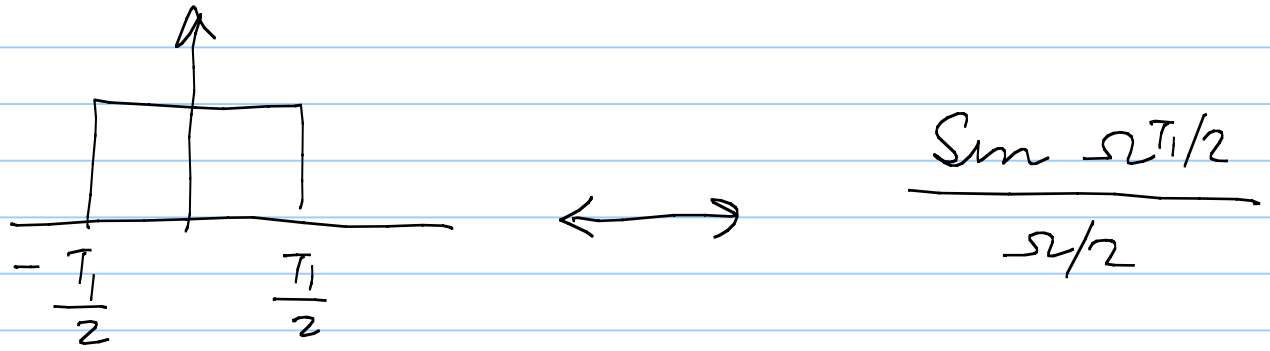
Ec 5142 26 Sep. 2011

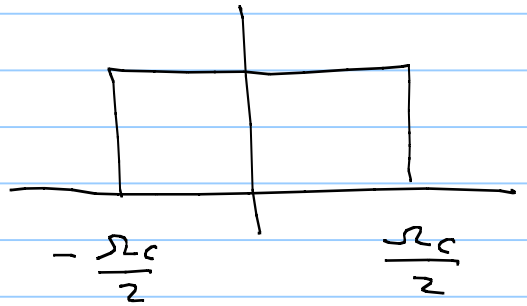
Note Title 26-09-2011

$x(t) \longleftrightarrow 2\pi x(-\Omega)$



$\longleftrightarrow \quad \frac{\sin \Omega T_1/2}{\Omega/2}$

$F_c \operatorname{sinc}(F_c t) \longleftrightarrow$



For real $x(t)$, $x(t) = x^*(t)$

$$X(\Omega) = X^*(-\Omega)$$

$$X(\Omega) = X_R(\Omega) + j X_I(\Omega)$$

$$X^*(-\Omega) = X_R(-\Omega) - j X_I(-\Omega)$$

$$\Rightarrow \quad X_R(\Omega) = X_R(-\Omega) \qquad \text{even}$$

$$X_I(-\Omega) = -X_I(\Omega) \qquad \text{odd}$$

$|X(\Omega)|$: even

$\Rightarrow X(\Omega)$: odd

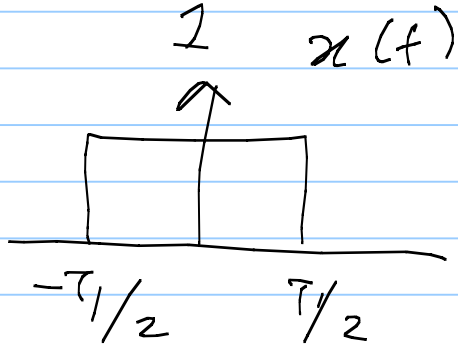
Differentiation:

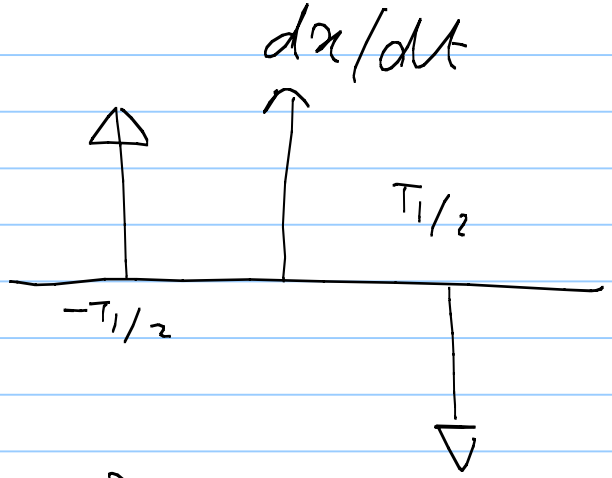
$$x(t) \longleftrightarrow X(\Omega)$$

$$x'(t) \longleftrightarrow j\Omega X(\Omega)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega$$

$$\frac{dx}{dt} \stackrel{?}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} j\Omega X(\Omega) e^{j\Omega t} d\Omega$$





$$\delta(t + T_1/2) - \delta(t - T_1/2)$$

$$e^{j\Omega T_1/2} - e^{-j\Omega T_1/2} \longleftrightarrow 2j \operatorname{Sin}\, \Omega T_1/2$$

$$x(t) \longrightarrow \frac{\operatorname{Sin}\, \Omega T_1/2}{\Omega/2}$$

$$x(at) \longleftrightarrow \frac{1}{|a|} X(\Omega/a)$$

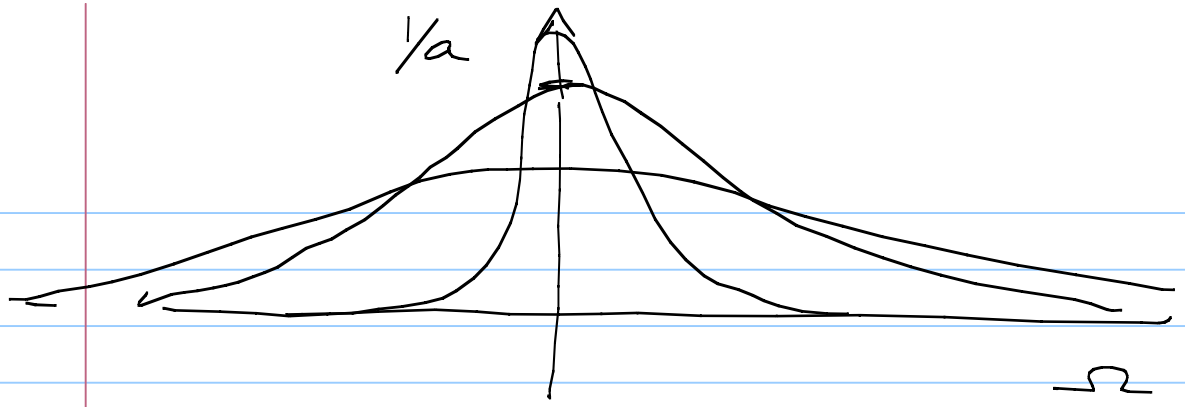
$$x(-t) \longleftrightarrow X(-\Omega)$$

$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\Omega}$$

$$u(t) \longleftrightarrow ?$$

$$e^{-at}u(t) \longleftrightarrow \frac{a}{a^2+\Omega^2} - \frac{j\Omega}{a^2+\Omega^2}$$

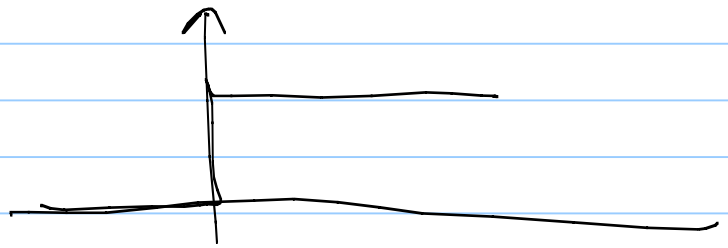
$$\lim_{a \to 0} \frac{a}{a^2+\Omega^2} - \frac{j\Omega}{a^2+\Omega^2} =$$



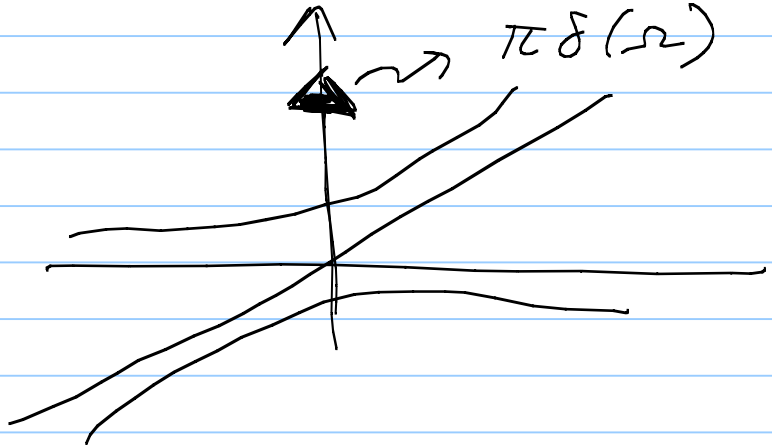
$$\int_{-\infty}^{\infty} \frac{a}{a^2+\Omega^2}\, d\Omega = \pi$$

indep. of a!

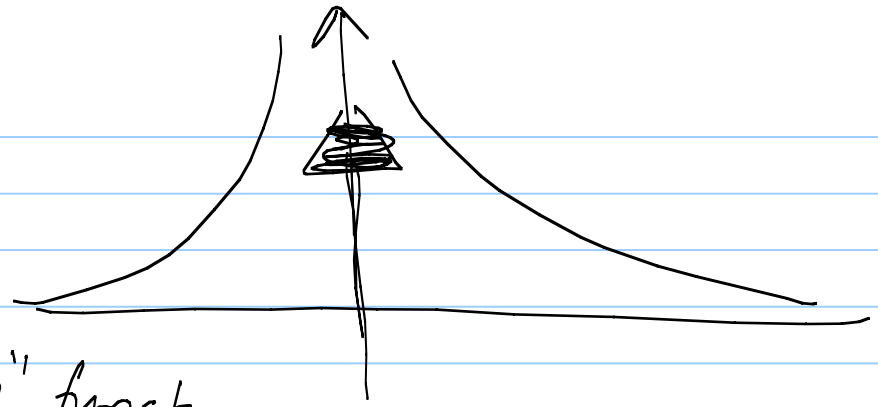
$$\lim_{a \to 0} \left(\frac{a}{a^2+\Omega^2} - \frac{j\Omega}{a^2+\Omega^2} \right) \longrightarrow \pi\,\delta(\Omega) + \frac{1}{j\Omega}$$



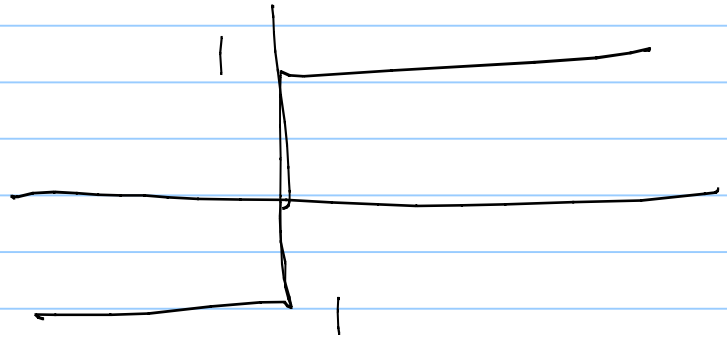
$\longleftrightarrow$



$|X(\Omega)|$



$\text{sgn}(t)$ = "signum" function

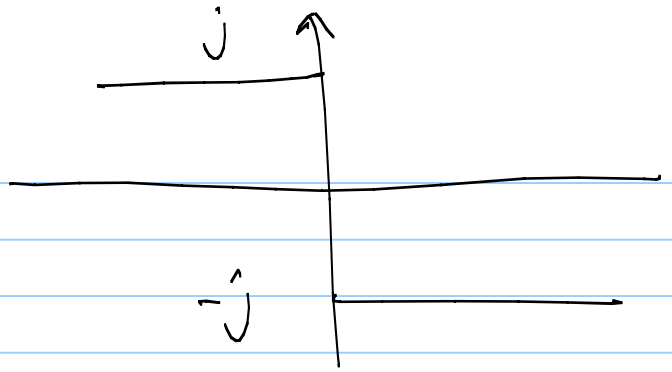


$$= 2\left[u(t) - 1/2\right]$$

$$= 2u(t) - 1$$

$$\longleftrightarrow \quad 2\pi\delta(\Omega) + \frac{2}{j\Omega} - 2\pi\delta(\Omega) = \frac{2}{j\Omega}$$

$$x(t) = \frac{1}{\pi t} \longleftrightarrow$$



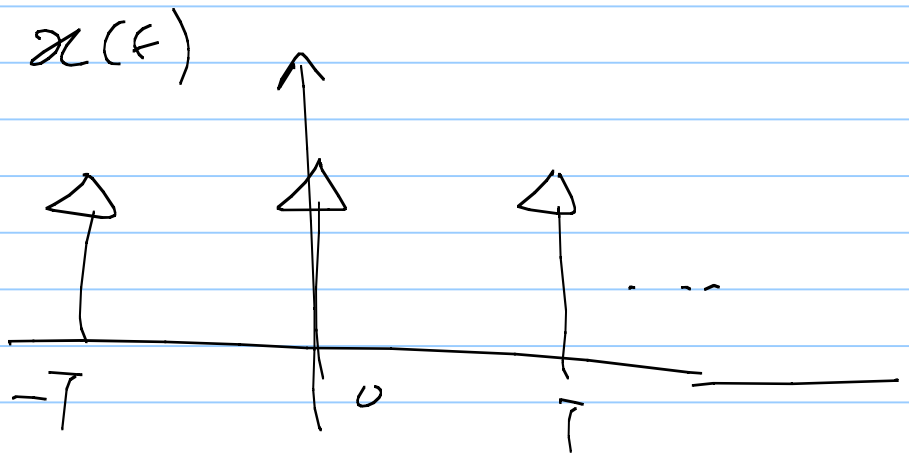
Other forms of $X(\Omega)$:

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j 2\pi F t} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j 2\pi F t} dF$$

$$\mathcal{F}\{\mathcal{F}\{x(t)\}\} = ?$$

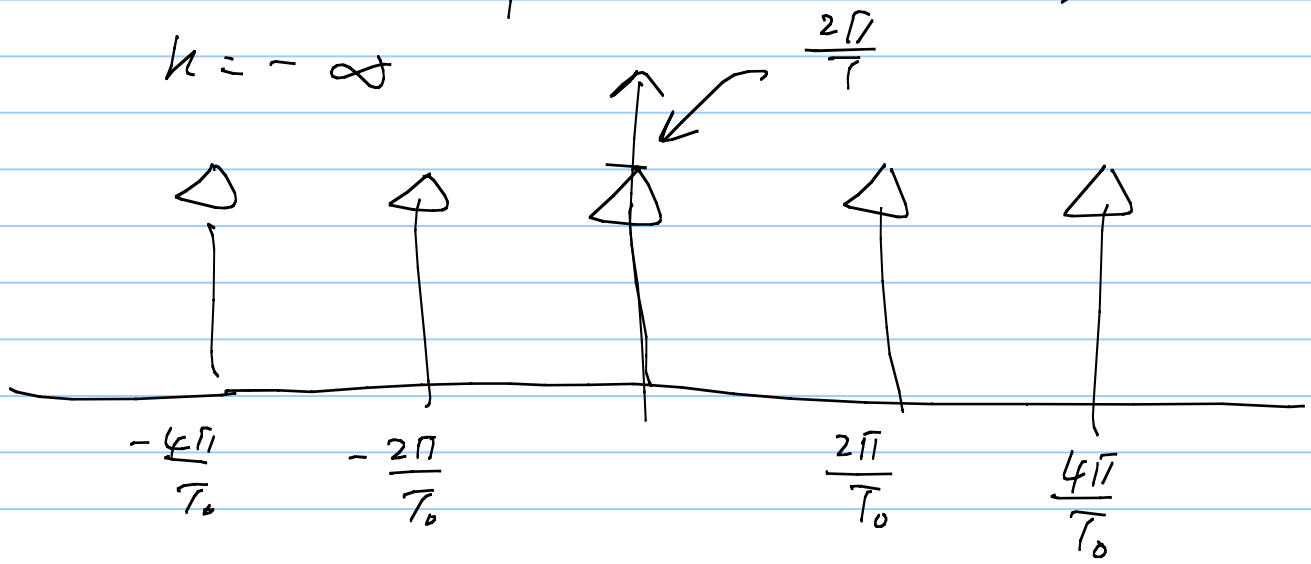
$$\mathcal{F}\mathcal{F}\mathcal{F}\mathcal{F}\{x(t)\} = ?$$



$$a_k = \frac{1}{T}$$

$$x(t) \quad \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jk\Omega_0 t}$$

$$\longleftrightarrow \quad \sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \delta(\Omega - k\Omega_0)$$



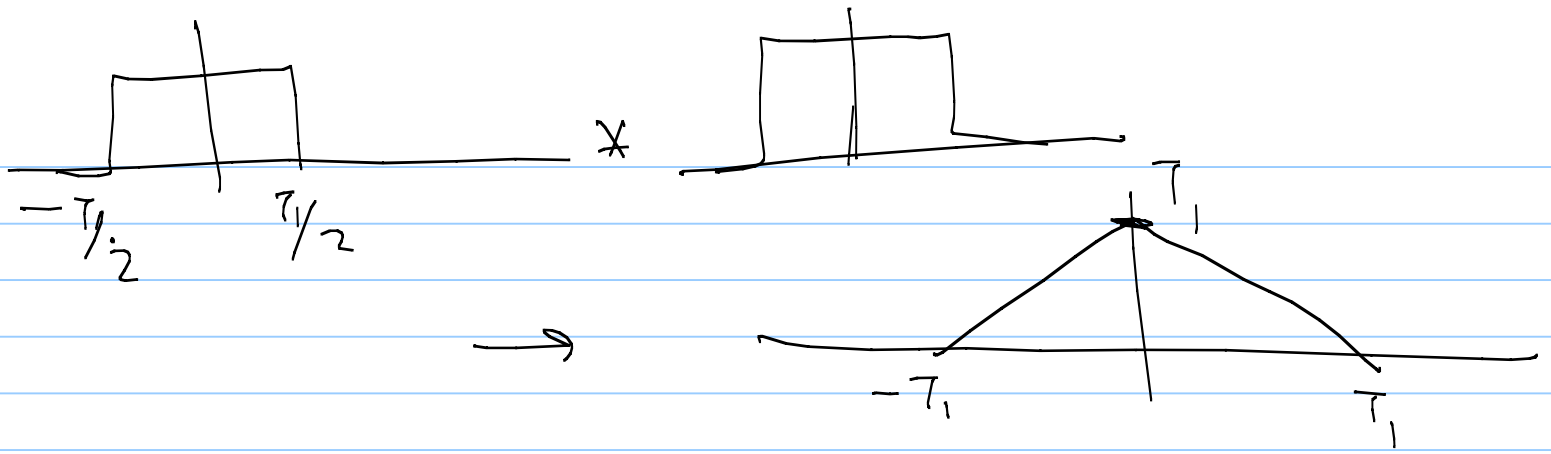
$$x(t) = e^{-t^2/2} \quad \longleftrightarrow \quad ?$$

$$\frac{1}{\sigma}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}}\,dt = 1$$

Convolution

$$x(t) * y(t) \quad \longleftrightarrow \quad X(\Omega)\,Y(\Omega)$$

$$x(t) \cdot y(t) \quad \longleftrightarrow \quad \frac{1}{2\pi}\, X(\Omega) * Y(\Omega)$$



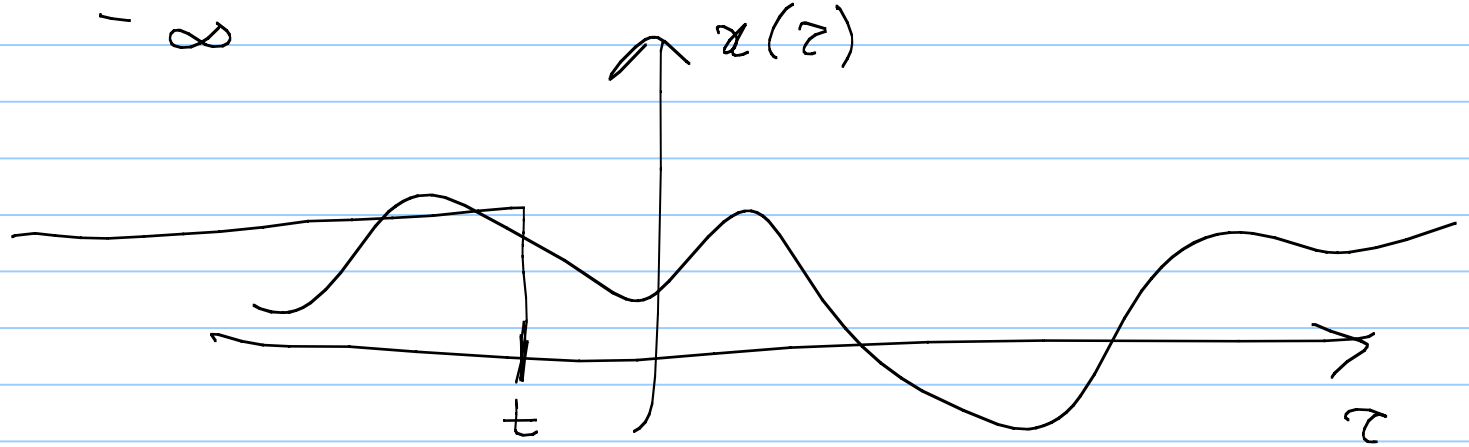
$$\longleftrightarrow \quad \left[\frac{\sin \Omega T_1/2}{\Omega/2}\right]^2$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega)\, Y^*(\Omega)\, d\Omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

Integration Property

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau = x(t) * u(t)$$



$$\longleftrightarrow \quad X(\Omega)\ U(\Omega)$$

$$X(\Omega)\left[\ \pi\delta(\Omega) + \frac{1}{j\Omega}\ \right]$$

$$= \pi X(0)\delta(\Omega) + \frac{X(\Omega)}{j\Omega}$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \quad \longleftrightarrow \quad \pi X(0)\delta(\Omega) + \frac{X(\Omega)}{j\Omega}$$

$$\frac{dy}{dt} = x(t)$$

$$j\Omega\left[\pi X(0)\delta(\Omega) + \frac{X(\Omega)}{j\Omega}\right]$$

$$= \underbrace{j\Omega\,\delta(\Omega)}_{0}\,\pi X(0) + X(\Omega)$$

$$= X(\Omega) \quad \text{as expected.}$$