EE 5330 30^{th} Jul., 2013

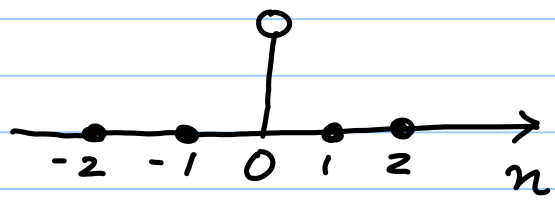
Note Title 30-07-2013

$e^{j\omega_0 n}$ & $e^{-j\omega_0 n}$ are independent

i.e., $\nexists$ $k \in \mathbb{C}$, s.t. $e^{j\omega_0 n} = k\, e^{-j\omega_0 n}$

$u[n]$: unit step (cf. with $u(t)$)

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

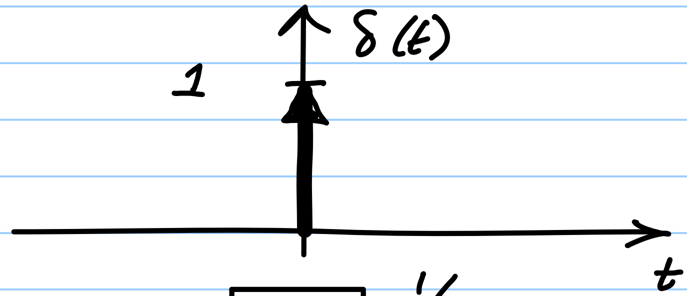


Cf. this with $\delta(t)$

$$\delta(t) = 0 \qquad t \neq 0$$

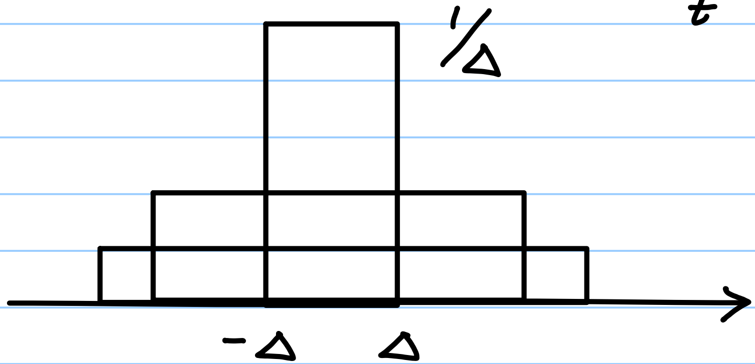
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$\delta(t) + \delta'(t)$ also satisfies these two equations



$\delta(0) = \infty$ wrong!

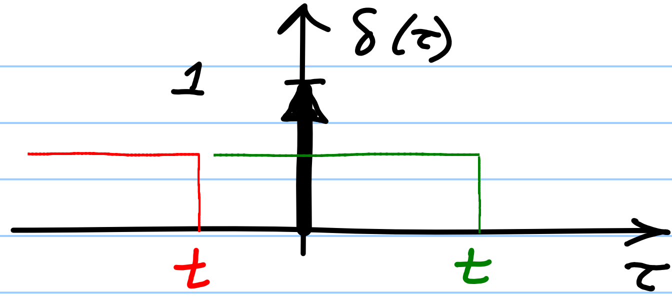
$\delta(0) = 1$ wrong!



$\delta(t)$ is a shorthand for limiting arguments

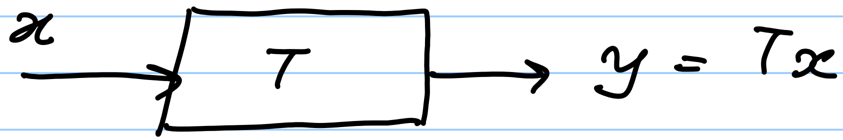
See *The Theory of Distributions: A Nontechnical Introduction* by J. Ian Richards and Heekyung K. Youn, Cambridge University Press (1995)

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



$$\frac{d}{dt} u(t) = \delta(t)$$

Systems



(a) Linearity : $a_1 x_1 + a_2 x_2 \longrightarrow a_1 y_1 + a_2 y_2$

(i) additivity (ii) homogeneity

These are two independent properties. (i) + (ii) $\Rightarrow$ linear

$$\sum_{k=1}^{N} a_k x_k \longrightarrow \sum_{k=1}^{N} a_k y_k$$

What about

$$\sum_{k=1}^{\infty} a_k x_k \longrightarrow \sum_{k=1}^{\infty} a_k y_k$$

Follows if the space is complete

(b) Time Invariance $\quad T\{x[n-n_0]\} = y[n-n_0]$

(c) Causality : O/p at $n = n_0$ depends only on i/p for time $n \leq n_0$

(d) Systems w/ memory : If o/p at $n = n_0$ depends only on i/p at $n = n_0$, the system is memoryless.

(e) <u>Stability</u> BIBO : bounded i/p $\rightarrow$ bounded o/p

If $|x[n]| < B_x < \infty$, then $T\{x[n]\} = y[n]$

is s.t. $|y[n]| < B_y < \infty$

We will focus on the class of Linear & Time Invariant (LTI) systems in this course.