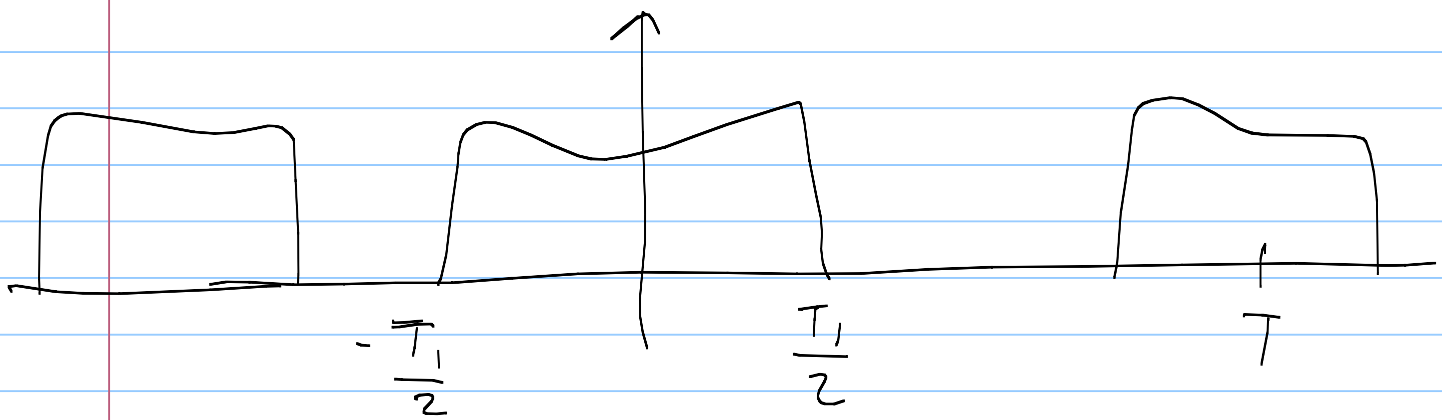
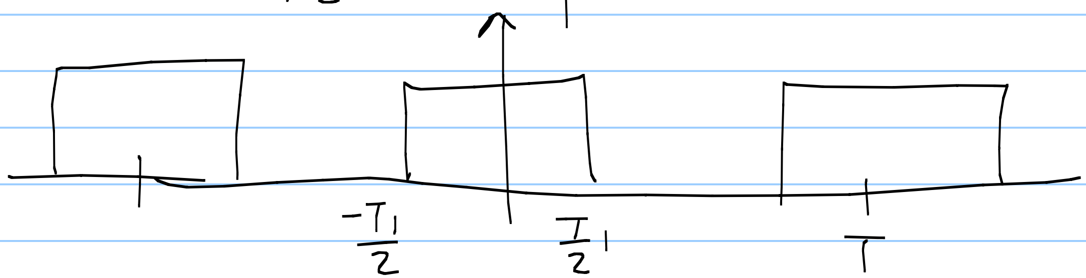
EC 5142 Sep. 19 2011

Note Title 19-09-2011



$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$



$$x(t) = \begin{cases} 1 & -T_1/2 \le t \le T_1/2 \\ 0 & T_1/2 < |t| < T/2 \end{cases}$$

$$a_k = \frac{1}{T} \frac{\sin \pi k T_1/T}{\pi k/T}$$

$$\frac{1}{T} \frac{\sin \Omega T_1/2}{\Omega/2} \Bigg|_{\Omega = k\frac{2\pi}{T}}$$

$$a_k T = \frac{\sin \pi k T_1/T}{\frac{\pi k}{T} T_1} T_1$$

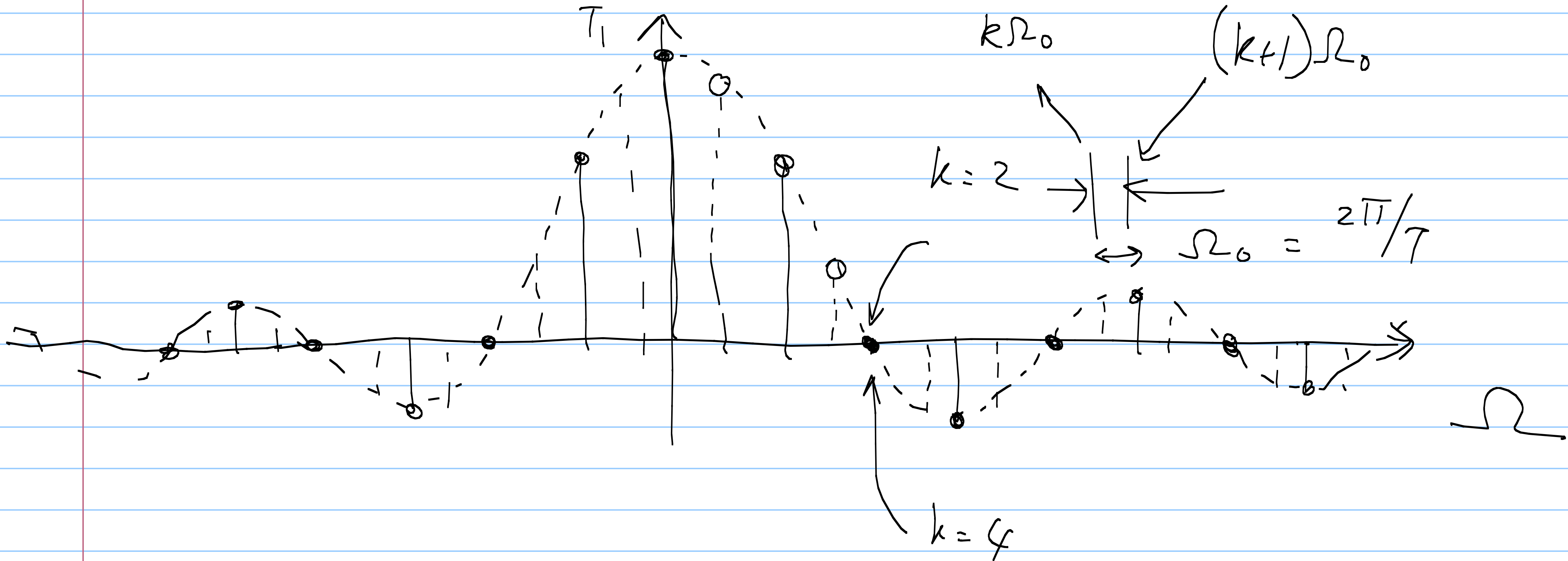
As $T \to \infty$, $a_k \to 0$

Let us consider $\frac{T_1}{T} = \frac{1}{2}$

$\frac{T_1}{T} = \frac{1}{4}$

Case 1) $$a_k T = \frac{\sin \pi k/2}{k\pi/2} \cdot T_1$$

Case 2) $a_k T = \dfrac{\sin \pi k/4}{k\pi/4} \cdot T_1$



$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\underbrace{k\frac{2\pi}{T}}\, t}\, dt$$

As $T \to \infty$, $a_k \to 0$, but $a_k T$ tends to a limit

$$a_k T \longrightarrow \int_{-\infty}^{\infty} \underbrace{x}(t)\, e^{-j\Omega t}\, dt = X(\Omega)$$

$$X(\Omega) \overset{\text{def}}{=} \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt \qquad \text{CTFT}$$

$$e^{j\Omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow e^{j\Omega_0 t} H(\Omega_0)$$

$$H(\Omega) = \int_{-\infty}^{\infty} h(t) e^{-j\Omega t} \, dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t} \qquad \Omega_0 = \frac{2\pi}{T}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} (a_k T) e^{jk\Omega_0 t}$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} (a_k T) e^{jk\Omega_0 t} \Omega_0$$

$$\longrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega \quad \text{as } \Omega_0 \to 0$$