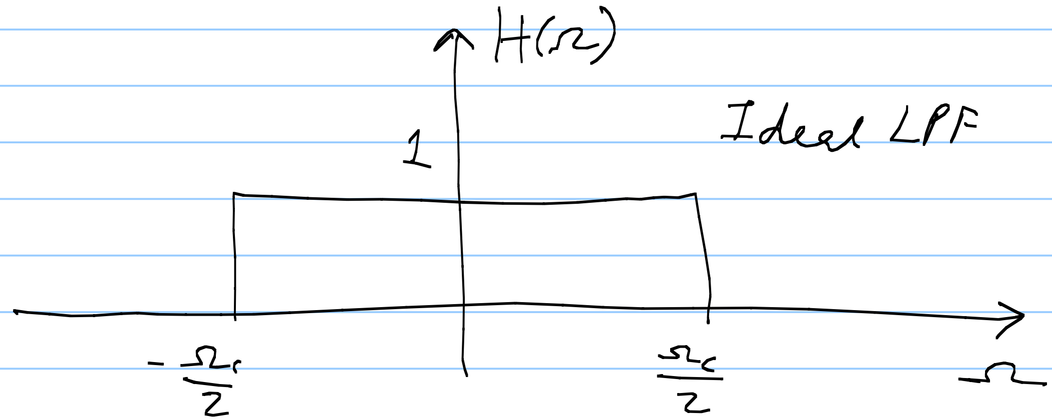
EC5142 Sep. 27 2011

Note Title 27-09-2011

Ideal Filters

Ideal LPF

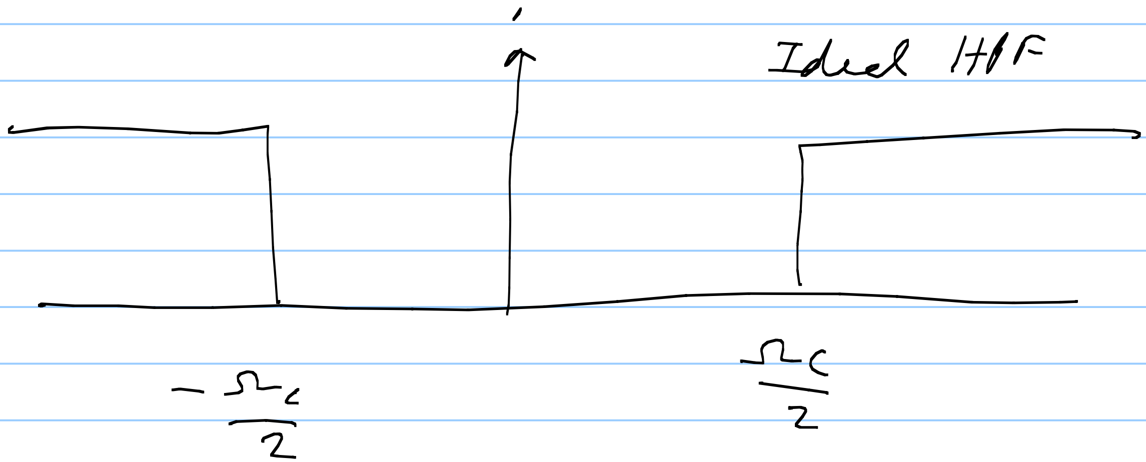


$\longleftrightarrow$ $F_c \, \mathrm{Sinc}(F_c t)$

$\downarrow$

non-causal

Ideal HPF



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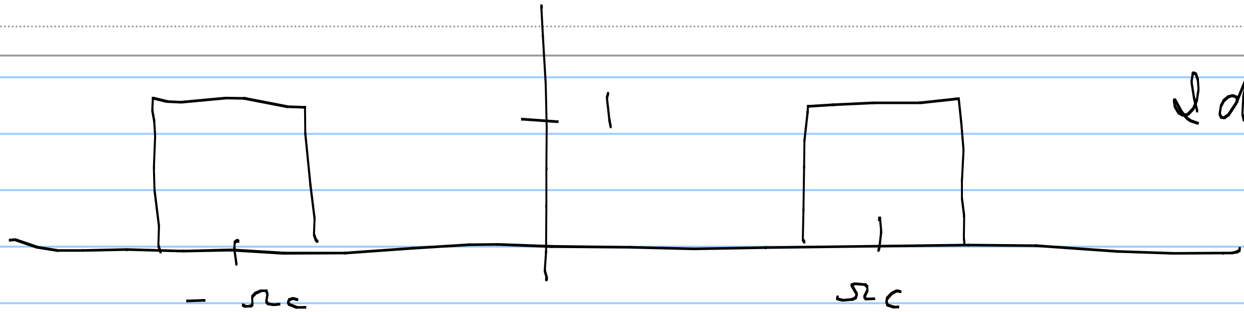


Ideal BPF

Ideal BSF

Ideal notch filter

$$x[n+N] = x[n]$$

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$N a_k = \sum_{n=\langle N \rangle} x[n]\, e^{-j \underbrace{k\omega_0}\, n} \qquad \omega_0 = \frac{2\pi}{N}$$

As $N \to \infty$

$$\boxed{N a_k \longrightarrow \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \equiv \underset{\text{DTFT}}{X(e^{j\omega})}}$$

$\hookrightarrow$ 2π periodic

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} (N a_k) e^{jk\frac{2\pi}{N}n} \qquad \omega_0 = \frac{2\pi}{N}$$

$$= \frac{1}{2\pi} \sum_{k=0}^{N-1} (N a_k) e^{jk\omega_0 n} \omega_0$$

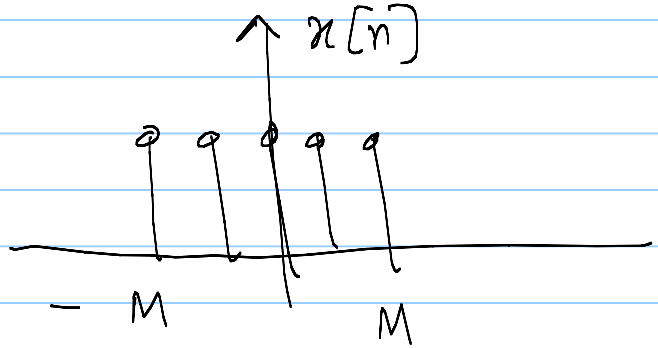
As $N \to \infty \quad \omega_0 \to 0$

IDTFT

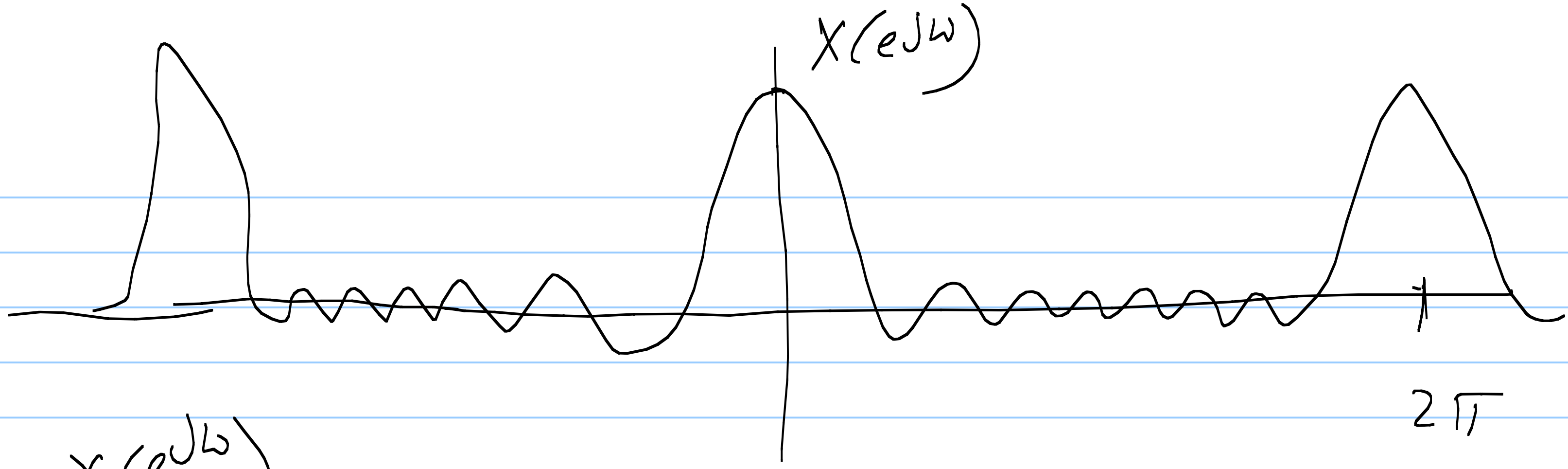
$$\boxed{\frac{1}{2\pi}\int_0^{2\pi} X(e^{jw})\, e^{jwn}\, dw = x[n]}$$

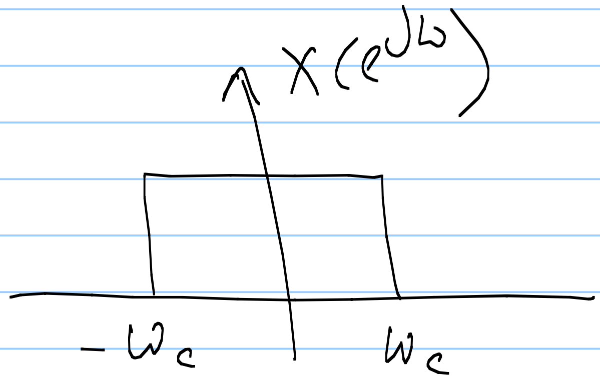
$$\boxed{X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}}$$

DTFT

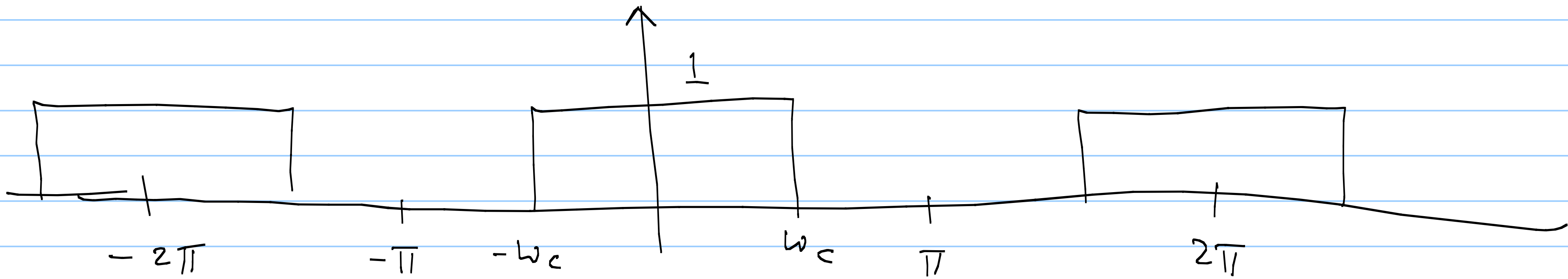


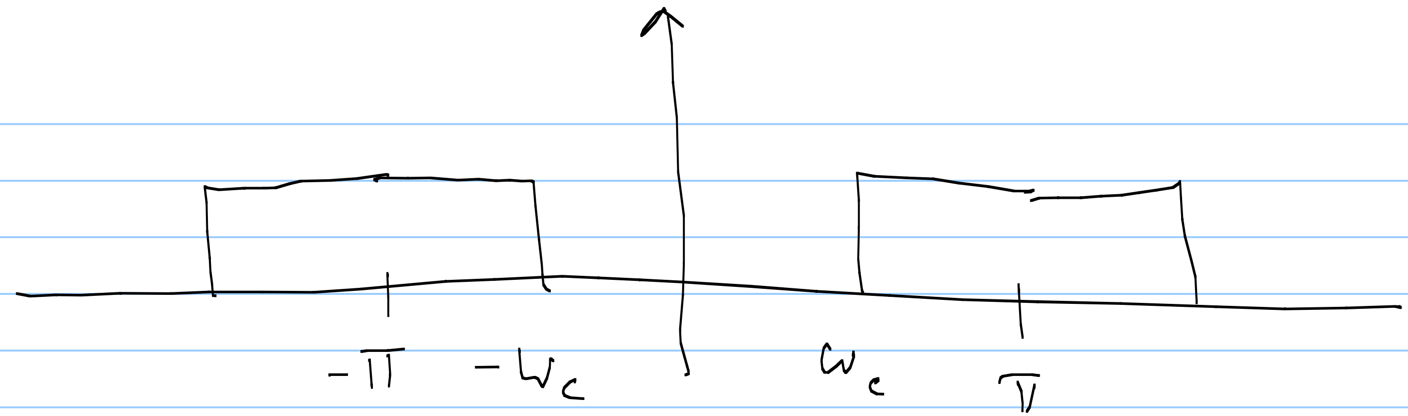
$$\longleftrightarrow \qquad X(e^{jw}) = \frac{\sin (2M+1) w/2}{\sin w/2}$$





$$\longleftrightarrow \quad x[n] = \frac{\sin \omega_c n}{\pi n}$$





$-\pi$
$-\omega_c$
ω_c
π