

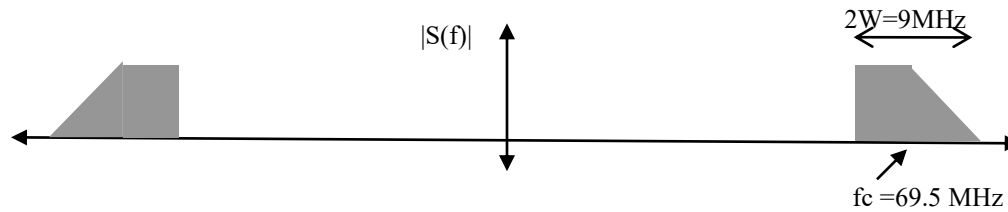
# EE 4140: Digital Communication Systems

Aug. 2025

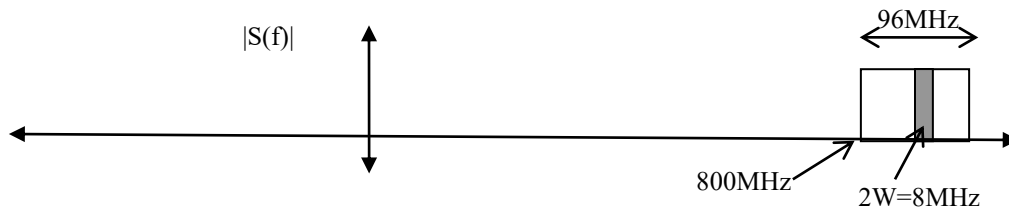
## Tutorial #1

KG / IITM

1. A low-pass signal of one-sided bandwidth of  $W=1.25\text{MHz}$  is sent as a DSB-SC signal. If the receiver uses an IF sampling scheme, with center frequency  $f_{\text{IF}} = 71\text{MHz}$ , determine the least sampling rate  $f_s$  required.
2. For the QCM signal with magnitude response as below, find the least possible band-pass sampling rate  $f_s$ . Make a rough plot of the frequency response of the sampled sequence around 0Hz. *Hint: Use both the band-edges (i.e.,  $f_c + W$  and  $f_c - W$  in order to decide the “lowest” sampling rate).*

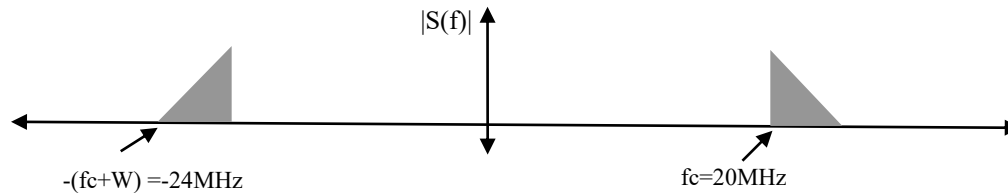


3. In the above problem, let us discuss two possible sources of error:
  - (a) Assume that the received signal has a phase offset of  $\theta$  radians; in other words,  $s(t) = m_1(t)\cos(2\pi f_c t + \theta) + m_2(t)\sin(2\pi f_c t + \theta)$ . Now, what will be the time-domain representation of the sampled sequence? For the special case when  $\theta = \pi/2$ , what will be the samples of the received signal?
  - (b) Instead, assume  $\theta = 0$  radians, but the incoming carrier frequency  $f_c'$  is offset by  $\Delta f$  Hz wrt to the sampling rate designed assuming  $f_c$ ; i.e.,  $f_c' = f_c + \Delta f$  and use this  $f_c'$  in place of  $f_c$  in the expression for  $s(t)$  above. What is then the expression for the time-domain samples after the band-pass sampling ADC in terms of your sampling rate  $f_s$  and the  $\Delta f$ ? Say, for  $\Delta f = 1\text{MHz}$ , will some of I-Q samples at least be undistorted? Justify your answer.
4. A QCM signal  $s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$  has the two message signals  $m_1(t)$  and  $m_2(t)$  of one-sided bandwidth of  $W_1=3\text{KHz}$  and  $W_2=4\text{KHz}$ , respectively, and take  $f_c=31\text{KHz}$ .
  - (a) Find the minimum band-pass sampling rate  $f_s=1/T_s$  that gives un-aliased samples of the two signals.
  - (b) Assuming that the spectrum of  $m_1(t)$  has a “triangular” shape between  $-3\text{KHz}$  to  $+3\text{KHz}$ , make a labeled, rough sketch of the spectrum of the samples  $m_1(kT_s)$  between  $-40\text{KHz}$  and  $+40\text{KHz}$ .
5. A dozen DSB-SC signals of one-sided (low-pass) bandwidth  $W = 4\text{MHz}$  (including a “guard-band of  $0.5\text{MHz}$ ”) are present between  $800\text{MHz}$  and  $896\text{MHz}$ , as shown below. Describe the operations (sampling, rate-conversion, filtering) that you need to do to recover Nyquist rate samples of the 7<sup>th</sup> DSB-SC signal (i.e., the signal present between  $848\text{MHz}$  and  $856\text{MHz}$ ).



6. In the question above, if the dozen signals were QCM, how does your answer change?

7. A real low-pass signal  $s(t)$  of one-sided bandwidth  $W=4\text{MHz}$  is sent as an upper-sideband only single-side band (SSB-SC) signal with magnitude spectrum as shown below. The receiver uses bandpass sampling.

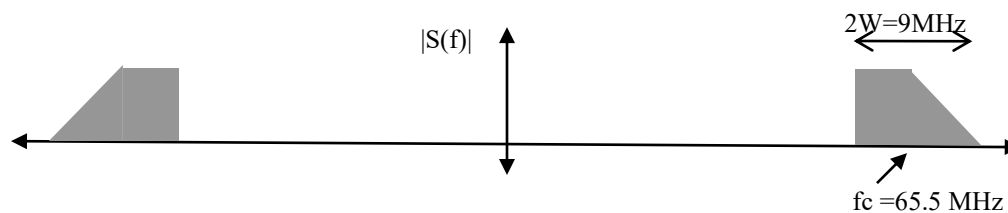


- Determine the least sampling rate  $f_s = 1/T_s$  required in MHz.
- Make a neat plot of the magnitude spectrum  $|S'(f)|$  of the sampled sequence  $s(kT_s)$  between 24MHz and -24MHz.

8. A real bandpass QCM signal  $s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$  with carrier frequency  $f_c=200\text{KHz}$  has the two message signals  $m_1(t)$  and  $m_2(t)$  of one-sided bandwidth of  $W=5\text{KHz}$  each. Let  $S(f)$ ,  $M_1(f)$ , and  $M_2(f)$  denote the (complex) Fourier transforms of  $s(t)$ ,  $m_1(t)$  and  $m_2(t)$ , respectively. Given  $M_1(f=2\text{KHz}) = a + jb$  and  $M_2(f=2\text{KHz}) = c + jd$  where  $j = \sqrt{-1}$ , determine the following in terms of  $a$ ,  $b$ ,  $c$ , and  $d$ . The number inside the brackets can be assumed to be in KHz.

- $M_1(-2)$  and  $M_2(-2)$
- $S(202)$  and  $S(198)$
- $S(-198)$

9. Consider a different QCM signal with magnitude response as below. *Note:* From the information given below, note that both the low-pass message signals have one-sided bandwidth of 4.5MHz each.



- Find the least possible band-pass sampling rate  $f_s$  for this choice of  $f_c$ .
- If you were allowed to modify (or design) the carrier frequency between 65MHz and 70MHz given these message signals, what would you choose as  $f_c$  to minimize the band-pass sampling rate? Justify your answer.