ldots N$

$\forall\ \sigma_k^2 = 1$

$$SINR_{(A)} = \frac{\text{sig. power}}{\text{intf. power} + \text{noise power}} = \frac{W^2 \cdot (\sqrt{P})^2 \cdot E[I_1^2(n)]}{(N-1) \times P \times \sigma_I^2 \times W + W \sigma_n^2}$$

$W^2 \cdot P \cdot \sigma_I^2 \to 1$

spreading factor

$$SINR = \frac{W \cdot P}{(N-1) \cdot P + \sigma_n^2};$$

$N \to$ users;

$\sigma_n^2 = (kT) \cdot \Delta f = \frac{1}{T_c}$

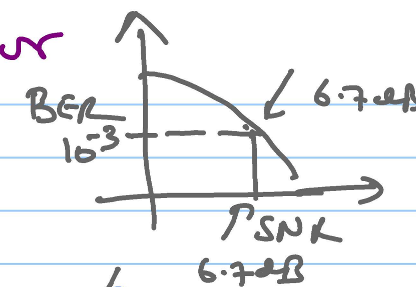
temp.

(*) Pole Capacity : $(N-1)P \gg \sigma_n^2$ → noise-floor

Infinite noise rise → N ✓

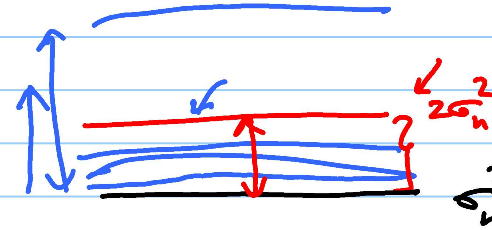
$W = 128$ ✓

SINR Target = 6 dB ⇒ 4



$$4 = \frac{128 \cdot P}{(N-1) \cdot P} \Rightarrow N = 32 + 1 = \boxed{33}$$

(*) Capacity for a finite noise rise



eg: Noise rise = 3 dB

$$4 = \frac{128 \cdot P}{\underbrace{(N-1) \cdot P + \sigma_n^2}_{\to\ 2\sigma_n^2}}$$

$$\boxed{(N-1)\,P = \sigma_n^2}$$

$$P = \sigma_n^2 / (N-1)$$

$$(N-1)P + \sigma_n^2 = 2 \cdot \sigma_n^2$$

$$4 = \frac{128 \cdot \cancel{\sigma_n^2}/(N-1)}{2\cancel{\sigma_n^2}} \; ; \; N-1 = \frac{128}{2 \cdot 4} = \frac{128}{8}$$

$$N = 16 + 1 = \boxed{17}$$

Revisit example: Noise rise of 6 dB → find N

(6 dB → 4)

$W = 128$;

$$SINR = \frac{W \cdot P}{(N-1)\cdot P + \sigma_n^2}$$

6 dB = 4

$$(N-1)P + \sigma_n^2 = 4\sigma_n^2$$

$$(N-1)P = 3\sigma_n^2$$

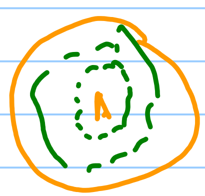
$$\sigma_n^2 = \frac{(N-1)P}{3}$$

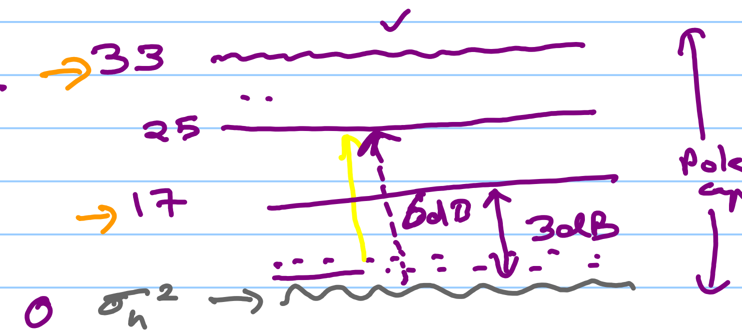
$$(N-1)P\left(1 + \frac{1}{3}\right)$$

$$\therefore \quad 4 = \frac{128}{(N-1)\cdot 4/3}$$

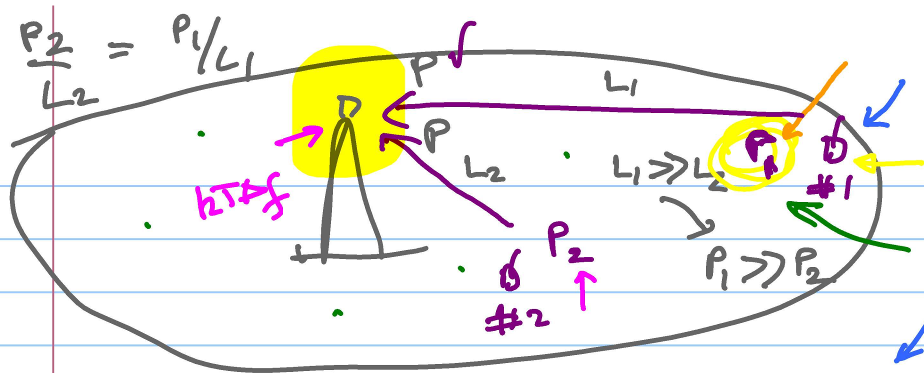
$$\Rightarrow \boxed{N = 25}$$

ROT





$\frac{P_2}{L_2} = P_1/L_1$



6dB $4 \approx 128$

$$SINR = \frac{N \cdot P}{(N-1)P + \sigma_n^2};$$

$t < T_1 \rightarrow N = 0;$

* Time T_1 → user #1 enters the Network

$N = 1$

$$4 = \frac{128 \cdot P}{0 + \sigma_n^2} \Rightarrow P = \frac{4\sigma_n^2}{128} = \boxed{\frac{\sigma_n^2}{32}}$$

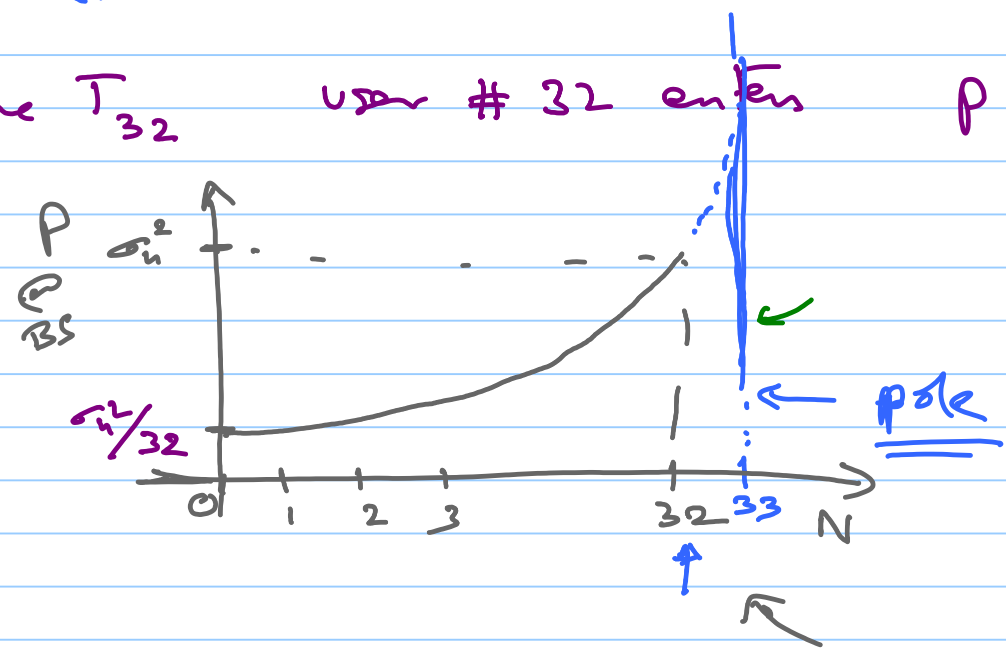
$P_1 = L_1 \cdot P$

* Time T_2 → user #2 enters

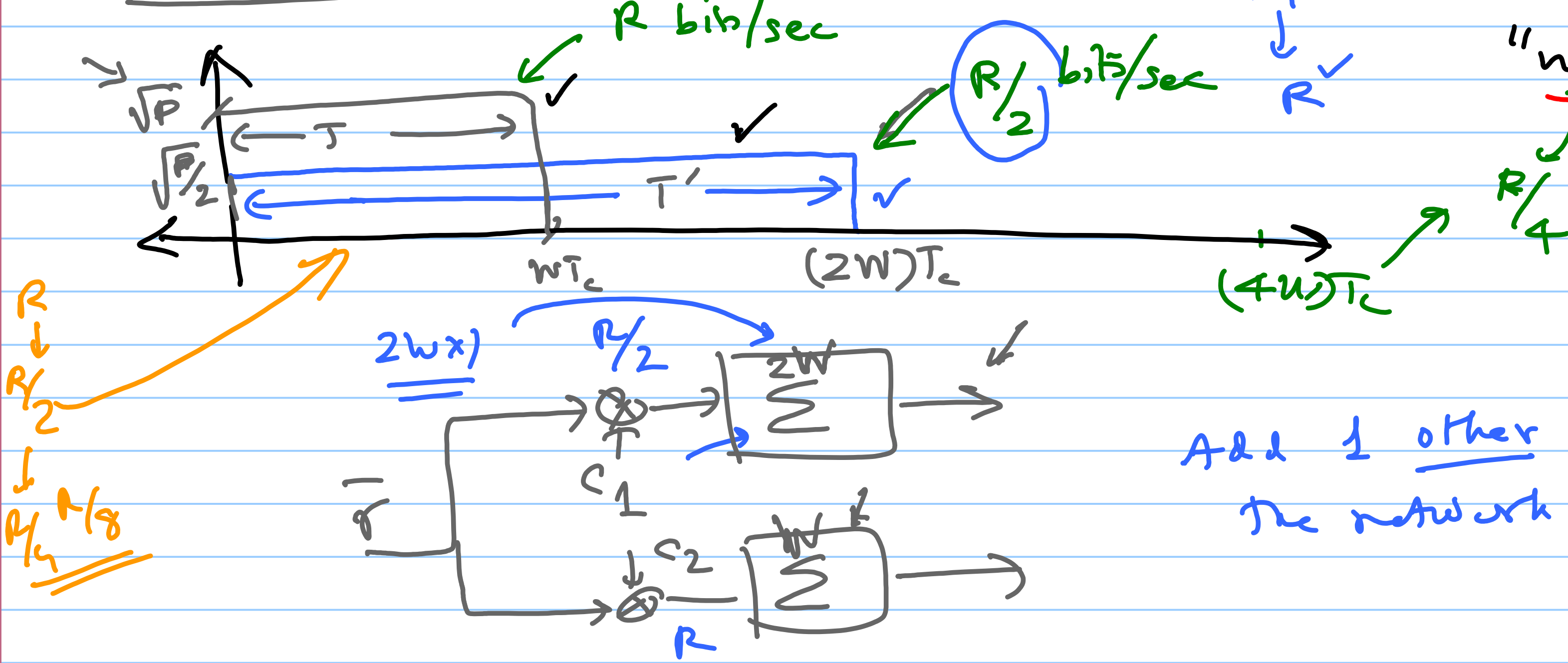
$$4 = \frac{128 \cdot P^2}{P + \sigma_n^2} \Rightarrow 4P + 4\sigma_n^2 = 128P \Rightarrow P = \frac{4\sigma_n^2}{124} = \boxed{\frac{\sigma_n^2}{31}}$$

* Time $T_{25} \rightarrow$ user # 25 enters $\quad P = \boxed{\dfrac{\sigma_n^2}{8}}$

* Time T_{32} user # 32 enters $\quad P = \sigma_n^2$



Rate Adaptation in DS-CDMA



$N_1 + N_2 \longleftrightarrow N \to R$

$N_2 \to R/2$ ✓

$N_1 \to R$ ✓

"narrowbanding"

Add 1 other user to the network at rate R/2

N_p

N_{p-1}

$N \rightarrow P$ watts $= NP$ — CDMA Power-Division !!

N_{p-1}, R

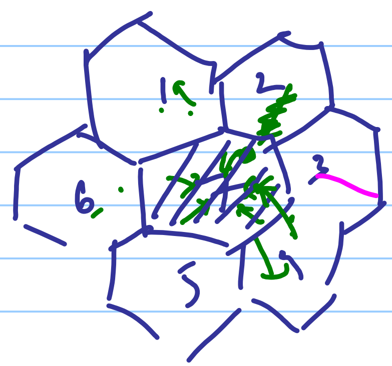
\# of users

2 users → $P/2$ → $P/2$, $P/2$: N + 1 → N + 2

4 users $P/2$

NP → P, P/2, P/4 ; P → R, P/2

$\frac{3P}{4}$: $2P/4 \rightarrow P/2$; $P/4 \rightarrow 3/4$



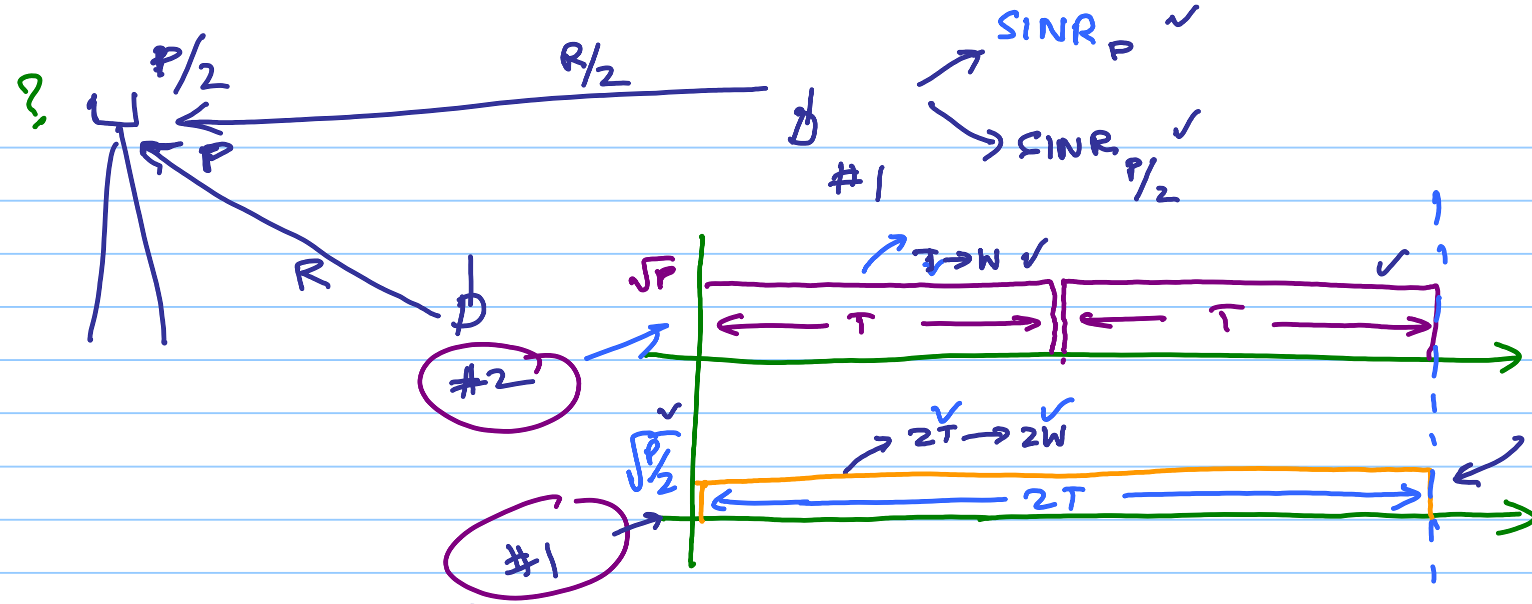
$$SINR = \frac{N \cdot P \times 2.5}{1.6 \times \left(\underbrace{(N-1)P}_{\text{in cell}} \right) + \sigma_n^2}$$

out-of cell intf. ← 1.6×

VAD

40%

4/10



$$SINR_{P\,(\#2)} = \frac{(W\sqrt{P})^2 = W^2 P}{\underbrace{1 \cdot W \cdot \left(\frac{P}{2}\right)}_{I} + \underbrace{W \cdot \sigma_n^2}_{N}} = \frac{WP}{\frac{P}{2} + \sigma_n^2} \; ; \rightarrow (1) \checkmark$$

$$SINR_{P/2} \ (\#1) = \frac{\left(2W \cdot \sqrt{P/2}\right)^2}{2W \cdot P + 2W\sigma_n^2} = \frac{2W \cdot \left(\frac{P}{2}\right) PW}{P + \sigma_n^2} ;$$

2

$$SINR_2 = \frac{PW}{P/2 + \sigma_n^2} ; \quad SINR_1 = \frac{PW}{P + \sigma_n^2} ;$$

N_p

$N \rightarrow$ even

Case 1:

$N \rightarrow N/2 \leftarrow P \rightarrow SINR_P$

$N \rightarrow N/2 \leftarrow P/2 \rightarrow SINR_{P/2}$

$\frac{N}{2} \cdot R + \frac{N}{2} \cdot \frac{R}{2} = \frac{3}{4} \cdot NR$

$\frac{N}{2} \cdot P + \frac{N}{2} \cdot \frac{P}{2} = \frac{3}{4}(NP)$

$P \rightarrow N = N_p - 1$

$= N \cdot P$

NR

-X- Case 2 : Add $N/2$ more users of Power $P/2$ to Case 1 and re-solve

$P > P/2$ — NP ← $SINR_P$ ✓, $SINR_{P/2}$ ✓

Total # of users: $(N/2 + N) \rightarrow 3N/2$

$\frac{N}{2} \cdot R + \frac{N \cdot R}{2} = NR$

$\frac{N}{2} - 1 \rightarrow P$

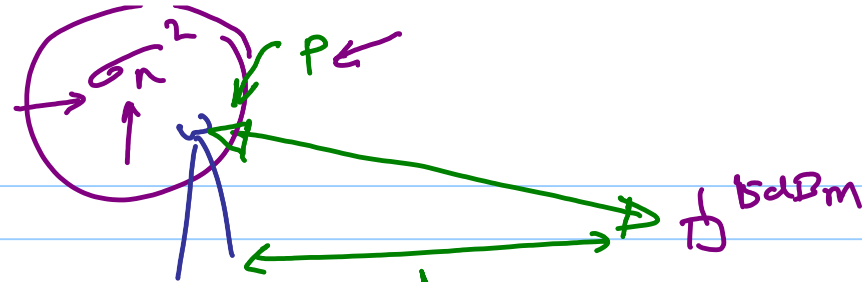
$\frac{N}{2} + \frac{N}{2} \rightarrow P/2$

$N/2 \rightarrow P$; $N-1 \rightarrow P/2$

"Code Division" ⟶ "Power Division"

"RAKE-finger management"

AC :



$P \rightarrow f(\sigma_n^2, W, N)$

SINR

$\frac{\sigma_n^2}{(\cdot)}$

Rx sensitivi. σ_n^2

$P_R = 15\,dBm + G_T + G_R - 10\log_{10}\left(\frac{4\pi}{\lambda}\right)^2 - 10\log_{10} d^n$; $n=2$

$SINR = \frac{W P_R}{\sigma_n^2 + (M-1)P}$

→ Schmidl-Cox

Norway NMT, AMPS — 1G

TDMA

→ IS-95 | IS-136

GSM + GPRS

cdma2000

2.5G

Multipath Propagation ⟷ DS-CDMA Tech.

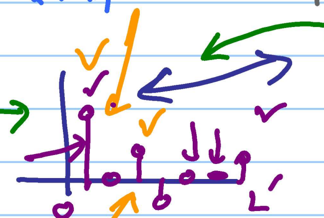
EE5141 Jan-Apr, 2022

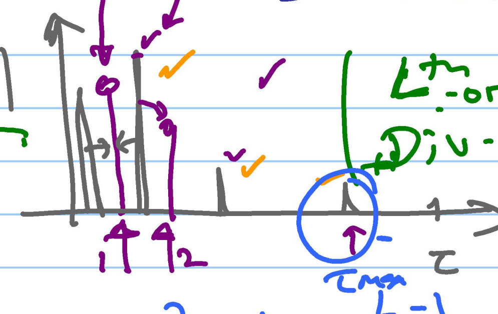
Lesson #7, Class 7 -- Mar. 29, 2022

TDMA

DS-CDMA

"Diffuse" multipath channel



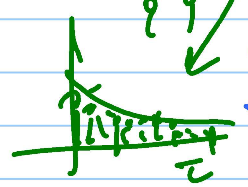


L^{th}-order Div.

WSS-US

Discrete multipath

$$h(t;\tau) = \sum_{i=0} \alpha_i(t)\,\delta(\tau - \tau_i(t))$$



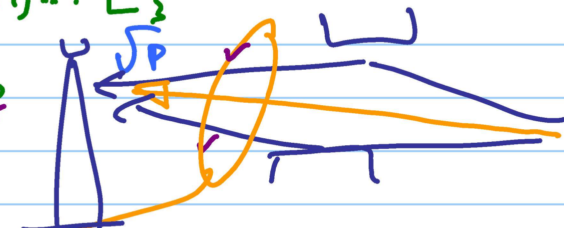
? RAKE Rx ?

$h_0 \dots h_{L'}$

$h_\ell \in \{0, 1, \dots L'\}$

T_c

multi-tap



$c_1(t)$

$s(t)$ $\sqrt{P_1}$

#1

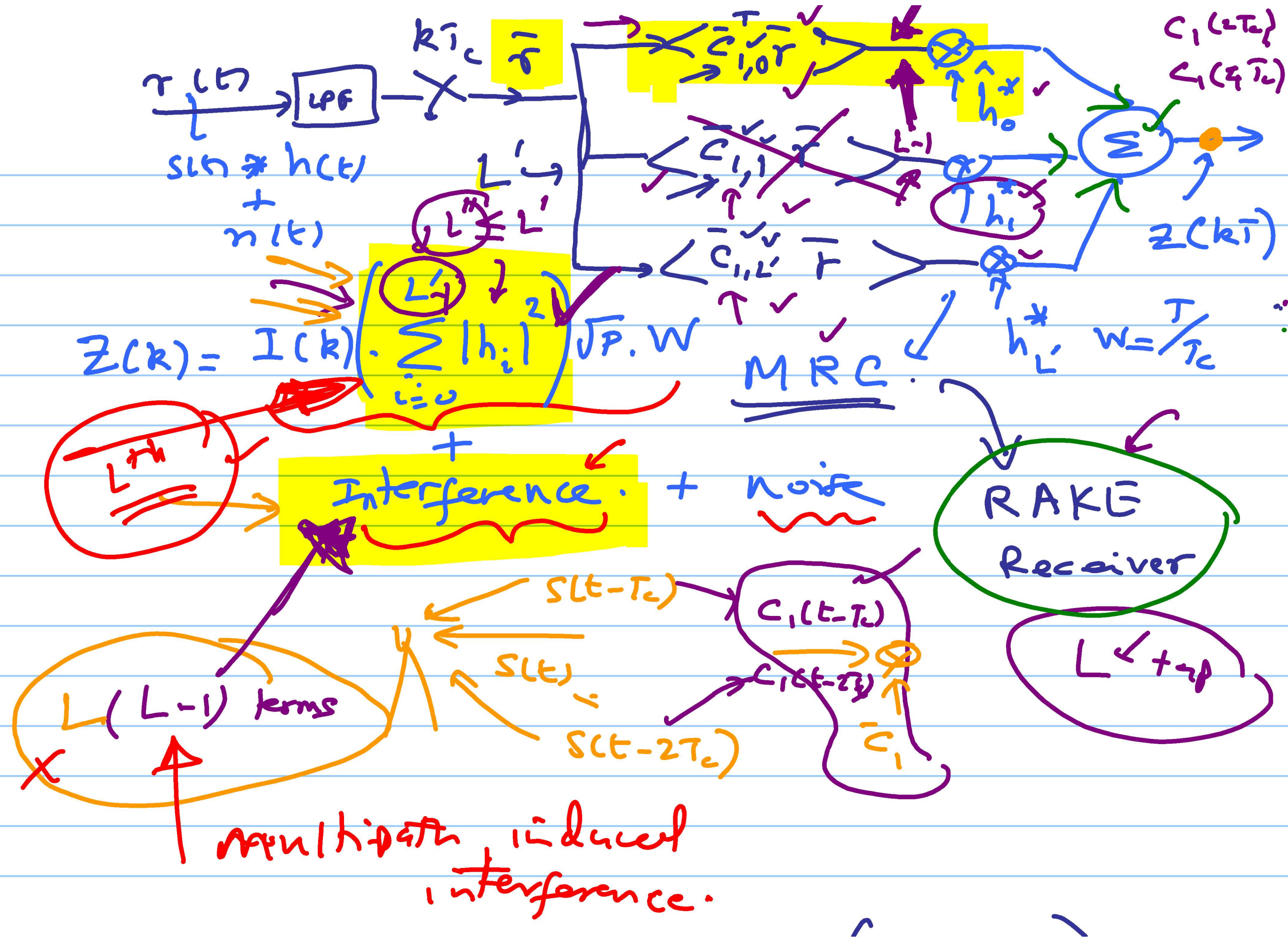
$r(t) * h(t;\tau)$

multi-path induced interference

$c_1(0)$

$c_1(T_c)$

RAKE → GRAKE



r(t)
LPF
kT_c
s(t) * h(t) + n(t)
L'
L* ≤ L'
MRC
RAKE Receiver
L' tap
w = T/T_c
z(kT)
Z(k) = I(k) · Σ_{i=0}^{L'} |h_i|^2 √P · W + Interference + noise
L(L−1) terms
multipath induced interference
s(t−T_c)
s(t)
s(t−2T_c)
c_1(t−T_c)

$$\text{SINR} = \frac{W \cdot P \cdot \left(\sum_{i=0}^{L-1} |h_i|^2 \right)}{L \cdot (L-1) \cdot P + \sigma_n^2}$$ ✓

→ Chip level Equalised DS-CDMA ✗

→ Block Modulated DS-CDMA ←