8th Nov 2025

① → Linear Equaliser (L-MMSE) ← FIR

① → Decision Feedback Equaliser || IIR ← non-linear

② → Fractionally Spaced Equalisation

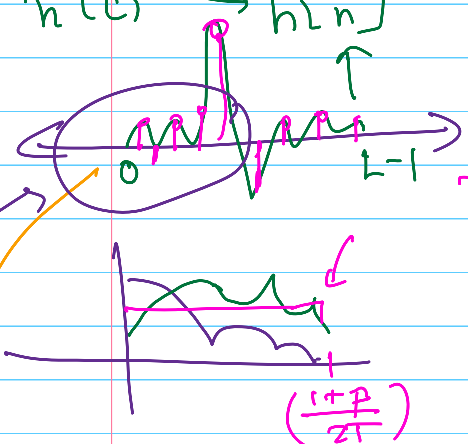
③ → Intro. to Block Modulation

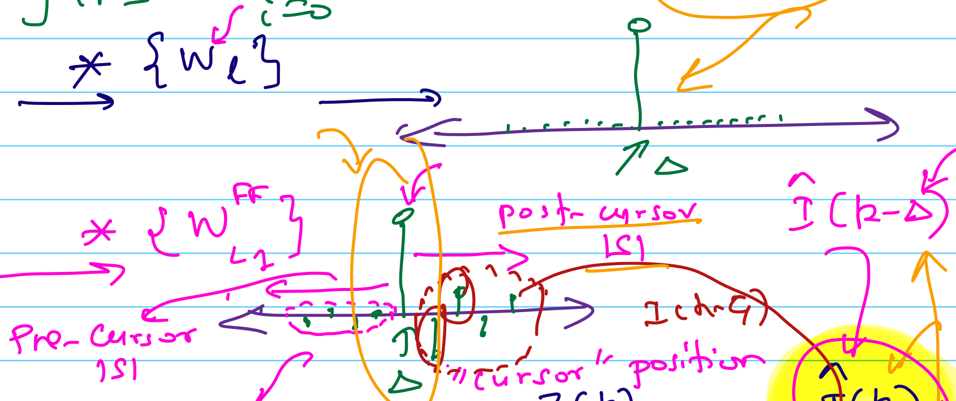
① DFE ← symbol-spaced DFE

h(t) → h[n]

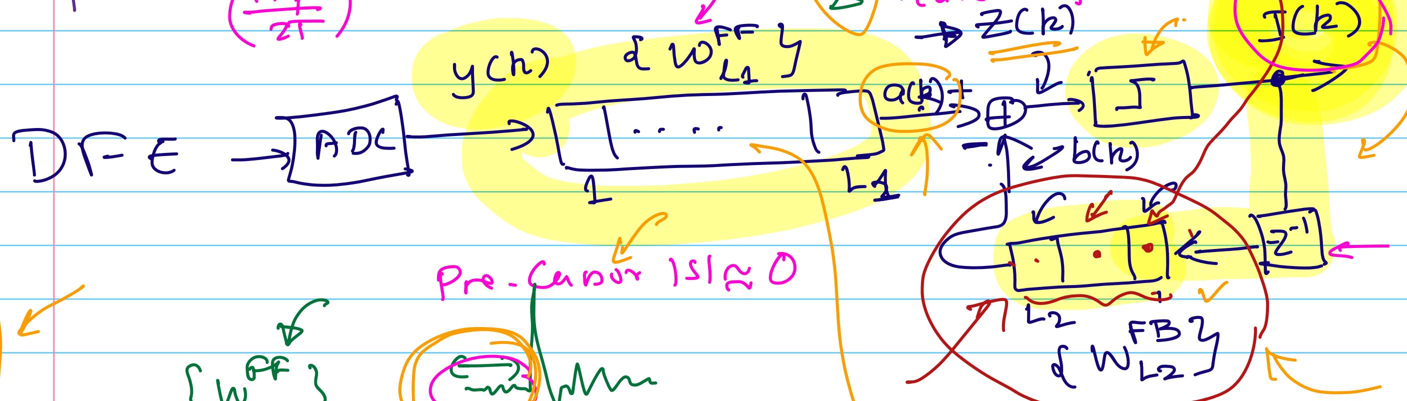
$$y[k] = \sum_{i=0}^{L-1} h_i\, I(k-i) + n(k)$$

$* \{w_l\}$



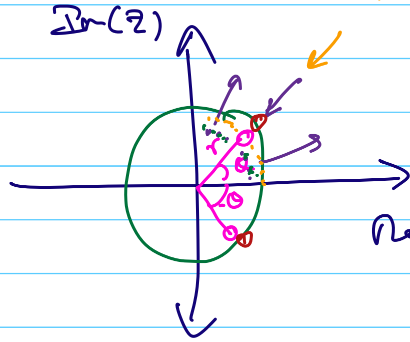


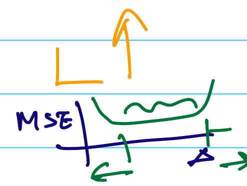
DFE → ADC



Pre-cursor ISI ≈ 0

$L \rightarrow \Delta \approx L/2$





$L_1 \rightarrow FF$

$L_2 \rightarrow FB$

Δ → ?

$\rightarrow \Delta \approx \frac{L_1}{2}$

DFE → Wiener-Hopf filter

$$LE \rightarrow \bar{W}_{LE} = R^{-1}\bar{p}, \quad R = E[\bar{y}\bar{y}^H]$$

$$\bar{y} = \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-L+1) \end{bmatrix} \Big\} L$$

$$\underbrace{\bar{W}}_{L_1+L_2} = R^{-1}\bar{P}$$

$\bar{P}$: $(L_1+L_2)\times 1$

R: $(L_1+L_2)\times(L_1+L_2)$

DFE

$$\bar{W} = \begin{bmatrix} W_1^{FF} \\ \vdots \\ W_{L_1}^{FF} \\ W_1^{FB} \\ \vdots \\ W_{L_2}^{FB} \end{bmatrix} \begin{matrix} \} L_1 \\ \\ \} L_2 \end{matrix} \quad (L_1+L_2)\times 1$$

$$\bar{x}(k) = \begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-L_1+1) \\ \hat{I}(k-1) \\ \hat{I}(k-2) \\ \vdots \\ \hat{I}(k-L_2) \end{bmatrix} \begin{matrix} \} L_1 \\ \\ \} L_2 \end{matrix}$$

$\hat{I}(k-\Delta-1)$

$\hat{I}(k-\Delta-L_2)$

$$R = E[\bar{x}(k)\,\bar{x}^H(k)]$$

$$\bar{p} = E[I^*(k)\,\bar{x}(k)]$$

$I^*(k-\Delta)$

L-tap channel $\rightarrow \{f_l\} \rightarrow$ VA

$\rightarrow$ LE $\rightarrow$ $\cancel{M}$ N-taps. ✓

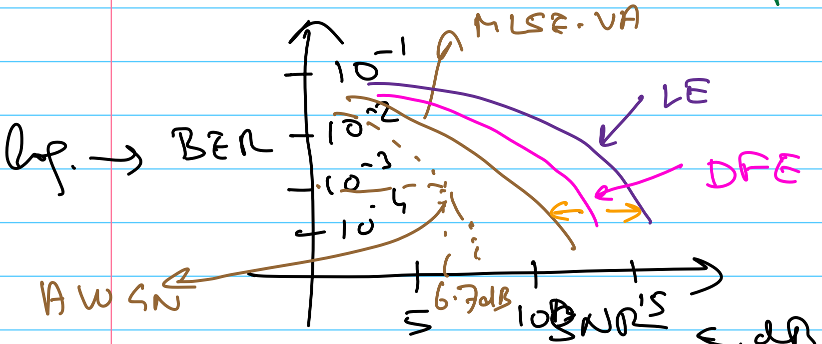
$\rightarrow$ DFE $\rightarrow$ $\cancel{M}$ N-taps

$M = L_1 + L_2$

M^3

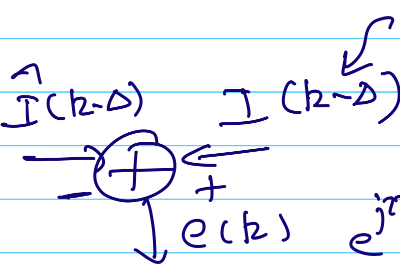
R

$M^2 \log(M)$



log. $\rightarrow$ BER

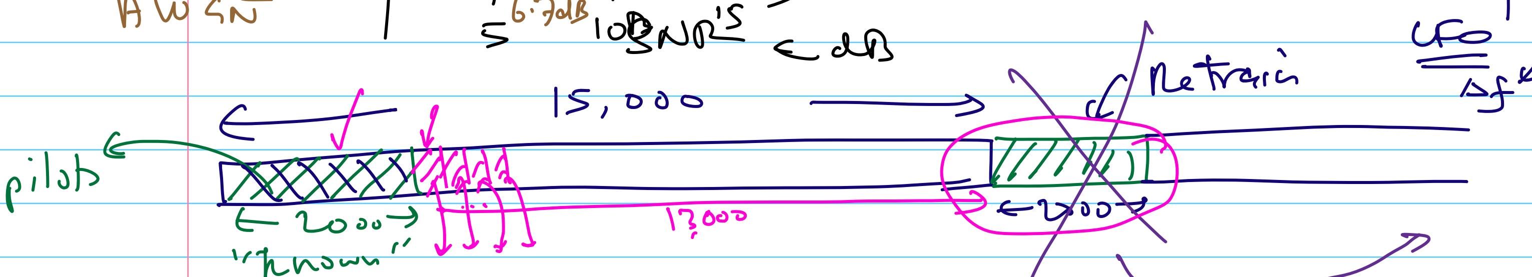
$\leftarrow$ dB



$e^{j2\pi\Delta f k}$

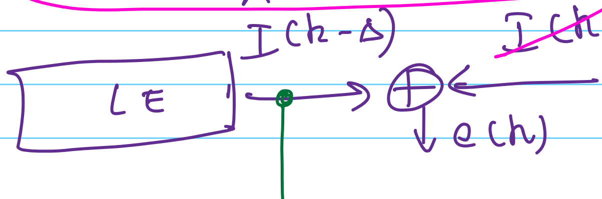
CFO

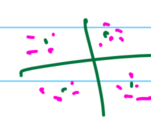
Δf



Decision-Aided Tracking $\rightarrow$ Error-Propagation

$I(k-\Delta)$





$\boxed{I} \longrightarrow \tilde{I}(k-\Delta)$

1+j −1−j
1−j −1+j

ergodic theory ||

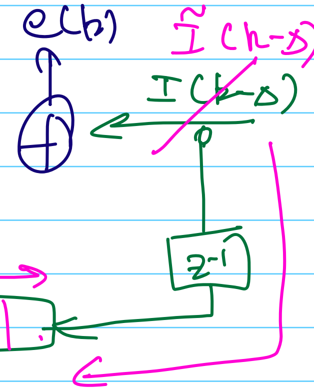
$$\bar{P} = E\big[\underbrace{I^*(k-\Delta)}_{\tilde{I}^*(k-\Delta)}\,\bar{y}(k)\big]$$

$\tilde{\bar{p}}$ $\sum_i$

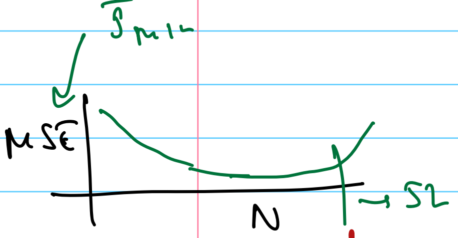
"Robust statistics"

"Boundary"

→ DFE ← ?



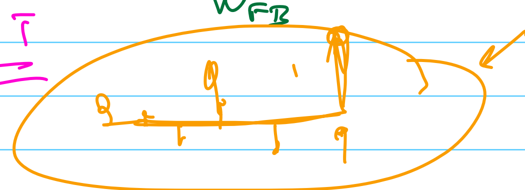
$\bar{J}_{min}$



$L_2 = ?$

$L_2 \uparrow_j \to L_2 T$

W_{FB}



M^{L-1}

order of LE

$\{f_l\}$ $l = 0 \dots L-1$ ← Memory of $(L-1)$

← DFE → $L_2 \approx L-1$

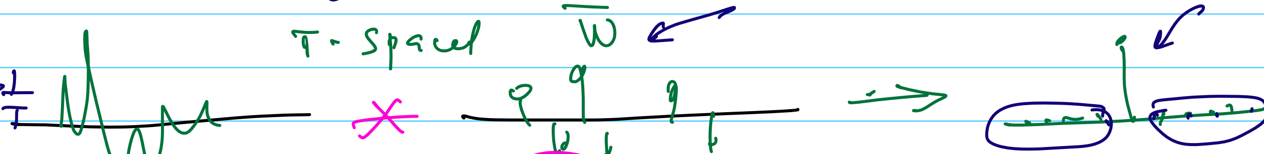
$\cancel{M} \approx 3L$ to $4L$
N

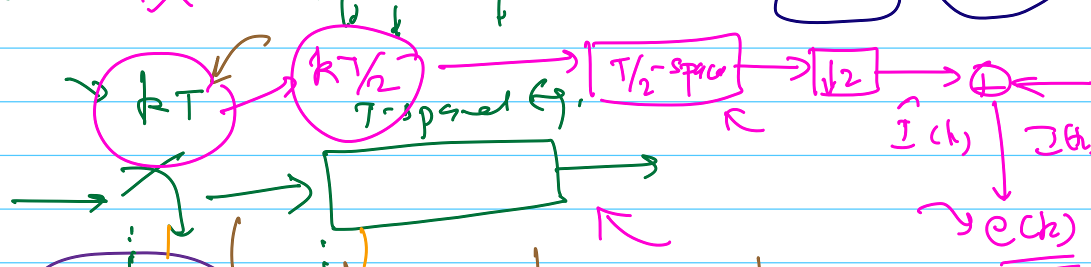
$\cancel{M} = L_1 + L_2$
N
↳ 2L to 3L

Fractionally Spaced Equalisation.

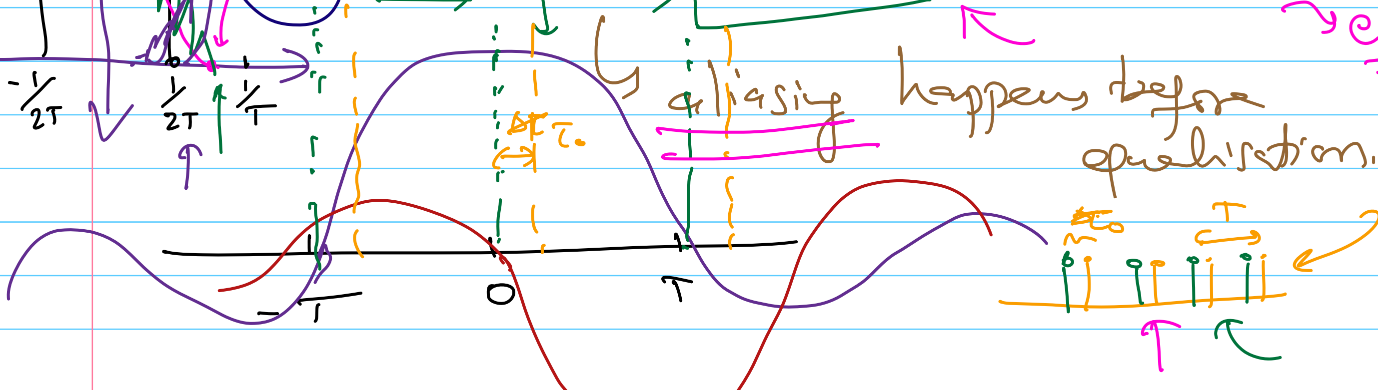
T-spaced $\bar{W}$

$X(f) = 0, \; |f| > \frac{1}{T}$





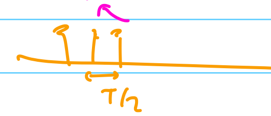
aliasing happens before equalisation.



$$Y_T(f) = \frac{1}{T}\sum_n X\left(f-\frac{n}{T}\right) \quad \text{"flat"}$$

$$\rightarrow Y_T(f) = \frac{1}{T}\sum_n X\left(f-\frac{n}{T}\right) e^{j2\pi\left(f-\frac{n}{T}\right)\tau_0}$$

$C_T(f)$ $\quad$ $2\pi f\tau_0$



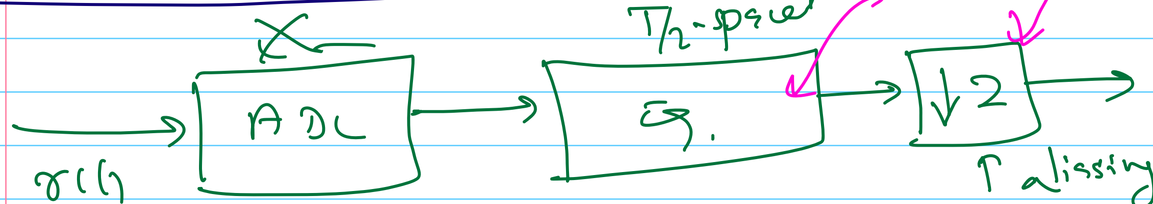
$$G_C^W \longrightarrow C_T(f) = \sum c_k e^{-j2\pi f kT}$$

Equalisation after aliasing

$$C_T(f)\cdot Y_T(f)$$

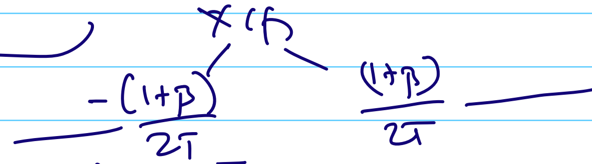
Fractionally - Spaced Equaliser (FSE)

FS-LE
FS-DFE



Equalisation before Aliasing

$$C_{T/2}(f)\, Y_{T/2}(f) = C_{T/2}(f)\underbrace{\sum_n X\left(f-\frac{n}{T/2}\right) e^{j2\pi\left(f-\frac{n}{T/2}\right)\tau_0}}$$



$$= C_{T/2}(f)\cdot X(f)\, e^{j2\pi f\tau_0} \qquad |f| < \frac{1}{T}$$

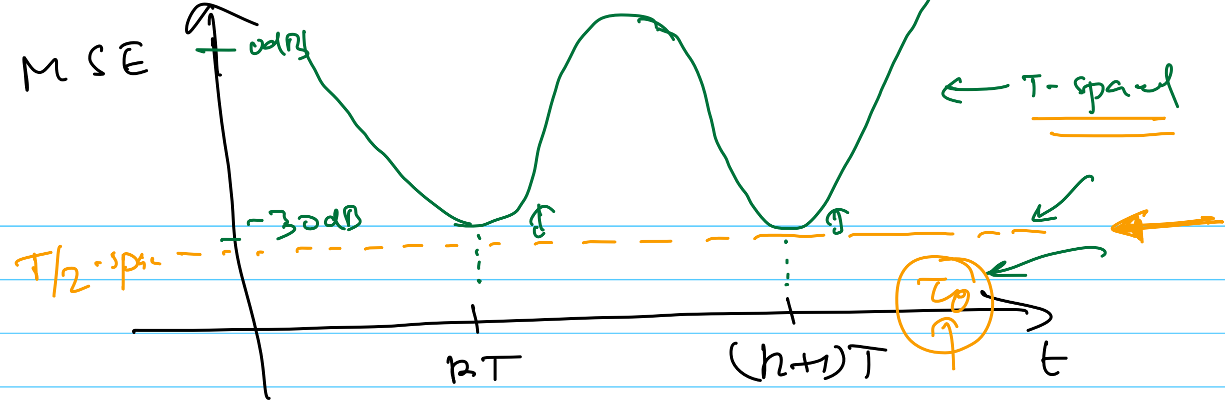
Nyquist.

Downsampling

$$\sum_n \underbrace{C_{T/2}\left(f-\frac{n}{T}\right)}\, X\left(f-\frac{n}{T}\right) e^{j2\pi\left(f-\frac{n}{T}\right)\tau_0}$$

within the summation

$$X_{RC}\left(f-\frac{n}{T}\right)$$



Symbol-by-Symbol Modulation

conventional single carrier

2G cellular TDMA

3G cellular DS-CDMA

c_1

c_N

4G → Block Modulation

IEEE → WiMAX → Single Carrier, Multi Carrier → OFDM

3G-PP → LTE → Multicarrier OFDM

PAPR

$N_{cp} > L$

6G

N/16 → N/32

Rx → ISI ← Delay Spread

MLSE → DFE → LE

sub-optimal VA

L

N + N_{cp}

N_{cp} N-QAM

N_{cp}

TX → Rx → FFT IFFT

$N_{cp} > L$

FDE → optimal

$\{f_0 \cdots f_{L-1}\}$