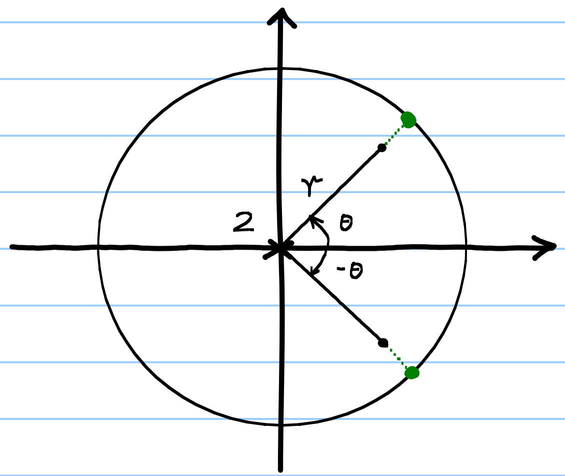
Note Title

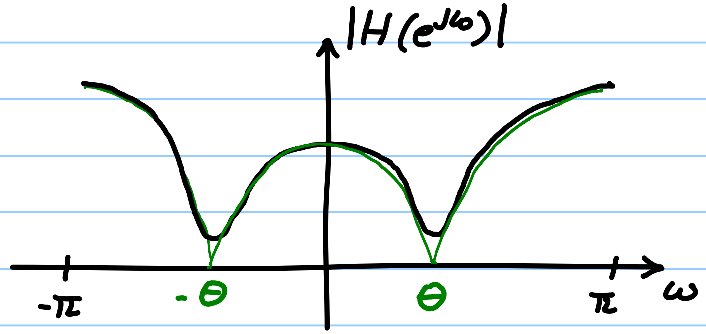
EE 5330 Sep. 19, 2013

19-09-2013

Notch Filter Used for removing one or more sinusoids.

Consider $H(z) = 1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}$

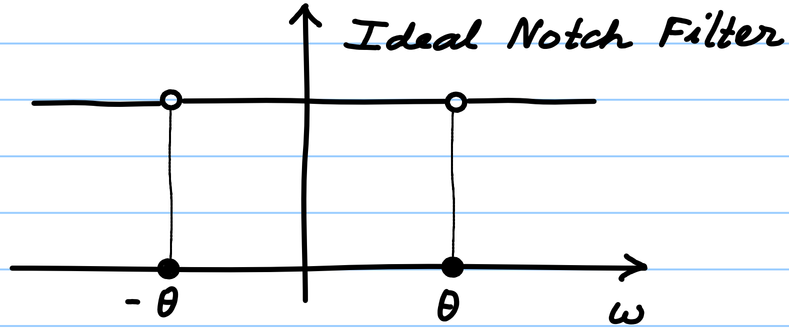




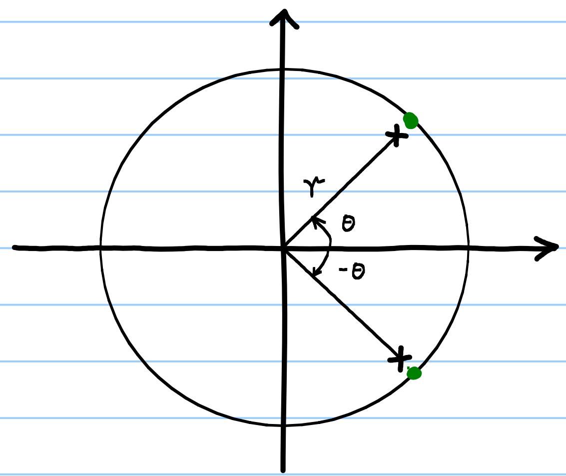
If $r = 1$, the zeros lie on the unit circle and the frequency response goes to zero at $\omega = \pm\theta$ [Green curve in the above fig.]

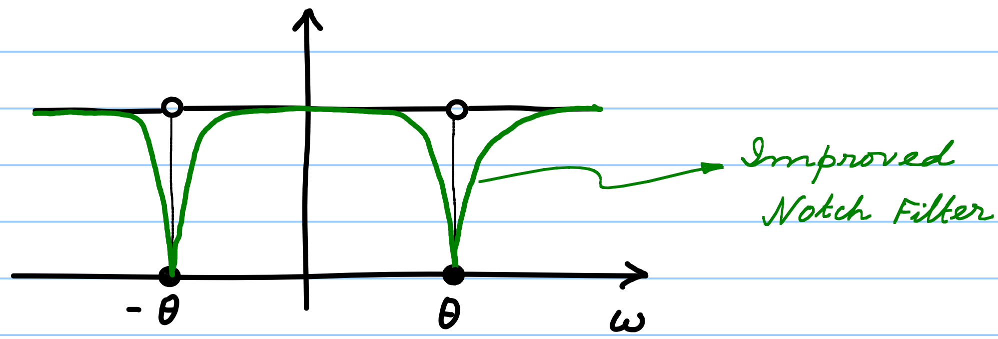
$$H(z) = 1 - 2\cos\theta\, z^{-1} + z^{-2}$$

While the frequency component at $\omega = \pm\theta$ is nulled, the notch filter's response is far from the ideal response shown below:



The given notch filter's response can be improved by adding poles at $re^{\pm j\theta}$ where r is close to 1.





Improved notch filter: $H(z) = \dfrac{1 - 2\cos\theta\, z^{-1} + z^{-2}}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}}$

In practice r cannot be made too close to 1 because of limitations imposed by finite precision effects.

Comb Filters

Consider the simple LPF given by $H(z) = \dfrac{1+z^{-1}}{2}$ ← ensures unity gain at $\omega = 0$

Then,

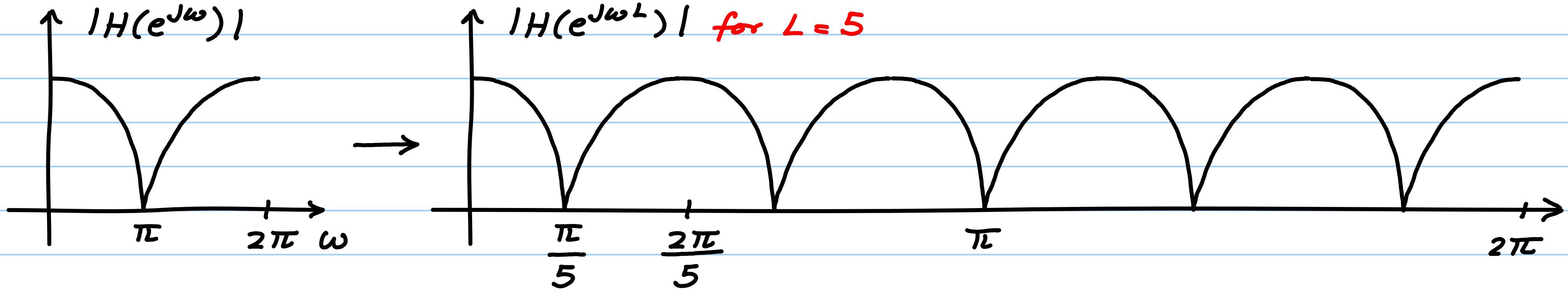
$$H(z^L) = \frac{1+z^{-L}}{2}$$

The roots are now the L^{th} roots of -1, i.e., $e^{j(2k+1)\pi/L}$

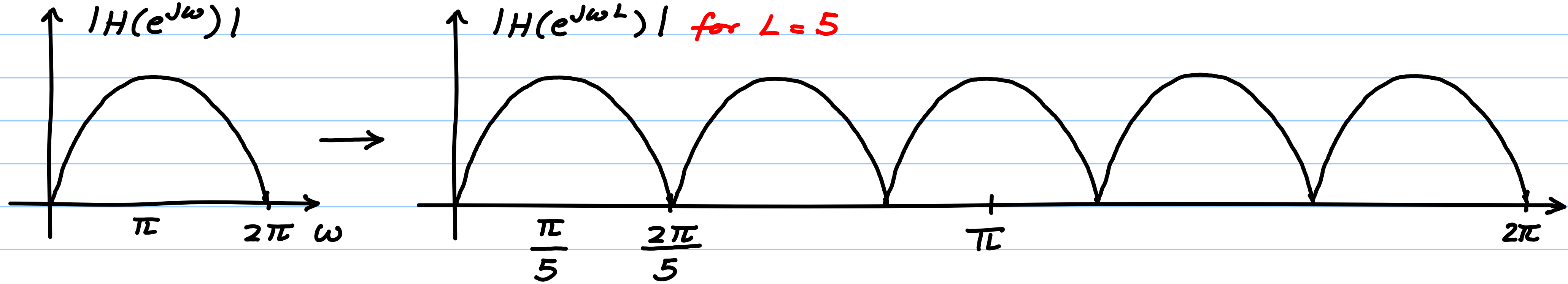
The peaks occur at $\omega = \dfrac{2\pi k}{L}$

Similarly, for a highpass filter, we start with $H(z) = \dfrac{1-z^{-1}}{2}$

and get $H(z) = \dfrac{1-z^{-L}}{2}$. Roots: L^{th} roots of 1



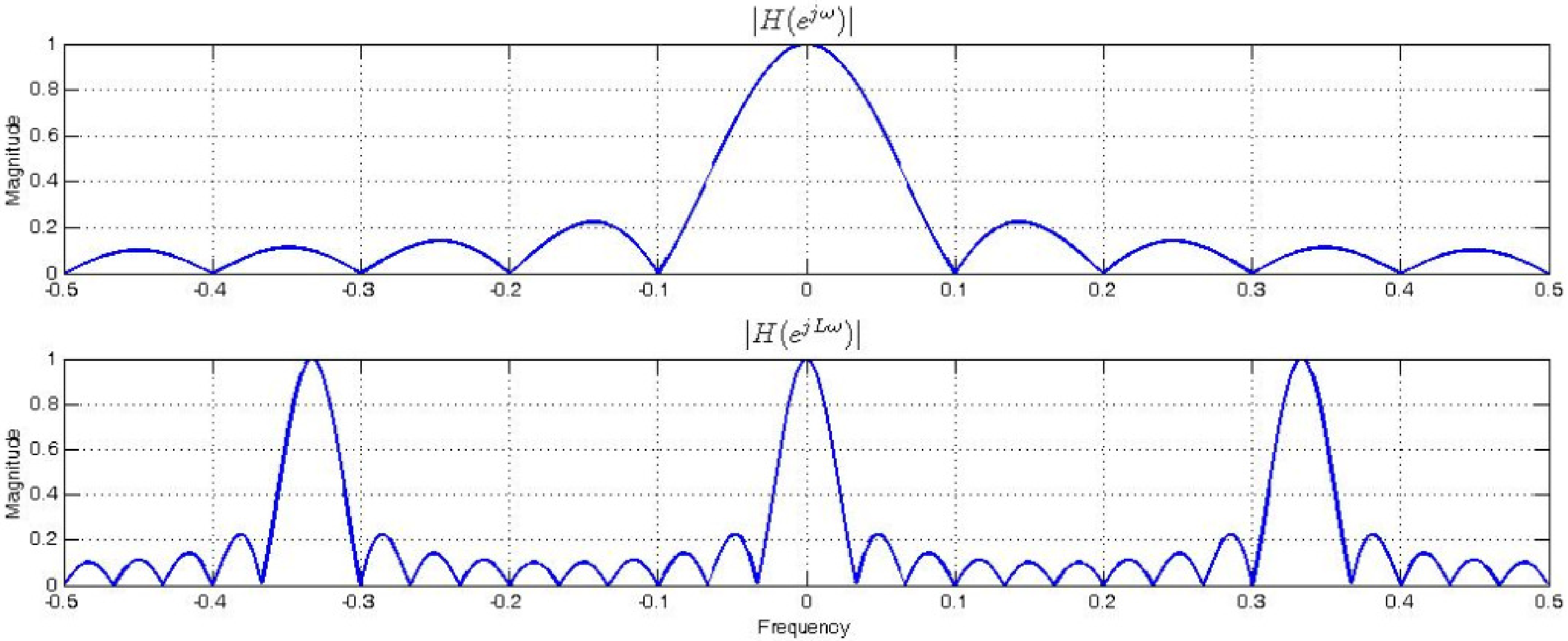
$|H(e^{j\omega})|$
π
2π
ω
$|H(e^{j\omega L})|$ for $L=5$
$\frac{\pi}{5}$
$\frac{2\pi}{5}$
π
2π



$|H(e^{j\omega})|$
π
2π
ω
$|H(e^{j\omega L})|$ for $L=5$
$\frac{\pi}{5}$
$\frac{2\pi}{5}$
π
2π

If $H(z) = \frac{1}{N} \frac{1-z^{-N}}{1-z^{-1}}$, then $|H(e^{j\omega})| = \left| \frac{\sin N\omega/2}{\sin \omega/2} \right|$

The plots for $N=10$ and $L=3$ are given below:



Phase Response

Recall that the standard form of $H(z)$ for a rational TF is
(transfer function)

$$H(z) = b_0 \frac{\prod_{l=1}^{M} (1 - z_l z^{-1})}{\prod_{k=1}^{N} (1 - p_k z^{-1})}$$

Hence,

$$\measuredangle H(e^{j\omega}) = \arg\{b_0\} + \sum_{l=1}^{M} \arg\{1 - z_l e^{-j\omega}\} - \sum_{k=1}^{N} \arg\{1 - p_k e^{-j\omega}\}$$

The form of a typical term is $\arg\{1 - re^{j\theta} e^{-j\omega}\}$

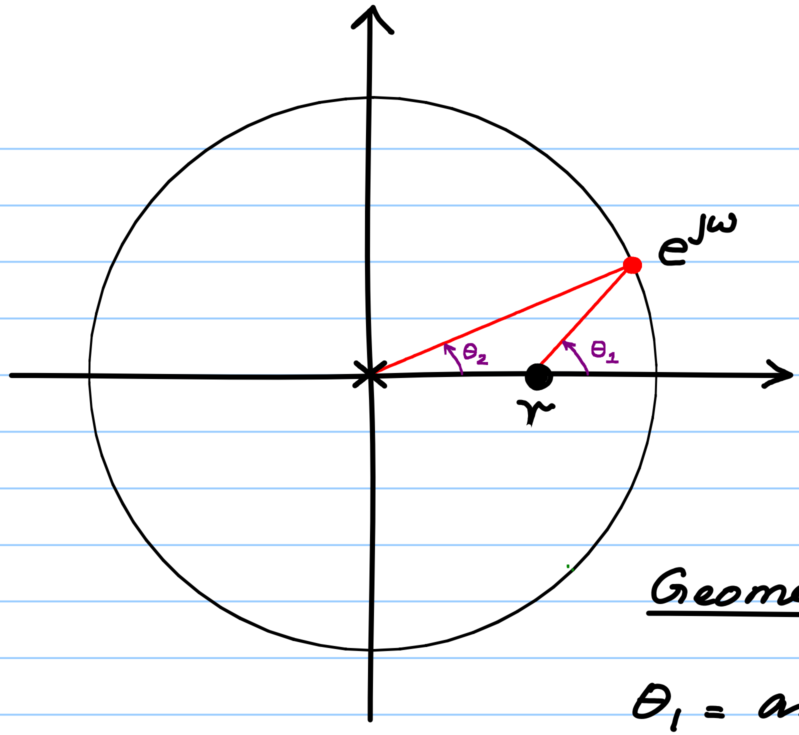
$$\arg\{1 - re^{j\theta}e^{-j\omega}\} = \arg\{1 - r\cos(\omega-\theta) + j\sin(\omega-\theta)\}$$

$$= \tan^{-1}\left[\frac{r\sin(\omega-\theta)}{1 - r\cos(\omega-\theta)}\right]$$

Note: $\tan^{-1}\left(\frac{3}{4}\right) \neq \tan^{-1}\left(\frac{-3}{-4}\right)$

We are interested in the OVERALL phase response. For a system with real-valued coefficients, the phase response will be an ODD function of ω.

Consider the case $0 < r < 1$ and $\theta = 0$, i.e., real-valued impulse response.



$$H(z) = 1 - r z^{-1}$$

$$\measuredangle H(e^{j\omega}) = \arg\{1 - r e^{-j\omega}\}$$

$$= \arg\{e^{j\omega} - r\} - \arg\{e^{j\omega}\}$$

Geometric Interpretation:

θ_1 = angle of the vector joining $e^{j\omega}$ & r

θ_2 = angle of the vector joining $e^{j\omega}$ & 0

Overall angle, i.e., phase response is

$\theta_1 - \theta_2$

Note: Trivial pole or zero i.e., pole or zero at the origin will contribute to the phase response.