Note Title

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Since the DTFT is nothing but the Fourier series expansion of the 2π-periodic frequency domain function, the following mean-square convergence theorem for Fourier Series is applicable.

The series $\sum_{n=-N}^{N} x[n]e^{-j\omega n}$ converges to $X(e^{j\omega})$ in the mean-square sense if $X(e^{j\omega})$ is square integrable over $[-\pi,\pi)$, i.e., $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega < \infty$.

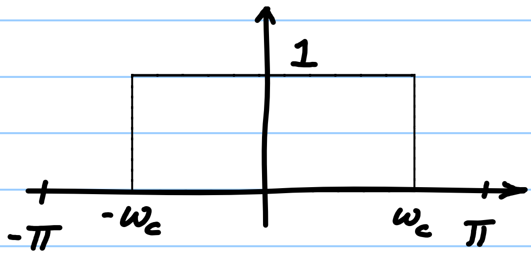
Let $X_N(e^{j\omega}) = \sum_{n=-N}^{N} x[n]e^{-j\omega n}$. MS convergence means

$$\int_{-\pi}^{\pi} |X_N(e^{j\omega}) - X(e^{j\omega})|^2 d\omega \to 0 \text{ as } N \to \infty.$$

Note that if $X(e^{j\omega})$ is square-integrable, then $x[n] \in \ell_2$ [why?]

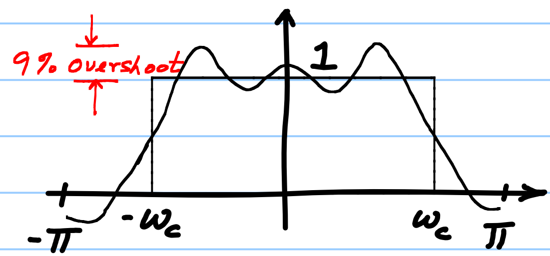
The lack of pointwise convergence but only mean-square convergence is illustrated through Gibbs phenomenon

$$\frac{\sin \omega_c n}{\pi n} \longleftrightarrow$$



$$\frac{\sin \omega_c n}{\pi n} \longleftrightarrow$$

$$-N \leq n \leq N$$



Relationship Between Laplace & Z-transforms

Let $x(t) \xleftrightarrow{\mathcal{L}} X(s)$

Define $$x_p(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\, \delta(t - nT)$$

$$X_p(s) = \mathcal{L}\{x_p(t)\} = \int_{-\infty}^{\infty} x_p(t) e^{-st} dt$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\, e^{-snT}$$

Recall $X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$

By letting $x(nT) \equiv x[n]$, we see that

$$X_p(s)\Big|_{z=e^{sT}} = X(z)$$

Note that, since $e^{sT} = e^{\left(s+j\frac{2\pi}{T}n\right)T}$, $X_p\left(s+j\frac{2\pi n}{T}\right) = X_p(s)$

The mapping e^{sT} maps (a) the left half of the s-plane to inside the unit circle, (b) the $j\Omega$ axis to the unit circle, and (c) the right half of the s-plane to outside the unit circle.

Horizontal lines in the s-plane get mapped to radial lines in the z-plane

Vertical lines in the s-plane get mapped to circles in the z-plane

$\Rightarrow$ vertical strips get mapped to annular regions.

The s-plane origin, i.e., $s=0$, gets mapped to $z=1$

Note that $s = \frac{1}{T} \ln z$. Since ln is a multivalued function,

a single point $z_1 = r_1 e^{j\theta_1}$ gets mapped to an infinite number of

points, i.e., $s = \frac{1}{T} \ln r e^{j\theta_1} = \frac{1}{T} \ln r e^{j(\theta_1 + 2n\pi)} = \frac{1}{T} \ln r + \frac{1}{T} j(\theta + 2\pi n)$