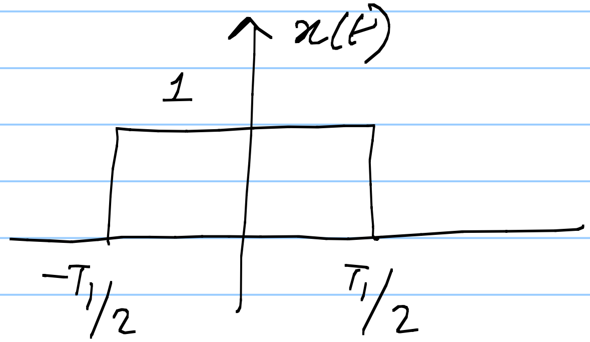
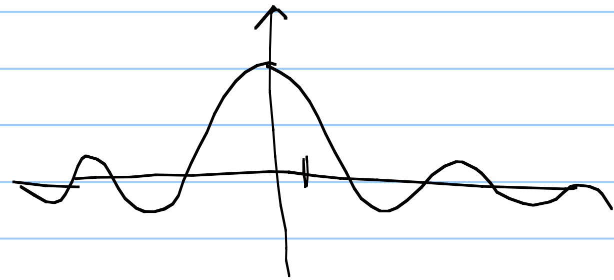
Note Title 21-09-2011

EC 5142 21 Sep. 2011

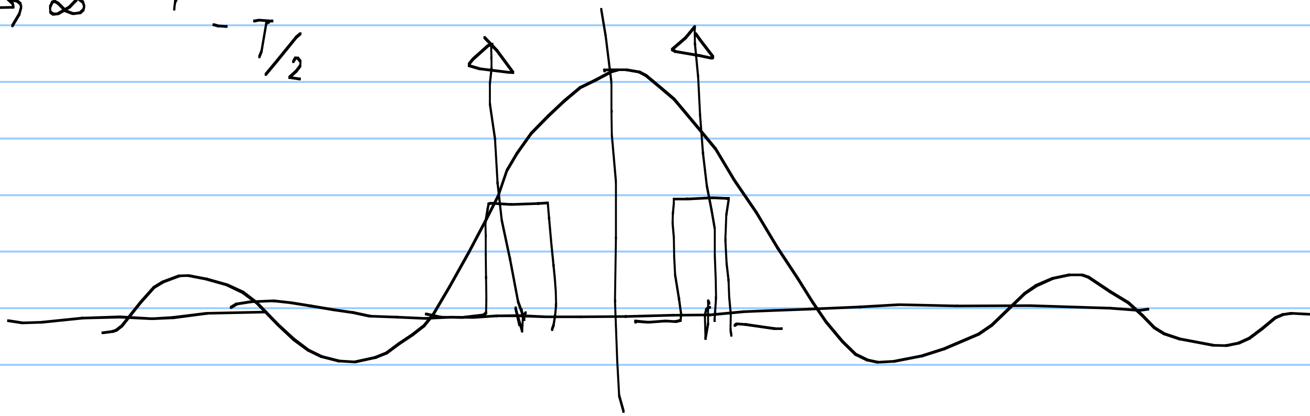


$$\longleftrightarrow \qquad \frac{\sin \Omega\, T_1/2}{\Omega/2}$$



$$X(0) = T_1$$

$$\bar{x}(t) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt = 0$$

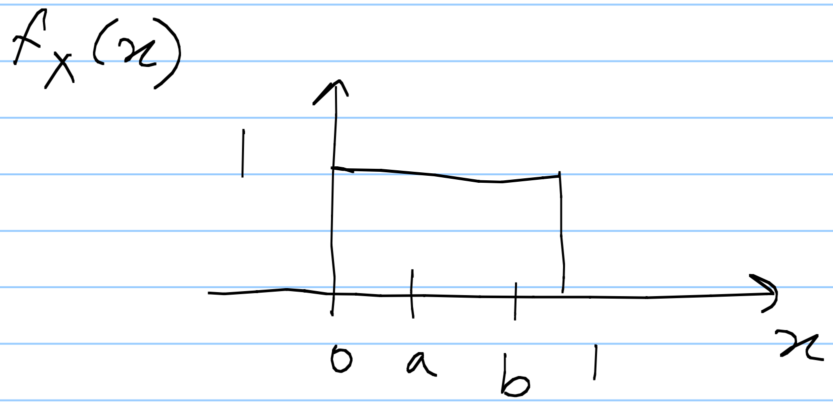


Note Title 21-09-2011

$$e^{j\Omega_0 t} \longleftrightarrow 2\pi\,\delta(\Omega-\Omega_0)$$

$$\cos\Omega_0 t \longleftrightarrow \pi\left[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)\right]$$

$$\sin\Omega_0 t \longleftrightarrow \frac{\pi}{j}\left[\delta(\Omega-\Omega_0)-\delta(\Omega+\Omega_0)\right]$$



$$0 \le a < b \le 1$$

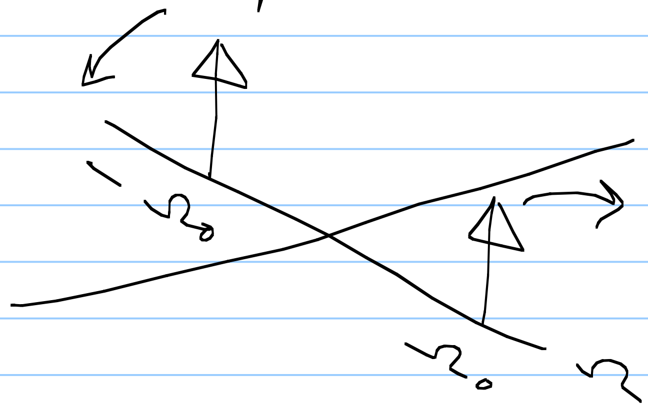
$$P[\, a \le X \le b \,] = b - a$$

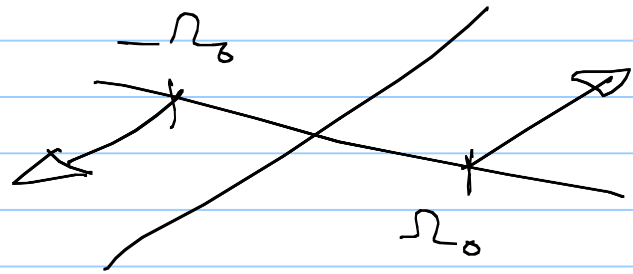
$$P[\, X = a \,] = 0$$

Properties

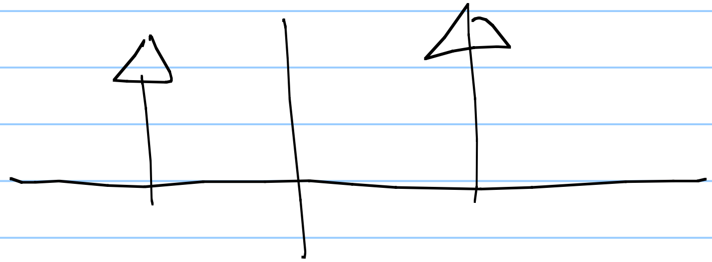
1) Linearity

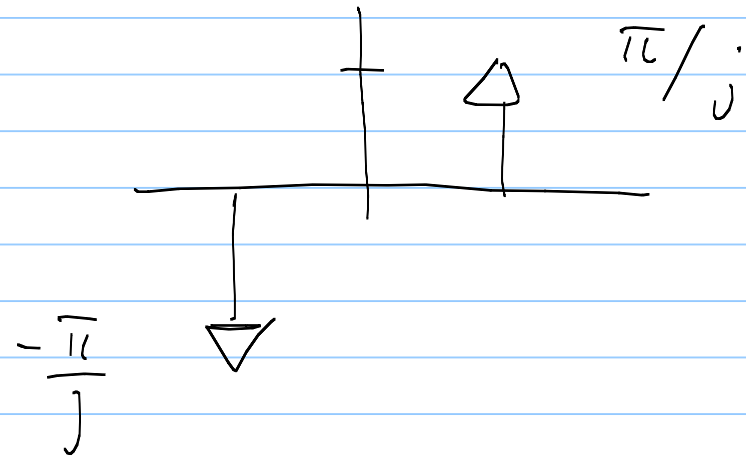
2) Time Shift : $x(t - t_0) \longleftrightarrow e^{-j\Omega t_0} X(\Omega)$



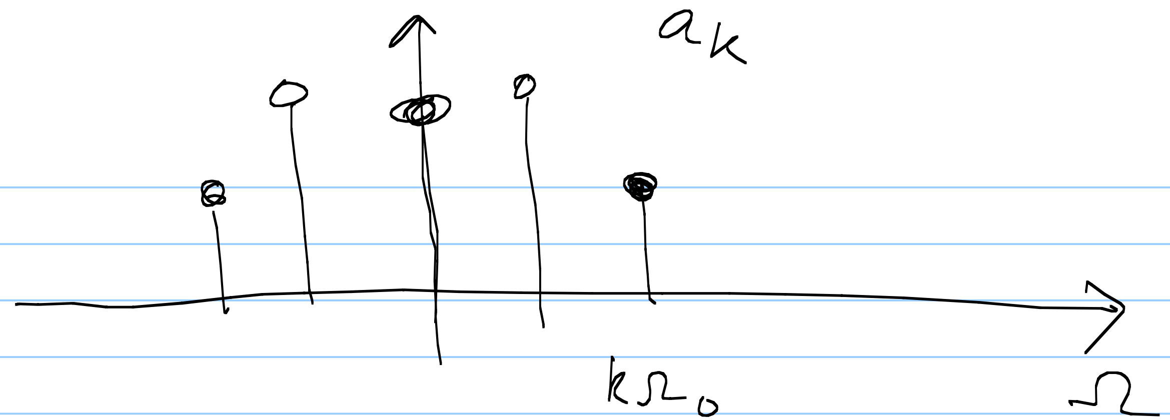


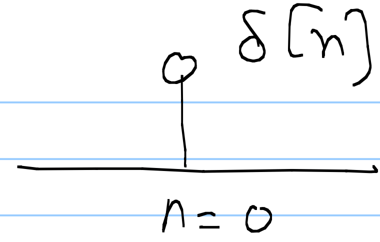
$$\cos \Omega_0 t \longleftrightarrow \pi \left[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0) \right]$$

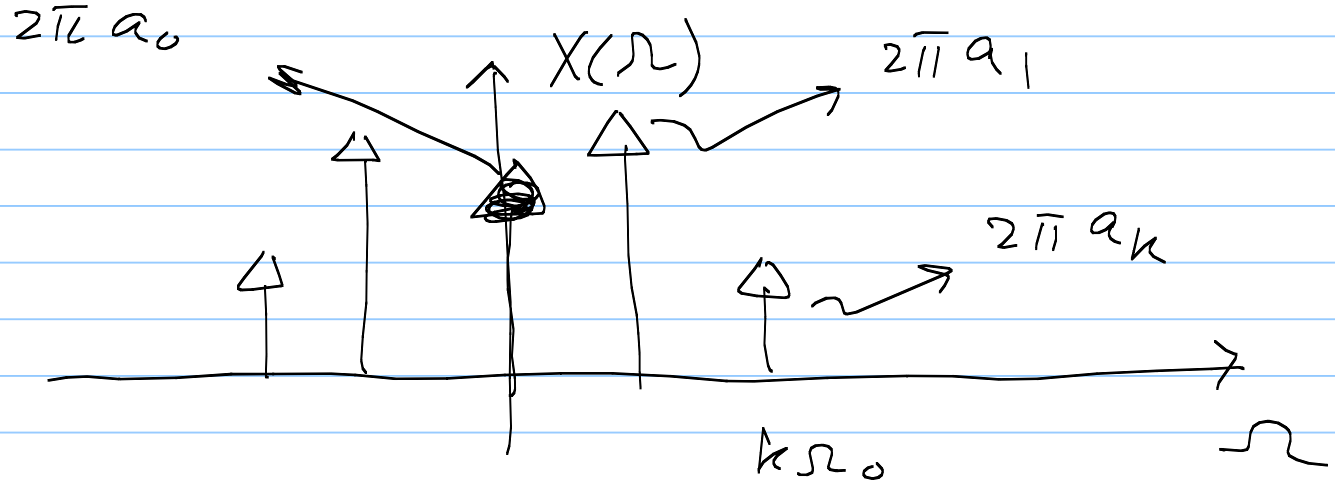


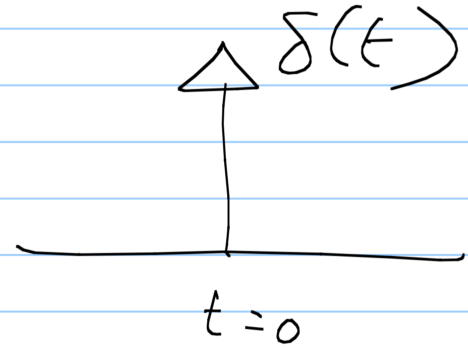


$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t} \longleftrightarrow \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\Omega - k\Omega_0)$$







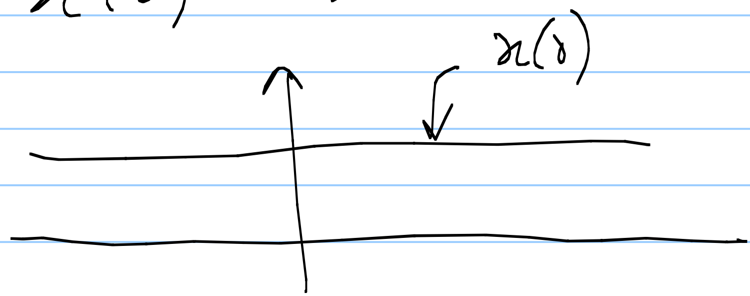


3) Modulation $\quad e^{j\Omega_0 t}\, x(t) \longleftrightarrow X(\Omega - \Omega_0)$

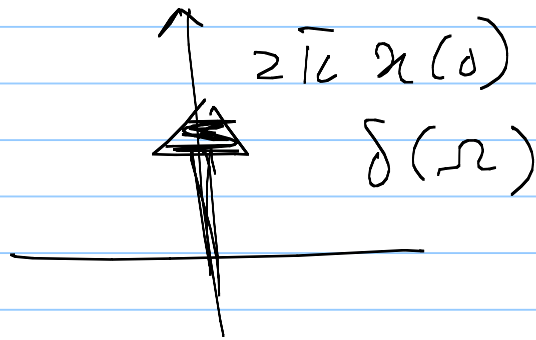
4) $y(t) = x(at) \longleftrightarrow \frac{1}{|a|} X(\Omega/a)$

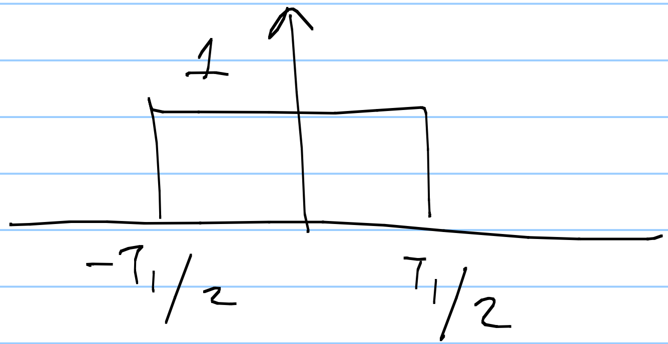
For $a = 0$

$$y(t) = x(0) \quad \forall t$$

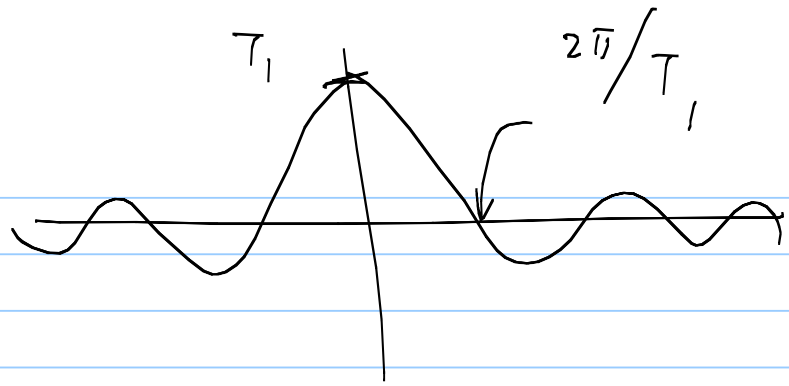


$\longleftrightarrow$





$$\longleftrightarrow \quad \frac{\sin \Omega T_1/2}{\Omega/2}$$



As $T_1 \to \infty$, $x(t) \to 1 \ \forall t$

$$\frac{\sin \Omega T_1/2}{\Omega/2} \longrightarrow 2\pi\,\delta(\Omega)$$

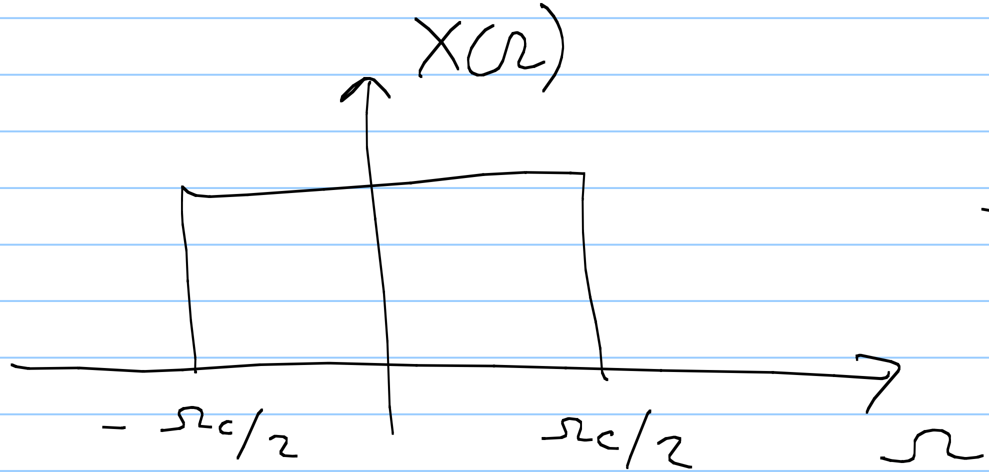
$$\frac{\sin \pi t}{\pi t} = \operatorname{sinc}(t)$$

5) Duality

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

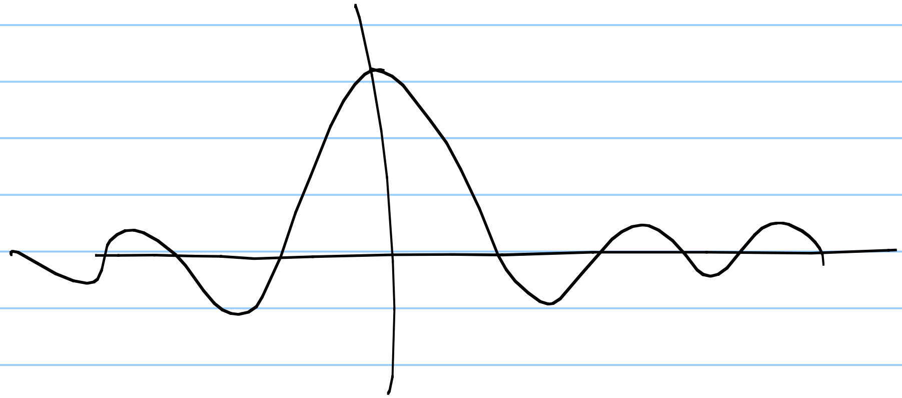
$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega$$

$X(\Omega)$

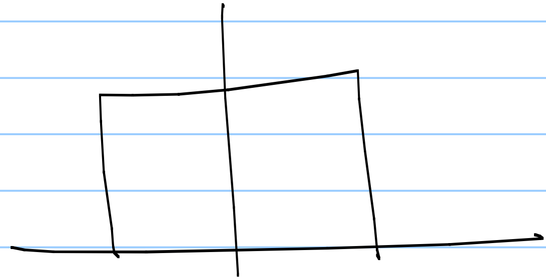


$$\frac{\Omega_c}{2\pi} = F_c$$

$$x(t) = \frac{1}{2\pi} \int_{-\Omega_c/2}^{\Omega_c/2} 1 \cdot e^{j\Omega t} \, d\Omega = F_c \, \mathrm{Sinc}(F_c t)$$



$\longleftrightarrow$



$$x(t) \longleftrightarrow X(\Omega)$$

$$y(t) = X(t) \longleftrightarrow 2\pi\, x(-\Omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, d\Omega$$

$$Y(\Omega) = \int_{-\infty}^{\infty} \underbrace{y(t)} e^{-j\Omega t}\, dt$$

$$\hookrightarrow \quad 2\pi x(\lambda) = \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega\lambda} \, d\Omega$$

$$= \int_{-\infty}^{\infty} X(t) e^{j\lambda t} \, dt$$

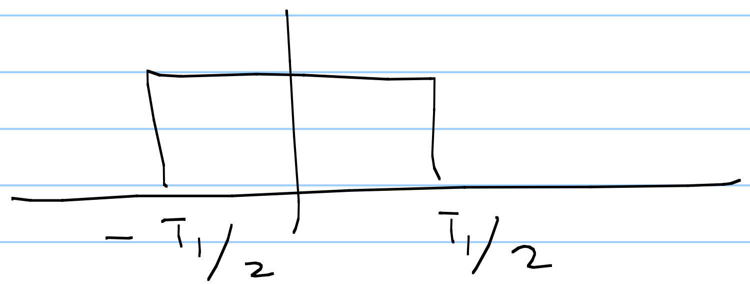
$$2\pi x(-\lambda) = \int_{-\infty}^{\infty} X(t) e^{-j\lambda t} \, dt$$

$$2\pi x(-\Omega) = \int_{-\infty}^{\infty} X(t) e^{-j\Omega t} \, dt$$

$$\mathcal{F}\{X(t)\} = 2\pi\, x(-\Omega)$$

$$X(t) \longleftrightarrow 2\pi\, x(-\Omega)$$

$x(t)$



$$\longleftrightarrow \quad \frac{\sin \Omega\, T_1/2}{\Omega/2} = X(\Omega)$$

$$x(t) = \frac{\sin t\, T_1/2}{t/2} \equiv \frac{\sin t\, \Omega_c/2}{t/2}$$

$$\frac{x(t)}{2\pi} = \frac{\sin t\, \Omega_c/2}{\pi t}$$

$$= \frac{\sin \pi F_c t}{\pi F_c t} \cdot F_c$$

$$= F_c \,\mathrm{sinc}(F_c t) \longleftrightarrow x(-\Omega)$$