

EC 5142 14 Sep. 2011

Note Title

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DFS & DTFS (Cont'd)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-jk \frac{2\pi}{N} n} \quad k = 0, 1, \dots, N-1$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi}{N} kn}$$

We can also define with $\frac{1}{\sqrt{N}}$ scale factor for

both DFT & IDFT

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j k \frac{2\pi}{N} n}$$

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{+j k \frac{2\pi}{N} n}$$

DTFS

$$x[n+N] = x[n]$$

$$x[n] = \sum_{k=0}^{N-1} a_k e^{j k \frac{2\pi}{N} n}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} nk}$$

$$X[k] = N a_k$$

$$y[n] = x[n]$$

Find FS coeffs of $y[n]$ & relate them to $x[n]$

$\Rightarrow$ Duality property

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j k \frac{2\pi}{N} n} \quad k = 0, 1, \dots, N-1$$

$$\begin{bmatrix}
 1 & 1 & 1 & \dots & 1 \\
 1 & e^{-j \frac{2\pi}{N}} & e^{-j 1 \cdot \frac{2\pi}{N} 2} & \dots & e^{-j 1 \cdot \frac{2\pi}{N} (N-1)} \\
 & \uparrow & & & \\
 & e^{-j k \frac{2\pi}{N} n} & & & \\
 \leftarrow & & \xrightarrow{n: 0 \rightarrow N-1} & & \\
 & \downarrow k = 0 \rightarrow (N-1) & & & \\
 1 & e^{-j (N-1) \frac{2\pi}{N} 1} & \dots & e^{-j (N-1) \frac{2\pi}{N} (N-1)} &
 \end{bmatrix}
 \begin{bmatrix}
 x[0] \\
 x[1] \\
 x[2] \\
 \vdots \\
 x[N-1]
 \end{bmatrix}
 =
 \begin{bmatrix}
 X[0] \\
 X[1] \\
 \\
 \\
 X[N-1]
 \end{bmatrix}$$

$N \times N$

$$\underline{W}_N \underline{x} = \underline{X}$$

$$N \times N \quad N \times 1 \quad N \times 1$$

$$e^{-j k \frac{2\pi}{N} n} = W_N^{nk}$$

$$\underline{x} = \underline{W}_N^{-1} \underline{X} = \frac{1}{N} \underline{W}_N^H \underline{X}$$