Note Title 31-07-2013

EE 5330 31st Jul., 2013

Among LTI systems, we focus on

$$y[n] = F\{x[n], x[n-1], \ldots, x[n-M], y[n-1], y[n-2], \ldots, y[n-N]\}$$

In particular, we consider the class

$$y[n] = -\sum_{k=1}^{N} a_k\, y[n-k] + \sum_{l=0}^{M} b_l\, x[n-l]$$

i.e., system represented by LCCDE.

For an LTI system, the impulse response completely characterizes the system.

The general conditions of causality, stability, etc. can be translated into conditions on the impulse response.

E.g.:

Causality: $h[n] = 0$ for $n < 0$

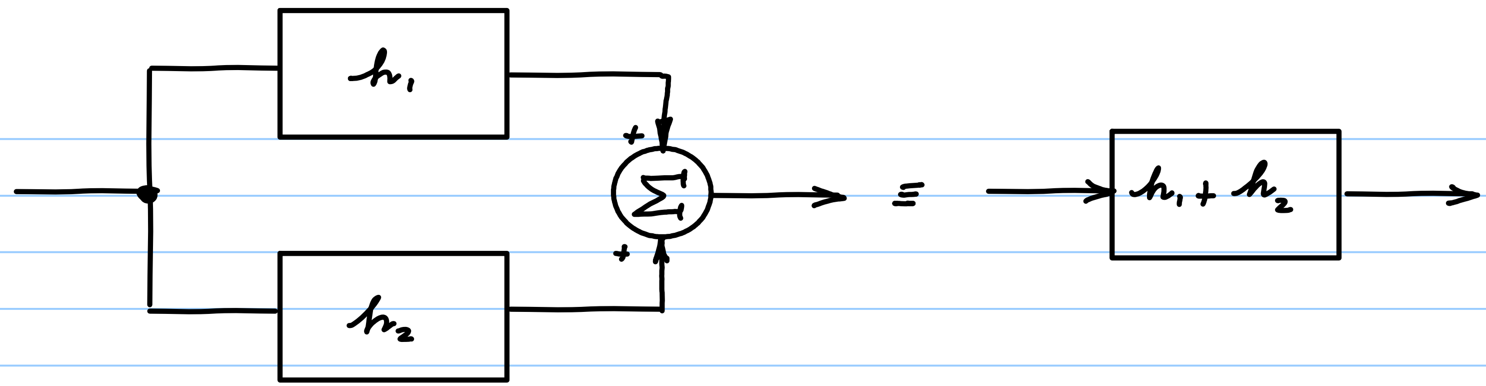
Stability: $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

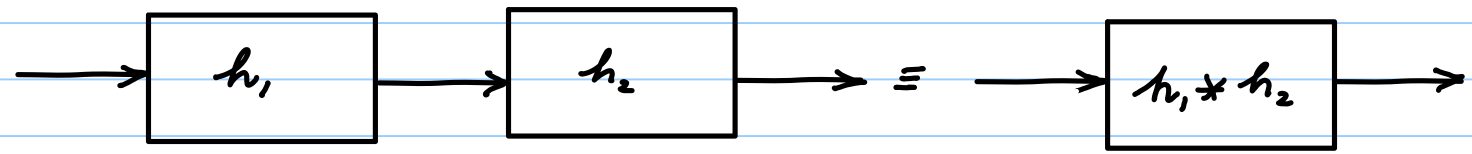
For an LTI system, knowing the impulse response means complete knowledge of the system.

Linearity and time invariance imply that the o/p to any input is given by the following convolution sum:

$$x[n] \longrightarrow \underset{LTI}{\boxed{h[n]}} \longrightarrow \qquad y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Parallel and cascade decomposition of systems are useful in Digital Filter implementation.





$$x * (y + z) = x * y + x * z \qquad \text{distributive}$$

$$x * (y * z) = (x * y) * z \qquad \text{associative}$$

Associativity holds if $x, y, z \in \ell_1$

ℓ_1 is the space of absolutely summable sequences:

$$\ell_1 = \left\{ x : \sum_{n=-\infty}^{\infty} |x[n]| < \infty \right\}$$

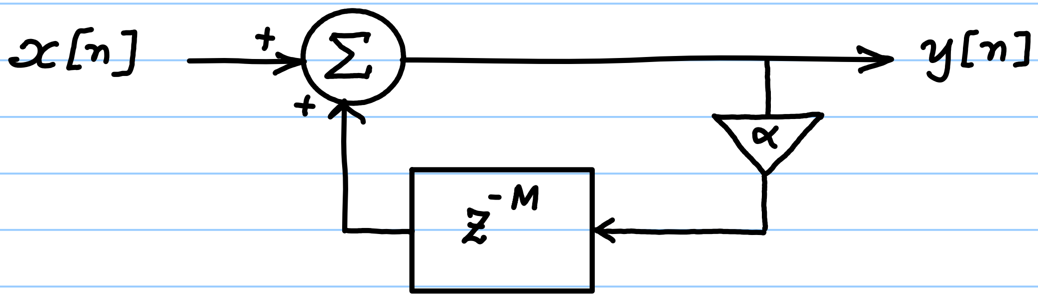
ℓ_2 is the space of square summable sequences:

$$\ell_2 = \left\{ x : \sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty \right\}$$

Is $\ell_1 \subset \ell_2$? That is, if $x \in \ell_1$, does it also belong to ℓ_2 ? Can you find y s.t. $y \in \ell_2$ but $y \notin \ell_1$?

A simple difference equation:

$$y[n] = \alpha\, y[n-M] + x[n]$$



Let $M = 100$, $\alpha = 0.98$ and $x[n] = \begin{cases} r & 0 \leq n \leq 99 \\ 0 & n \geq 100 \end{cases}$

where $r \in \mathcal{U}[-1,1]$

Sounds like a plucked string when played at $F_s = 44.1$ kHz !

Take a look at the Karplus-Strong algorithm for more details