EC 5142 Sep. 20 2011

Note Title 20-09-2011

$$\boxed{X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt} \equiv X(j\Omega)$$

$$\lim_{T\to\infty} a_k T = \lim_{T\to\infty} \int_{-T/2}^{T/2} x(t)\, e^{-j\overbrace{k\Omega_0}^{\Omega_k} t}\, dt$$

$x(t+T) = x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$$ Synthesis eqn.

As $T \to \infty$ $\qquad \Omega_0 = \frac{2\pi}{T}$

$$x(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} a_k T \, e^{j \overbrace{k\Omega_0}^{\Omega_k} t}$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} (a_k T) \, e^{j\Omega_k t} \, \Omega_0$$

$$\Omega_0 = (k+1)\Omega_0 - k\Omega_0 \longrightarrow \Delta\Omega$$

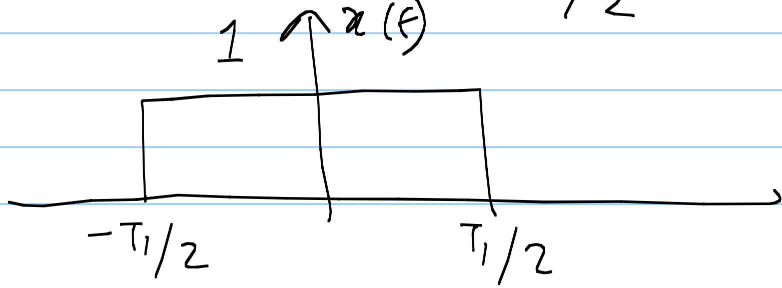
$$x(t) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} (a_k T) \, e^{j\Omega_k t} \, \Delta\Omega$$

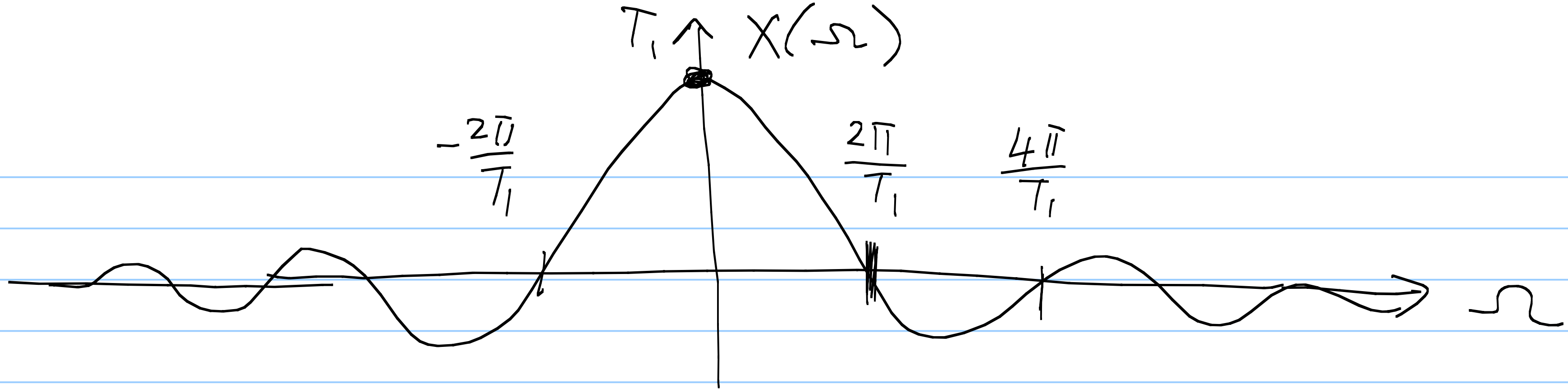
As $T \to \infty$,

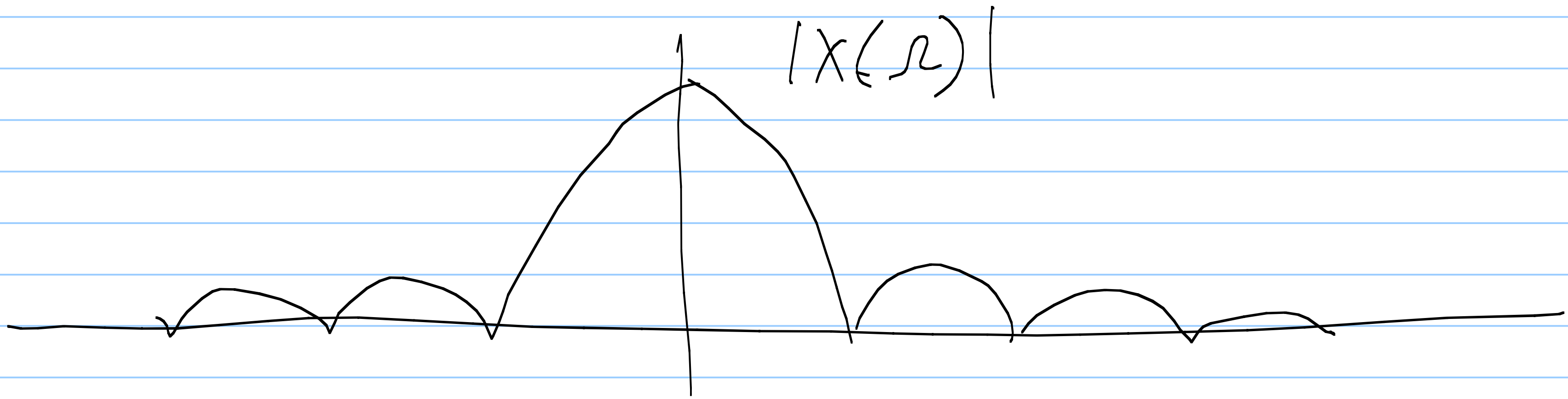
$$\boxed{\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, d\Omega = x(t)}$$

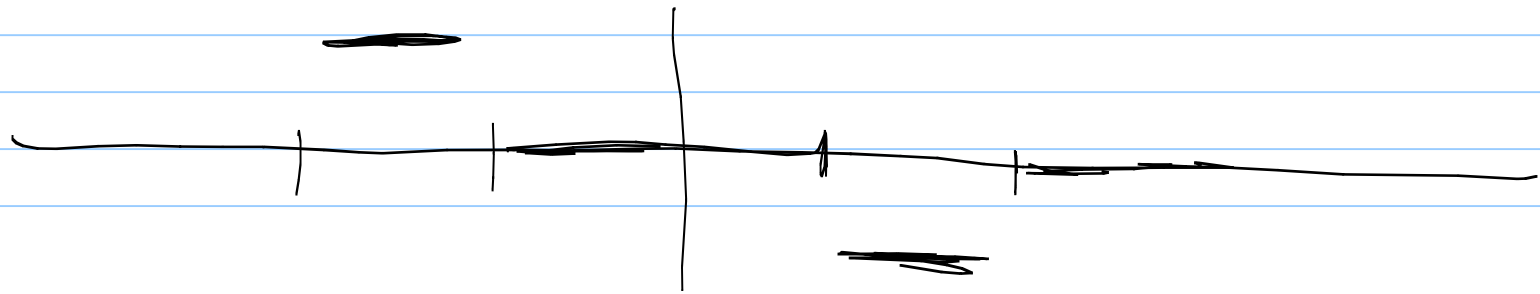
If $x(t) = \begin{cases} 1 & |t| < T_1/2 \\ 0 & \text{otherwise} \end{cases}$

$$X(\Omega) = \int_{-T_1/2}^{T_1/2} 1 \cdot e^{-j\Omega t}\, dt = \frac{\sin \Omega T_1/2}{\Omega/2}$$




T_1
$X(\Omega)$
$-\frac{2\pi}{T_1}$
$\frac{2\pi}{T_1}$
$\frac{4\pi}{T_1}$
Ω


$|X(\Omega)|$



$x(t) = x^*(t)$ real-valued

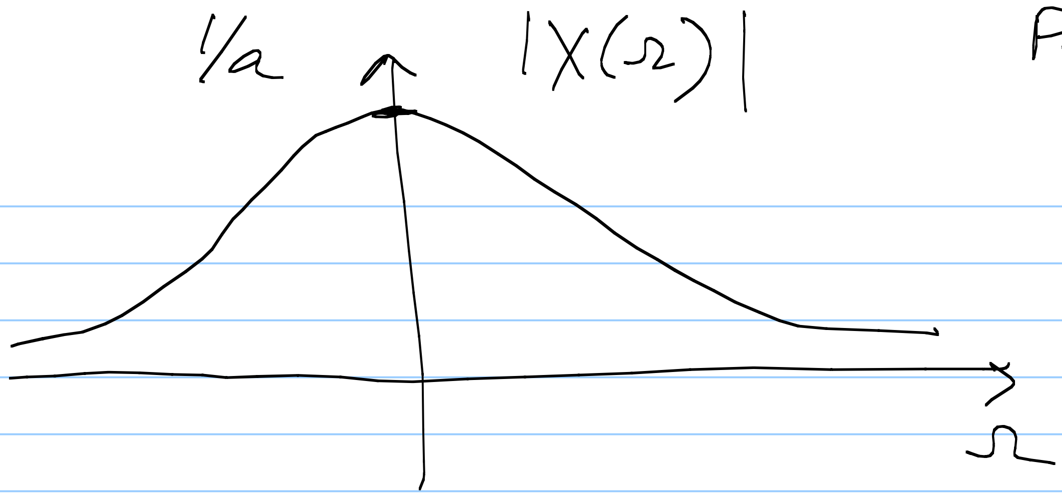
$$X(\Omega) = X^*(-\Omega)$$

$\Rightarrow$ $|X(\Omega)|$: even function of Ω mag. spectrum

$\measuredangle X(\Omega)$: odd " " phase spectrum

$$\mathcal{F}\left\{e^{-at}u(t)\right\} = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-j\Omega t}\,dt$$

$$= \frac{1}{a+j\Omega} \qquad \mathrm{Re}\{a\} > 0$$



For real a,

$$|X(\Omega)| = \frac{1}{\sqrt{a^2 + \Omega^2}}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{a + j\Omega} e^{j\Omega t} \, d\Omega$$

$$X(0) = \int_{-\infty}^{\infty} x(t) \, dt$$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, d\Omega$$

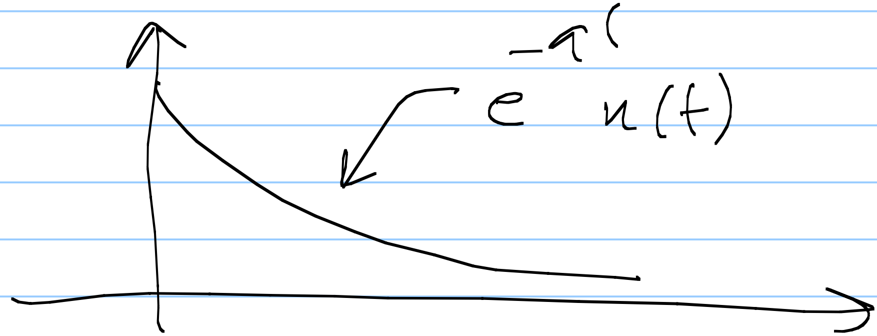
$$x(t)\Big|_{t=0} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{a}{a^2+\Omega^2} - j\,\frac{\Omega}{a^2+\Omega^2}\right] d\Omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{a^2}{a^2+\Omega^2}\, d\Omega - \underbrace{\frac{j}{2\pi}\int_{-\infty}^{\infty}\frac{\Omega}{a^2+\Omega^2}\, d\Omega}_{=0}$$

$$\underset{T \to \infty}{Lt} \int_{-T}^{T} \frac{\Omega}{a^2+\Omega^2}\, d\Omega = 0$$

Cauchy Principal Value

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{a}{a^2+\Omega^2}\, d\Omega = \frac{1}{2\pi} \tan\left(\Omega/a\right)\Big|_{-\infty}^{\infty}$$

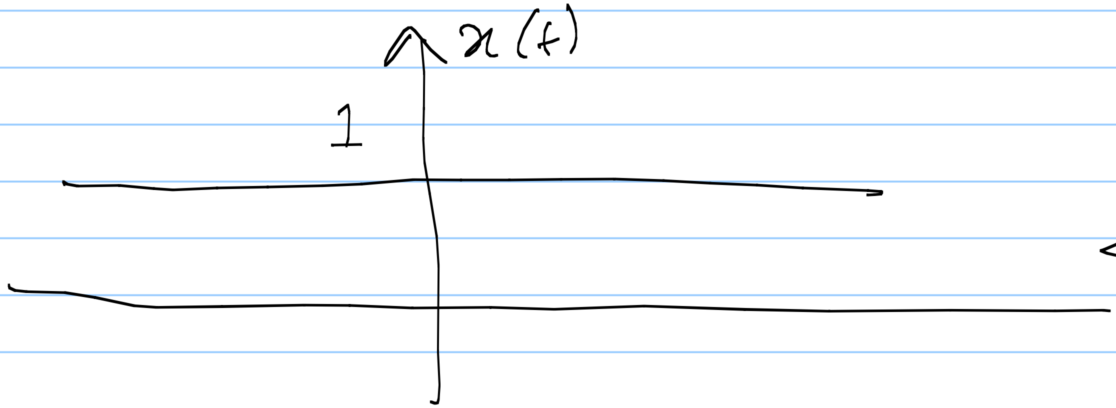


$e^{-at} u(t)$

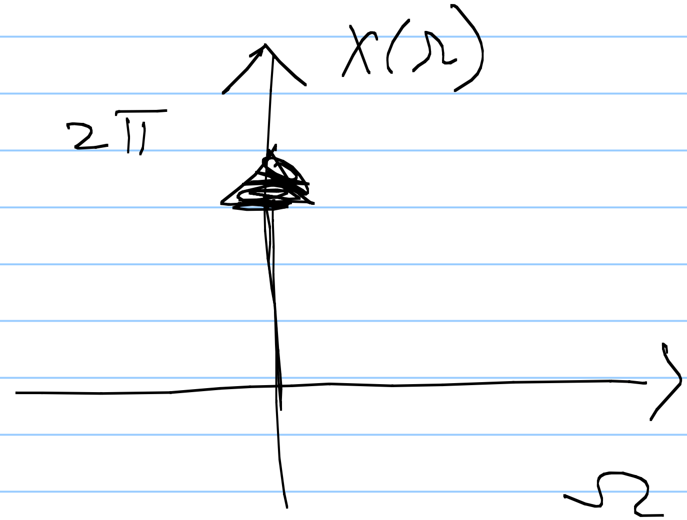
$$= \frac{1}{2\pi}\, \pi = \frac{1}{2}$$

$$X(\Omega) = 2\pi\,\delta(\Omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\,\delta(\Omega)\,e^{j\Omega t}\,d\Omega = 1 \quad \forall t$$



$\longleftrightarrow$



$$X(\Omega) = \int_{-\infty}^{\infty} 1 \cdot e^{-j\Omega t}\, dt = 2\pi\,\delta(\Omega)$$

$$1 \longleftrightarrow 2\pi\,\delta(\Omega)$$

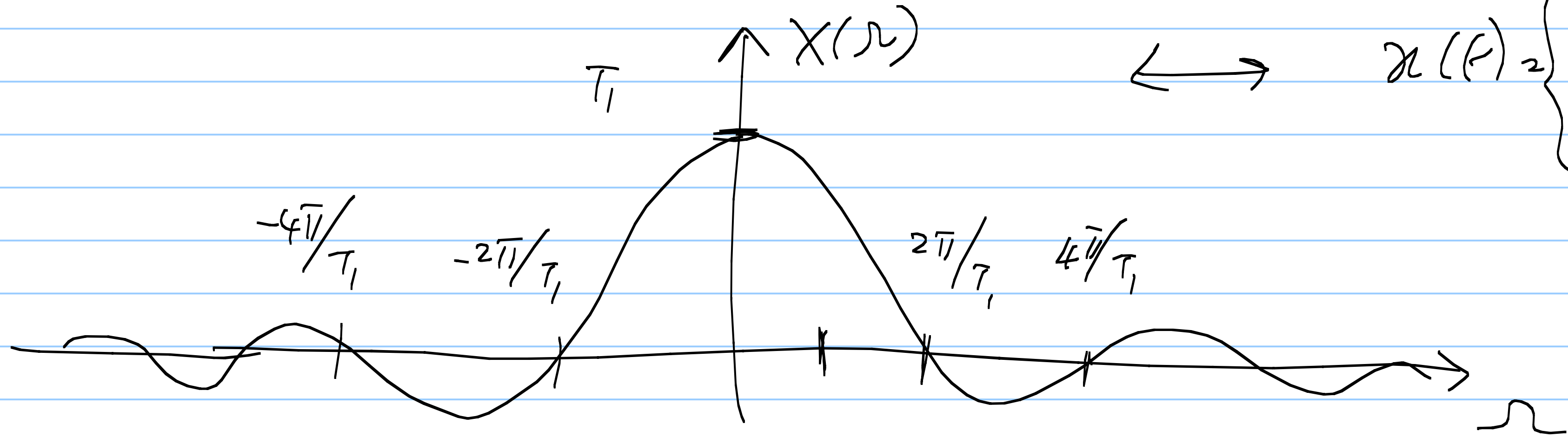
$$x(t - t_0) \longleftrightarrow e^{-j\Omega t_0} X(\Omega)$$

$$e^{j\Omega_0 t} x(t) \longleftrightarrow X(\Omega - \Omega_0)$$

$$\downarrow$$

$$e^{j\Omega_0 t} \cdot 1 \longleftrightarrow 2\pi\,\delta(\Omega - \Omega_0)$$

$$a_1 x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 X_1(\Omega) + a_2 X_2(\Omega)$$



$$\longleftrightarrow \quad x(t) = \begin{cases} 1 & |t| < T_1/2 \\ 0 & \text{otherwise} \end{cases}$$

What is the DC value of $x(t)$?

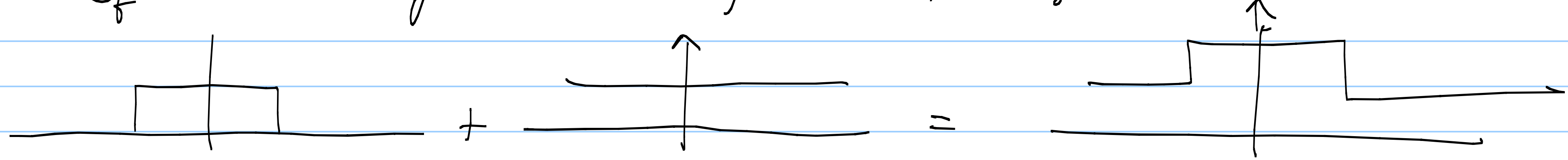
$$\overline{x(t)} = \mathop{Lt}_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt$$

$$= \mathop{Lt}_{T \to \infty} \frac{1}{T} \int_{-T_1/2}^{T_1/2} 1 \, dt$$

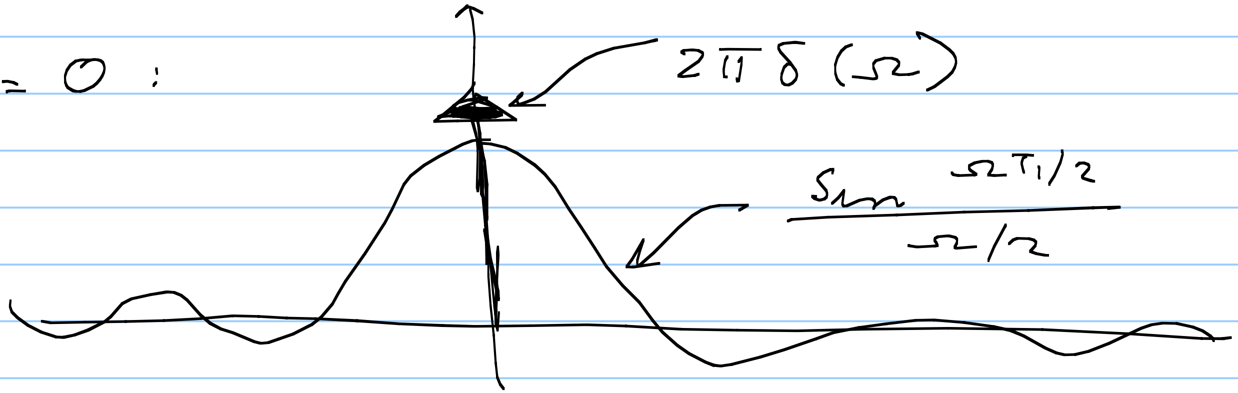
$$= \mathop{Lt}_{T \to \infty} \frac{T_1}{T} = 0$$

DC value is actually zero. Hence if $X(0)$ is non zero, it doesn't mean a pure tone of that frequency is present.

If DC component is present, signal will be



Spectrum will now contain an extra impulse at $\Omega = 0$:



$$X(\Omega) = 2\pi\delta(\Omega) + \frac{\sin \Omega T_1/2}{\Omega/2}$$