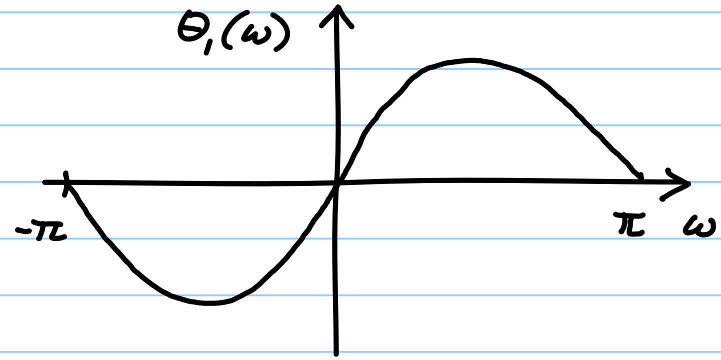
Note Title

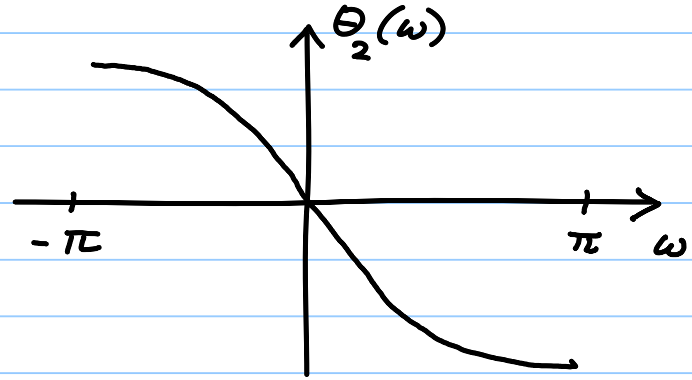
EE 5330 Sep. 26, 2013

26-09-2013

$$H_1(z) = 1 - \frac{1}{2} z^{-1}$$



$$H_2(z) = -\frac{1}{2} + z^{-1}$$



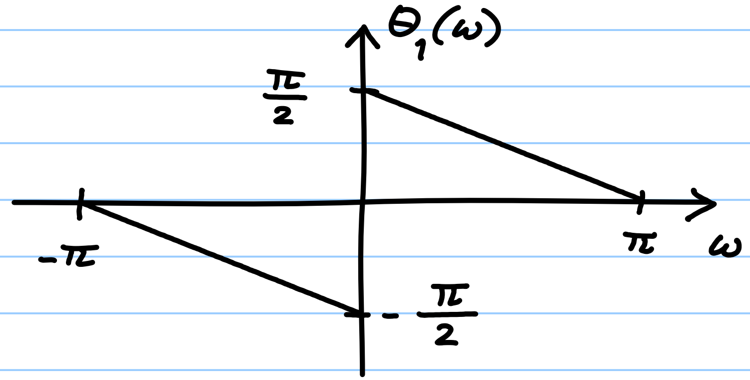
From our previous result,

$$|H_1(e^{j\omega})| = |H_2(e^{j\omega})|$$

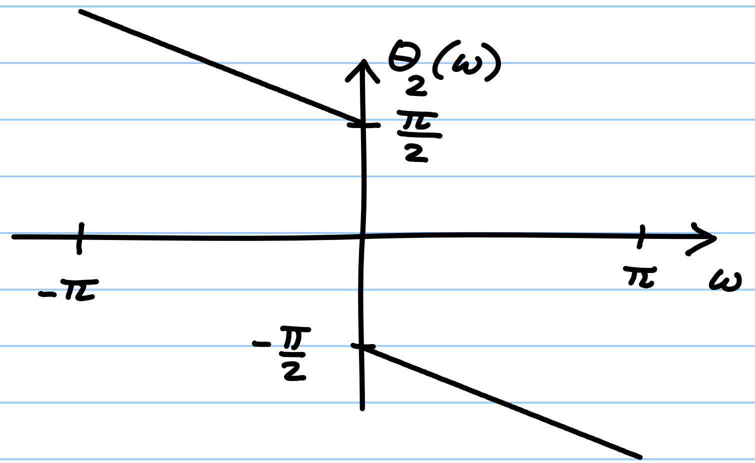
but $\theta_1(\omega)$ and $\theta_2(\omega)$ are very different.

Another example:

$$G_1(z) = 1 - z^{-1}$$



$$G_2(z) = -1 + z^{-1}$$



Since $-1+z^{-1} = -(1-z^{-1})$, the multiplication by -1 results in a shift by π in the phase

Since $1-az^{-1}$ and $-a^*+z^{-1}$ have identical magnitude response,

$$H(z) = \frac{-a^* + z^{-1}}{1-az^{-1}}$$

has unit magnitude response. This can also be seen from

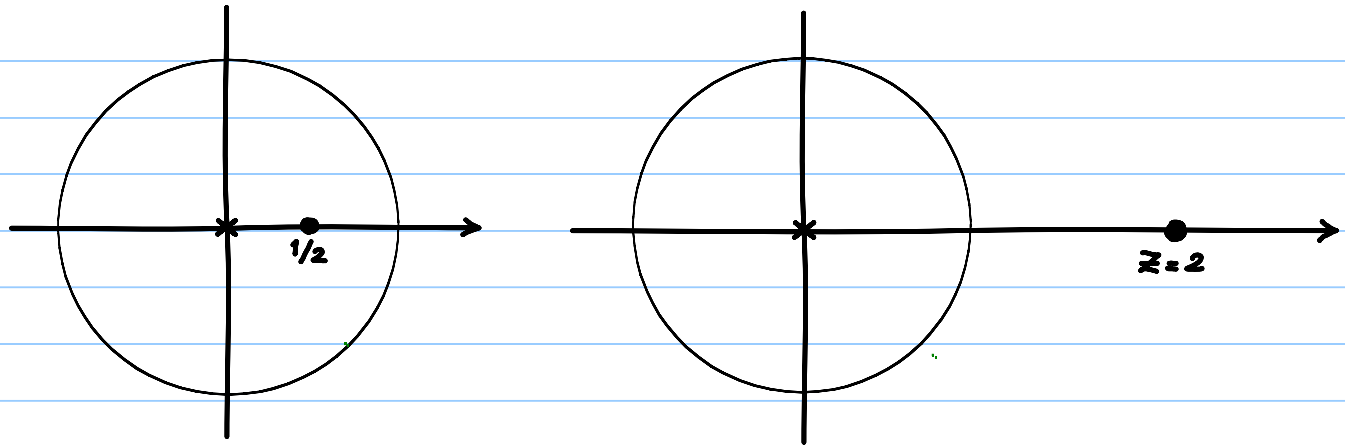
$$H(e^{j\omega}) = \frac{-a^* + e^{-j\omega}}{1-ae^{-j\omega}} = \frac{e^{-j\omega} - a^*}{e^{-j\omega}(e^{j\omega}-a)} \Rightarrow \left| \frac{1}{e^{-j\omega}} \frac{(e^{j\omega}-a)^*}{(e^{j\omega}-a)} \right| = 1$$

A filter with unit or constant magnitude response is called as an $\boxed{\text{ALLPASS Filter}}$.

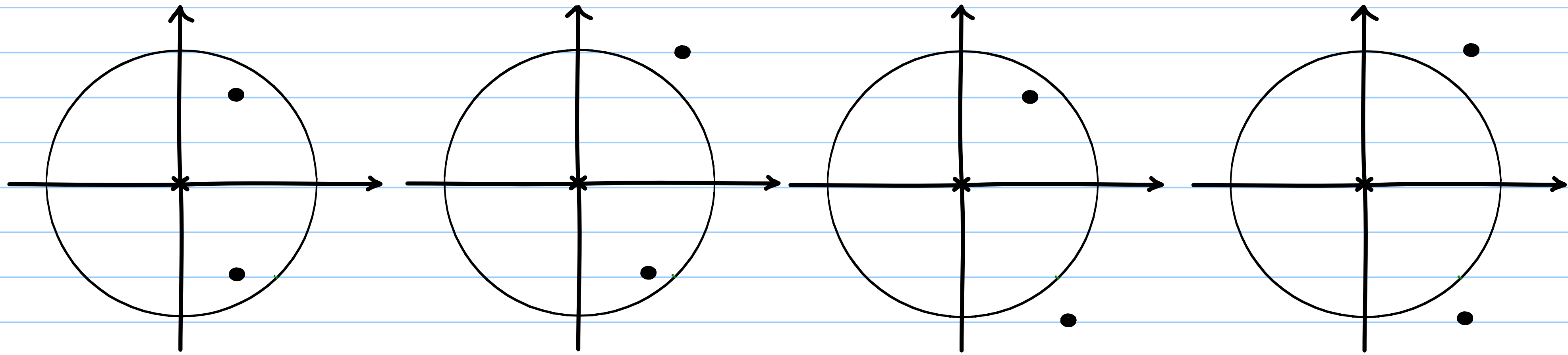
Note that both $H_1(z)$ and $H_2(z)$ are causal filters. Nevertheless, knowing the magnitude response does not help us in determining the phase response. However, if the filter transfer function is rational, whether or not the system is causal, for a given magnitude response, the number of choices for the phase response is fixed, provided the filter order is specified.

In the case of $1-\frac{1}{2}z^{-1}$, the only other system with identical magnitude response is $-\frac{1}{2}+z^{-1}$.

For $H(z) = 1 - \frac{1}{2} z^{-1}$ there are **TWO** systems with identical mag. response



For $H(z) = (1 - re^{j\theta} z^{-1})(1 - re^{-j\theta} z^{-1})$, there are **FOUR** such possibilities:



For a system with N poles and M zeros, can you guess how many possibilities exist?

In some practical cases, we are given $|H(e^{j\omega})|^2$ and required to find $H(z)$.

Recall that $H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}}$ and $|H(e^{j\omega})|^2 = H(z)H^*(1/z^*)\Big|_{z=e^{j\omega}}$

A general expression for $H(z)$ is $b_0 \dfrac{\prod_{\ell=1}^{M}(1-c_\ell z^{-1})}{\prod_{k=1}^{N}(1-d_k z^{-1})}$

Assuming $b_0 \in \mathbb{R}$,

$$H^*(1/z^*) = b_0 \frac{\prod_{\ell=1}^{M} (1 - c_\ell^* z)}{\prod_{k=1}^{N} (1 - d_k^* z)}$$

Hence,

$$C(z) = H(z)\, H^*(1/z^*)$$

$$= b_0^2 \frac{\prod_{\ell=1}^{M} (1 - c_\ell z^{-1})(1 - c_\ell^* z)}{\prod_{k=1}^{N} (1 - d_k z^{-1})(1 - d_k^* z)}$$

If c_k is a zero of $H(z)$, c_k is also a zero of $C(z)$. In addition $1/c_k^*$ is also a zero of $C(z)$. Similarly, d_k and $1/d_k^*$ are the poles of $C(z)$.

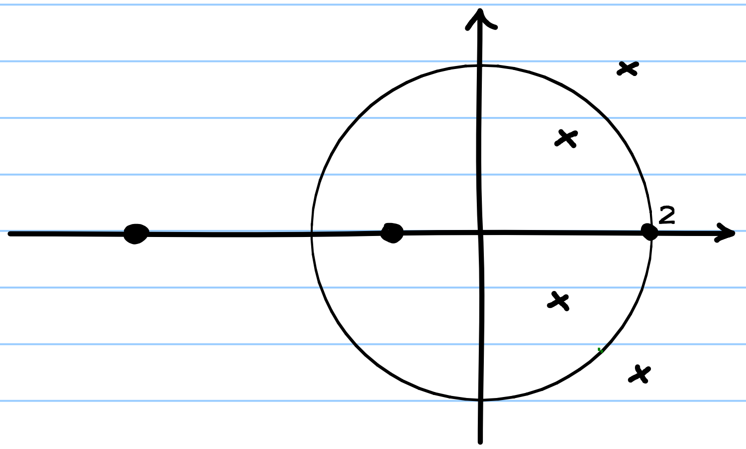
Example

$$H_1(z) = \frac{(1-z^{-1})(1+2z^{-1})}{(1-0.8e^{j\pi/4}z^{-1})(1-0.8e^{-j\pi/4}z^{-1})}$$

$$H_2(z) = \frac{(1-z^{-1})(2+z^{-1})}{(1-0.8e^{j\pi/4}z^{-1})(1-0.8e^{-j\pi/4}z^{-1})}$$

Verify that $H_1(z)H_1^*(1/z^*) = H_2(z)H_2^*(1/z^*) = C(z)$

The pole-zero plot of $C(z)$ is given below:



Problem: We cannot go from $C(z)$ to $H(z)$ in a unique manner. If we assume causal & stable systems, then the poles of $H(z)$ have to be inside the unit circle. But the zeros can be anywhere!

Exercise How many different $H(z)$'s give rise to the given $C(z)$?