# Ec 5142 15th Sep. 2011

Note Title 15-09-2011

$$\underset{N\times 1}{\underline{X}} = \underset{N\times N}{\underline{W}}\ \underset{N\times 1}{\underline{x}}$$

$$\underline{W} \equiv \underline{W}_N$$

$$\underline{x} = \underline{W}^{-1}\,\underline{X}$$

$$\underline{W}^{-1} = \frac{1}{N}\underline{W}^{H}$$

$$\underline{W} = \left[ e^{-j k \frac{2\pi}{N} n} \right]$$

($k$ down the rows, $n$ across the columns)

$$\underline{x}^H \underline{x} = \sum_{n=0}^{N-1} |x[n]|^2$$

$$\underline{X}^H \underline{X} = (\underline{W}\,\underline{x})^H (\underline{W}\,\underline{x})$$

$$= \underline{x}^H \underline{W}^H \underline{W}\,\underline{x}$$

$$= \underline{x}^H N \underline{W}^{-1} \underline{W}\,\underline{x}$$

$$= N\,\underline{x}^H \underline{x}$$

$$\underline{x}^H \underline{x} = \frac{1}{N} \underline{X}^H \underline{X}$$

In DTFS, for real-valued $x[n]$

$$a_{-k} = a_k^*$$

$$a_{-k} = a_{N-k}$$

For DFT, $X[-k] = X^*[k]$

$$X[-k] = X[N-k]$$

Notation to show periodicity explicitly:

$$x[n] \equiv x[\langle n \rangle_N]$$

$$\langle n \rangle_N \equiv n \bmod N$$

$$x_k \equiv x[k]$$

$$x[0] = x[N] = x[-N] = x[2N] = x[-2N] = \cdots$$

$$x[-1] = x[N-1]$$

$$X[0] = X[N]$$

$$X[-1] = X[N-1]$$

$$x[-n] = x[N-n]$$

$$x[n] \circledast h[n] = \sum_{k=0}^{N-1} x[k]\, h[n-k] = y[n]$$

$$y_0 = x_0 h_0 + x_1 h_{-1} + x_2 h_{-2} + \cdots + x_{N-1} h_{-(N-1)}$$

$$h_{-1} \equiv h_{N-1} \qquad h_{-2} \equiv h_{N-2} \qquad h_{-(N-1)} \equiv h_1$$

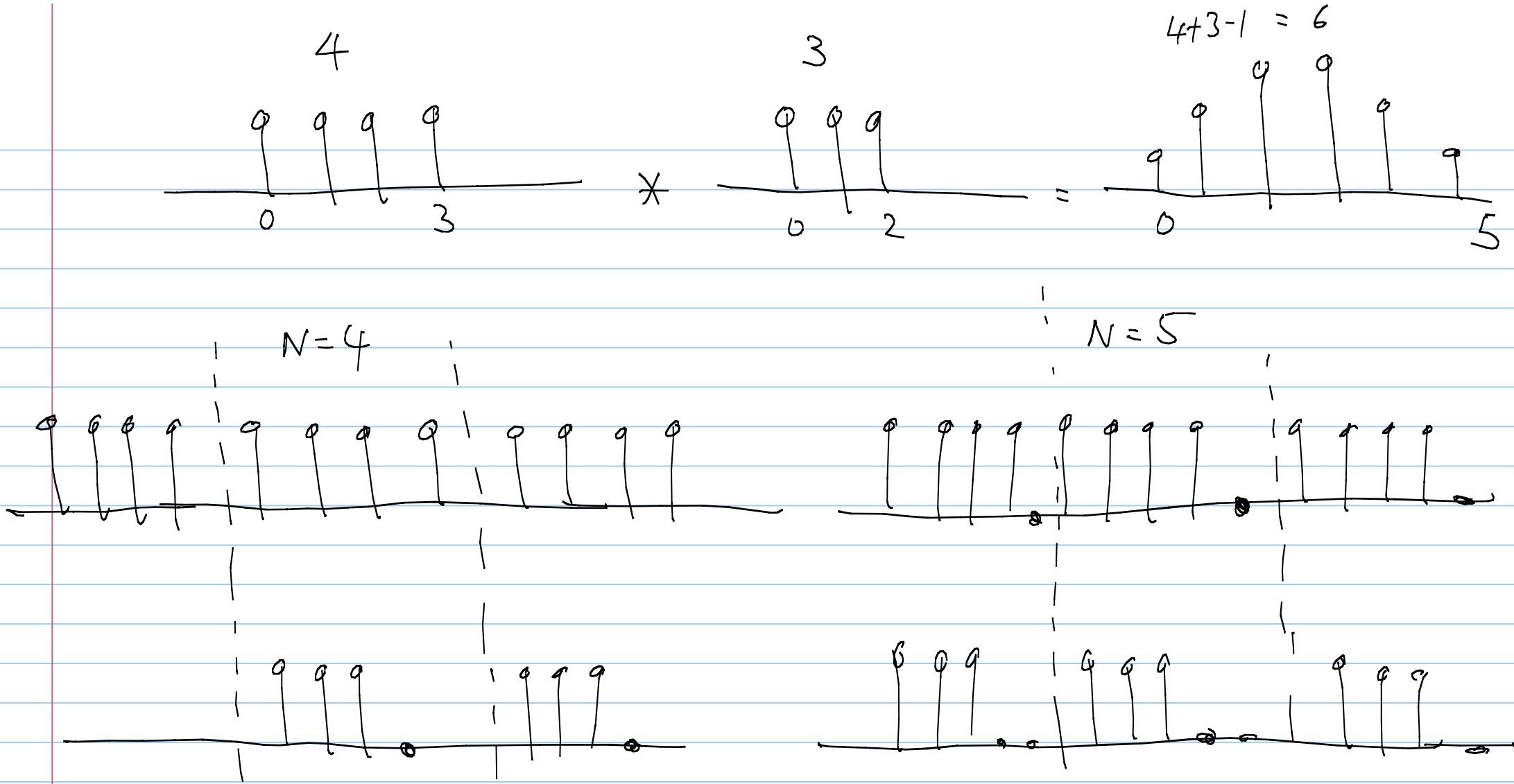
$$\begin{bmatrix} h_0 & h_{N-1} & h_{N-2} & \cdots & h_1 \\ h_1 & h_0 & h_{N-1} & \cdots & h_2 \\ & & & & \\ & & & & \\ h_{N-1} & h_{N-2} & & & h_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{N-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \\ \\ y_{N-1} \end{bmatrix}$$

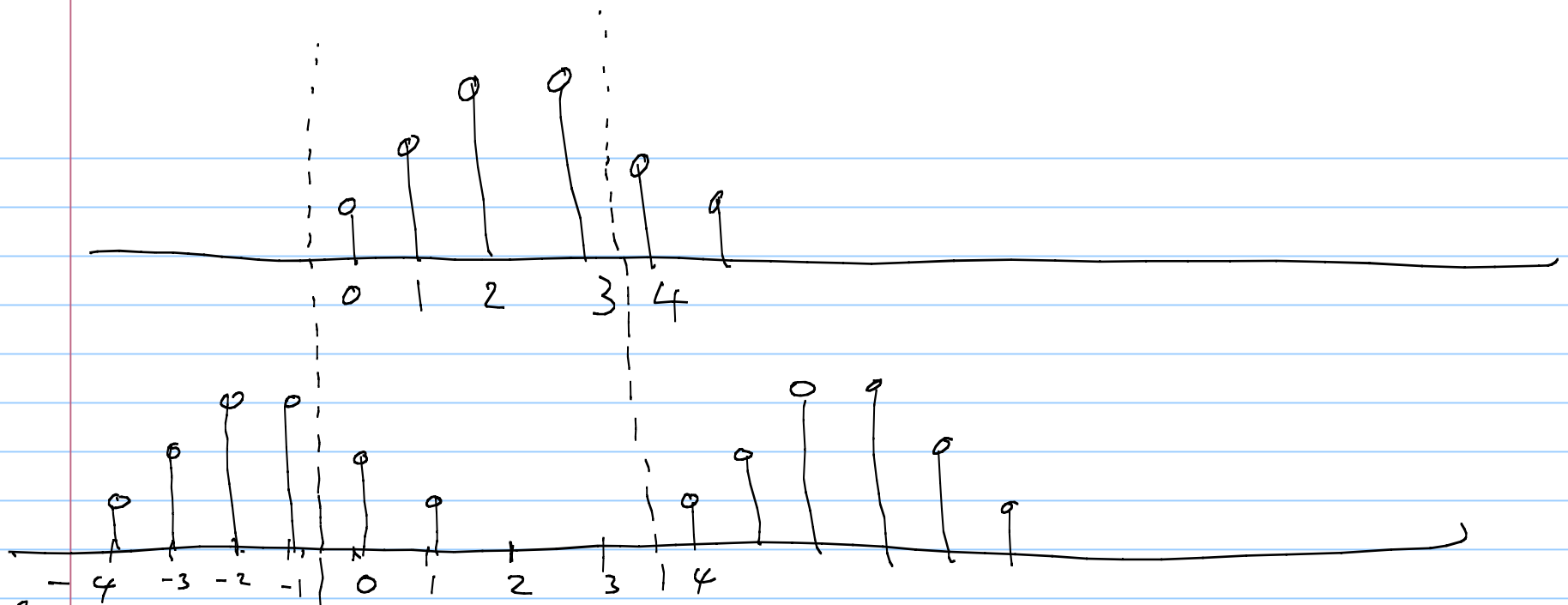
$2^{nd}$ row is a circular shift of first row

To get $x[n] \circledast h[n]$, the indirect approach:

take the DFTs of $x[n]$ & $h[n]$, multiply in the transform domain, & then take inverse transform

$$x[n] \circledast h[n] \longleftrightarrow X[k]\,H[k]$$

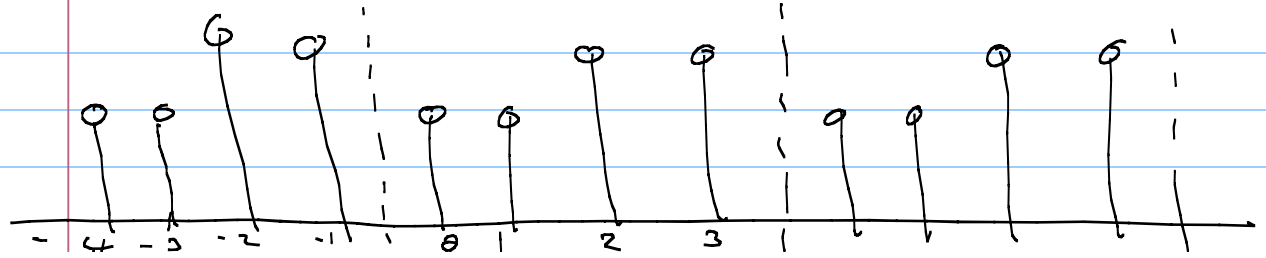

4
0
3
3
0
2
4+3-1 = 6
0
5
N=4
N=5




first repetition to the left.

Tail interferes with the first 2 samples

first repetition to the right does not interfere with values in [0, 3]




result of circular convolution for N = 4

If the repetitions should not overlap then we must choose $N \geq P + Q - 1$. In such a situation, over $[0, N-1]$, the results of linear & circular convolutions are identical. If $N = P + Q - 1$, the repetitions abut each other. If $N > P + Q - 1$, then zeros occur between the repetitions.